Subject Name: Digital Communication systems

Stream: ECE

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Total contact hour- 40

Credits: 3

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Prerequisite: Analog Communication, Probability & Statistics

Module-I

Probability Theory and Random Processes:

1.1 Introduction:

Numerousevents are happening in our daily life. Many events can't be predicted with total certainty. The best we can say is how likely they are to happen, using the idea of probability.

For example, Tossing a Coin

When a coin is tossed, there are two possible outcomes:

heads (H) or tails (T)

We say that the probability of the coin landing H is ½

And the probability of the coin landing T is ½ Fig1.1 Tossing a Coin

Another example Throwing Dice

When a single die is thrown, there are six possible outcomes: 1, 2, 3, 4, 5, 6.The probability of any one of them is

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The probability of any one of them is _

1.2 Probability:

In general:Probability of an event happening = ______________________

Example: the chances of rolling a "4" with a die

Number of ways it can happen: 1 (there is only 1 face with a "4" on it)

Total number of outcomes: 6 (there are 6 faces altogether)

So the probability = _

Example: there are 5 marbles in a bag: 4 are blue, and 1 is red. What is the probability that a blue marble gets picked?

Number of ways it can happen: 4 (there are 4 blues)

Total number of outcomes: 5 (there are 5 marbles in total)

So the probability = = 0.8

Probability Line

We can show probability on a Probability Line

Probability is always between 0 and 1

Probability is Just a Guide since it does not tell us exactly what will happen, it is just a guide.

Example: toss a coin 100 times, how many Heads will come up?

Probability says that heads have a ½ chance, so we can expect 50 Heads.

But when we actually try it we might get 48 heads, or 55 heads ... or anything really, but in most cases, it will be a number near 50.

Some words have special meaning in Probability:

Experiment or Trial: an action where the result is uncertain.

Tossing a coin, throwing dice, seeing what pizza people choose are all examples of experiments.

Sample Space: all the possible outcomes of an experiment. Probability of sample space is 1.

Example: choosing a card from a deck

There are 52 cards in a deck (not including Jokers)

So the Sample Space is all 52 possible cards: {Ace (Tekka) of Hearts, 2 of Hearts, etc...} The Sample Space is made up of Sample Points:

Sample Point: just one of the possible outcomes Example:

Deck of Cards

the 5 of Clubs is a sample point, the King of Hearts is a sample point

"King" is not a sample point. As there are 4 Kings that is 4 different sample points.

Event: a single result of an experiment Example

Events:

Getting a Tail when tossing a coin is an event Rolling a "5" is an event.

An event can include one or more possible outcomes:

Choosing a "King" from a deck of cards (any of the 4 Kings) is an event.

Rolling an "even number" (2, 4 or 6) is also an event.

The Sample Space is all possible outcomes.

A Sample Point is just one possible outcome.

And an Event can be one or more of the possible outcomes.

Example: Alex wants to see how many times a "double" comes up when throwing 2 dice.

Each time Alex throws the 2 dice is an Experiment.

It is an Experiment because the result is uncertain.

The Event Alex is looking for is a "double", where both dice have the same number. It is made up of these 6 Sample Points:

{1,1} {2,2} {3,3} {4,4} {5,5} and {6,6}

The Sample Space is all possible outcomes (36 Sample Points):

{1,1} {1,2} {1,3} {1,4} ... {6,3} {6,4} {6,5} {6,6}

These are Alex's Results:

Experiment	Is it a Double?
{3,4}	No
{5,1}	No
{2,2}	Yes
{6,3}	No
...	...

Practically after 100 Experiments, Alex has 19 "double" Events ... is that close to what you would expect?

1.3 Conditional probability:

Given two events A and B, from the sigma-field of a probability space, with P(B) > 0, the conditional probability of A given B is defined as the quotient of the probability of the joint of events A and B, and the probability of B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This may be visualized as restricting the sample space to B. The logic behind this equation is that if the outcomes are restricted to B, this set serves as the new sample space. Note that this is a definition but not a theoretical result. We just denote the quantity $\frac{P(A \cap B)}{P(B)}$ as $P(A|B)$ and call it the conditional probability of A given B.

1.4 Communicationexample:

At present communication systems play a central role in our lives. Every day, we use our cell phones, access the internet, use our TV remote controls, and so on. Each of these systems relies on transferring information from one place to another. For example, when you talk on the phone, what you say is converted to a sequence of 00's or 11's called information bits. These information bits are then transmitted by your cell phone antenna to a nearby cell tower as shown in Figure 1.2.

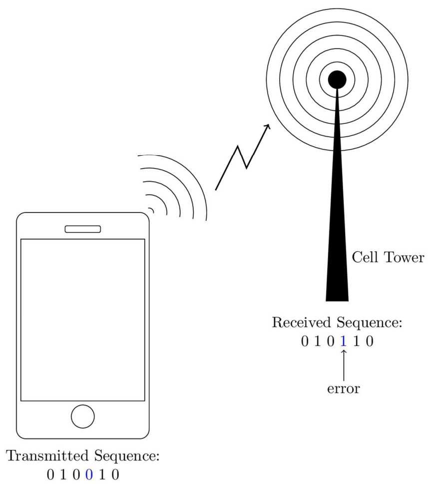


Fig.1.2 - Transmission of data from a cell phone to a cell tower.

The problem that communication engineers must consider is that the transmission is always affected by noise. That is, some of the bits received at the cell tower are incorrect. For example, your cell phone may transmit the sequence "010010□,""010010□," while thesequence"010110□""010110□"might be received at the cell tower. In this case, the fourth bit is incorrect. Errors like this could affect the quality of the audio in your phone conversation.The noise in the transmission is a random phenomenon. Before sending the transmission, we do not know which bits will be affected. It is as if someone tosses a (biased) coin for each bit and decides whether or not that bit will be received in error.

Probability theory is used extensively in the design of modern communication systems in order to understand the behavior of noise in these systems and take measures to correct the errors.

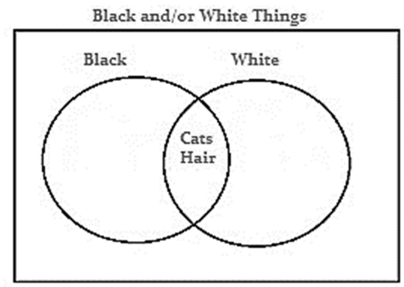
This example shows just one application of probability. You can pick almost any discipline and find many applications in which probability is used as a major tool. Randomness is prevalent everywhere, and probability theory has proven to be a powerful way to understand and manage its effects.

1.5 Jointprobability:

A joint probability is a statistical measure where the likelihood of two events occurring together and at the same point in time are calculated. Joint probability is the probability of event Y occurring at the same time event X occurs.

Notation for joint probability takes the form:

P(X∩Y) or P(X and Y) or P(XY), which reads as the joint probability of X and Y.



Joint probability is also called the intersection of two (or more) events. The intersection can be represented by a Venn diagram. A Venn diagram intersection shows the intersection of events A and B happening together.

Fig 1.3: Venn diagram for joint probability

1.6 Statisticalindependence:

Twoevents are independent, statistically independent, if the occurrence of one does not affect the probability of occurrence of the other.

Two events A and B are independent if their joint probability equals the product of their probabilities:

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B).$$

Why this defines independence is made clear by rewriting with conditional probabilities:

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B) \Leftrightarrow \mathrm{P}(A) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \mathrm{P}(A \mid B).$$

and similarly

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B) \Leftrightarrow \mathrm{P}(B) = \mathrm{P}(B \mid A).$$

Thus, the occurrence of B does not affect the probability of A, and vice versa. Although the derived expressions may seem more intuitive, they are not the preferred definition, as the conditional probabilities may be undefined if P(A) or P(B) are 0. Furthermore, the preferred definition makes clear by symmetry that when A is independent of B, B is also independent of A.

1.7 Randomvariable-continuous and discrete:

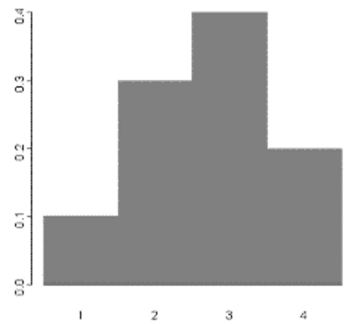
The outcome of an experiment need not be a number, for example, the outcome when a coin is tossed can be 'heads' or 'tails'. However, we often want to represent outcomes as numbers. A random variable is a function that associates a unique numerical value with every outcome of an experiment. The value of the random variable will vary from trial to trial as the experiment is repeated.

Example

Suppose a variable X can take the values 1, 2, 3, or 4.

The probabilities associated with each outcome are described by the following table:

Outcome	1	2	3	4
Probability	0.1	0.3	0.4	0.2



The probability that X is equal to 2 or 3 is the sum of the two probabilities: P(X = 2 or X = 3) = P(X = 2) + P(X = 3) = 0.3 + 0.4 = 0.7. Similarly, the probability that X is greater than 1 is equal to 1 – P(X = 1) = 1 - 0.1 = 0.9, by the complement rule.

This distribution may also be described by the probability histogram shown to the right:

There are two types of random variable - discrete and continuous.

A random variable has either an associated probability distribution (discrete random variable) or probability density function (continuous random variable).

1.8 Discrete Random Variable:

A discrete random variable is one which may take on only a countable number of distinct values such as 0, 1, 2, 3, 4, ... Discrete random variables are usually (but not necessarily) counts. If a random variable can take only a finite number of distinct values, then it must be discrete. Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.

Example

A coin is tossed ten times. The random variable X is the number of tails that are noted. X can only take the values 0, 1, ..., 10, so X is a discrete random variable.

1.9 Continuous Random Variable:

A continuous random variable is one which takes an infinite number of possible values. Continuous random variables are usually measurements. Examples include height, weight, the amount of sugar in an orange, the time required to run a mile.

Example

A light bulb is burned until it burns out. The random variable Y is its lifetime in hours. Y can take any positive real value, so Y is a continuous random variable.

1.10 CumulativeDistribution Function:

All random variables (discrete and continuous) have a cumulative distribution function. It is a function giving the probability that the random variable X is less than or equal to x, for every value x.

Formally, the cumulative distribution function F(x) is defined to be: $F(x) = P(X \leq x)$ for $-\infty < x < \infty$

For a discrete random variable, the cumulative distribution function is found by summing up the probabilities as in the example below.

For a continuous random variable, the cumulative distribution function is the integral of its probability density function.

Example

Discrete case: Suppose a random variable X has the following probability distribution p(xi):

xi	0	1	2	3	4	5
p(xi)	1/32	5/32	10/32	10/32	5/32	1/32

This is actually a binomial distribution: Bi (5, 0.5) or B (5, 0.5). The cumulative distribution function F(x) is then:

xi	0	1	2	3	4	5
F(xi)	1/32	6/32	16/32	26/32	31/32	32/32

F(x) does not change at intermediate values. For example:

F(1.3) = F(1) = 6/32

F(2.86) = F(2) = 16/32

1.11 Probability Density Function

The probability density function of a continuous random variable is a function which can be integrated to obtain the probability that the random variable takes a value in a given interval.

More formally, the probability density function, f(x), of a continuous random variable X is the derivative of the cumulative distribution function F(x):

$$f(x) = \frac{d}{dx} F(x)$$

Since $F(x) = P(X \leq x)$ it follows that:

$$\int f(x)\,dx = F(b) - F(a) = P(a < X < b)$$

If f(x) is a probability density function then it must obey two conditions: that the total probability for all possible values of the continuous random variable X is 1:

$$\int f(x)\,dx = 1$$

that the probability density function can never be negative: f(x) > 0 for all x

1.12 Continuous Random Variable– Uniform, Rayleigh, Gaussian and Rician distribution

i) Uniform:

A random variable X is said to be uniformly distributed in the interval $a \leq x \leq b$ if,

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & elsewhere \end{cases}$$

A plot of $f_X(x)$ is shown below

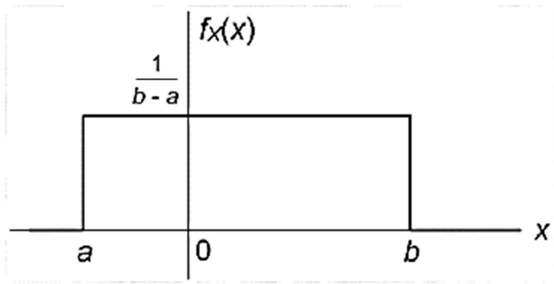


Fig. Uniform PDF

It is easy show that

$$E[X] = \frac{a+b}{2} \text{ and } \sigma_X^2 = \frac{(b-a)^2}{12}$$

Note that the variance of the uniform PDF depends only on the width of the interval $(b-a)$. Therefore, whether X is uniform in $(-1, 1)$ or $(2, 4)$, it has the same variance, namely $\frac{1}{3}$.

ii) Rayleigh:

An RV X is said to be Rayleigh distributed if,

$$f_X(x) = \begin{cases} \frac{x}{b} \exp\left(-\frac{x^2}{2b}\right), & x \geq 0 \\ 0 & , \textit{elsewhere} \end{cases}$$

where b is a positive constant,

A typical sketch of the Rayleigh PDF is shown below

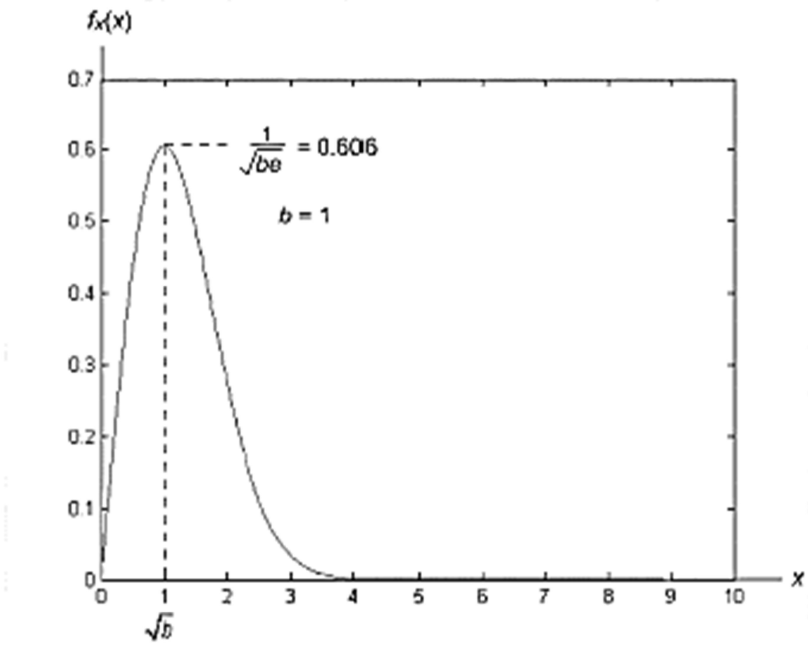


Fig.Rayleigh PDF

Rayleigh PDF frequently arises in *radar* and communication problems. We will encounter it later in the study of narrow-band noise processes.

iii) Gaussian

By far the most widely used PDF, in the context of communication theory is the Gaussian (also called *normal*) density, specified by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_X} \exp\left[-\frac{(x - m_X)^2}{2\sigma_X^2}\right], \quad -\infty < x < \infty$$

where m_X is the mean value and σ_X^2 the variance. That is, the Gaussian PDF is completely specified by the two parameters, m_X and σ_X^2. We use the symbol $N(m_X, \sigma_X^2)$ to denote the Gaussian density[1]. In appendix A2.3, we show that $f_X(x)$ as given by above Eq. is a valid PDF.

As can be seen from the Fig. 2.21, The Gaussian PDF is symmetrical with respect to m_X.

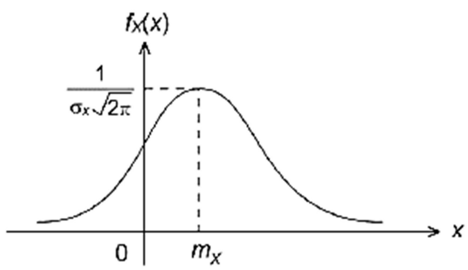


Fig.Gaussian PDF

[1] In this notation, $N(0, 1)$ denotes the Gaussian PDF with zero mean and unit variance. Note that if X is $N(m_x, \sigma_x^2)$, then $Y = \left(\frac{X - m_x}{\sigma_x}\right)$ is $N(0, 1)$.

Hence $F_X(m_X) = \int_{-\infty}^{m_X} f_X(x)\, dx = 0.5$

Consider $P[X \geq a]$. We have,

$$P[X \geq a] = \int_a^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_X} \exp\left[-\frac{(x - m_X)^2}{2\sigma_X^2}\right] dx$$

This integral cannot be evaluated in closed form. By making a change of variable

$z = \left(\frac{x - m_X}{\sigma_X}\right)$, we have

$$P[X \geq a] = \int_{\frac{a - m_X}{\sigma_X}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$= Q\left(\frac{a - m_X}{\sigma_X}\right)$$

where $Q(y) = \int_y^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$

Note that the integrand on the RHS of Eq. 2.57 is $N(0, 1)$.

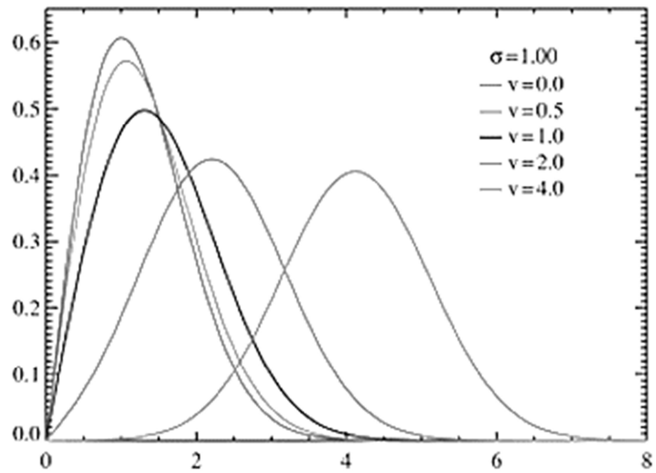
iv) Rician distribution

Rician distribution is the probability distribution of the magnitude of a circular bivariate normal random variable with potentially non-zero mean. It was named after Stephen O. Rice.

Two shape parameters define the Rician distribution: ν and σ. These, along with a third number, m, define the qualities of the probability density function. "m" controls the horizontal location of the distribution's maximum value. The distribution is unimodal with thin tails (the tails decrease exponentially for large x-values).

The probability density function formula is:

$$f(x \mid \nu, \sigma) = \frac{x}{\sigma^2} \exp\left(\frac{-(x^2 + \nu^2)}{2\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right),$$



PDF of the Rician distribution. Image: PAR-commonswikiWikimedia Commons. $I_0(z)$ is a modified Bessel function of the first kind with order zero.

The distribution is valid for real positive numbers. In other words, the interval for this distribution is {a, ∞) where a is any real positive number (real numbers are numbers that can be found on the number line).

1.13 Mean:

For a random variable X When we know the probability 'p' of every value 'x' we can calculate the Expected Value (Mean) of X:

$\mu = \Sigma xp$

Note: Σ is Sigma Notation, and means to sum up.

To calculate the Expected Value first multiply each value by its probability and then sum them up.

x	1	2	3	4	5	6
p	0.1	0.1	0.1	0.1	0.1	0.5
xp	0.1	0.2	0.3	0.4	0.5	3

$\mu = \Sigma xp = 0.1+0.2+0.3+0.4+0.5+3 = 4.5$

The expected value or mean is 4.5

Note: this is a weighted mean: values with higher probability have higher contribution to the mean.

Variance: Var(X) The

Variance is:

$Var(X) = \Sigma x^2p - \mu^2$ To calculate the Variance: square each value and multiply by its probability sum them up and we get Σx^2p then subtract the square of the Expected Value μ^2 Example continued:

x	1	2	3	4	5	6
p	0.1	0.1	0.1	0.1	0.1	0.5
x^2p	0.1	0.4	0.9	1.6	2.5	18

$\Sigma x^2p = 0.1+0.4+0.9+1.6+2.5+18 = 23.5$

$Var(X) = \Sigma x^2p - \mu^2 = 23.5 - 4.52 = 3.25$

The variance is 3.25

Standard Deviation: σ

The Standard Deviation is the square root of the Variance:

$\sigma = \sqrt{Var(X)}$ $\sigma = \sqrt{Var(X)} =$

$\sqrt{3.25} = 1.803...$

The Standard Deviation is 1.803...

Let's have another example!

(Note that we run the table downwards instead of along this time.)

You plan to open a new McDougals Fried Chicken, and found these stats for similar restaurants:

Percent	Year's Earnings
20%	\$50,000 Loss
30%	\$0
40%	\$50,000 Profit
10%	\$150,000 Profit

Using that as probabilities for your new restaurant's profit, what is the Expected Value and Standard Deviation?

The Random Variable is X = 'possible profit'.

Sum up xp and x^2p:

Probability p	Earnings (\$'000s) x	xp	x^2p
0.2	-50	-10	500
0.3	0	0	0
0.4	50	20	1000
0.1	150	15	2250
$\Sigma p = 1$		$\Sigma xp = 25$	$\Sigma x^2p = 3750$

$\mu = \Sigma xp = 25$

$\text{Var}(X) = \Sigma x^2p - \mu^2$

$= 3750 - 252$

$= 3750 - 625 = 3125$ $\sigma = \sqrt{3125} = 56$ (to nearest whole number) But remember these are in thousands of dollars, so:

μ = \$25,000 σ

= \$56,000

So you might expect to make \$25,000, but with a very wide deviation possible.

Let's try that again, but with a much higher probability for \$50,000:

Example (continued):

Now with different probabilities (the \$50,000 value has a high probability of 0.7 now):

Probability Earnings (\$'000s)

p	x	xp	x^2p
0.1	-50	-5	250
0.1	0	0	0
0.7	50	35	1750
0.1	150	15	2250
Σp = 1	Sums:	Σxp = 45	Σx2p = 4250

$\mu = \Sigma xp = 45$

$Var(X) = \Sigma x^2p - \mu^2$

$= 4250 - 452$

$= 4250 - 2025$ $= 2225$ $\sigma = \sqrt{2225} = 47$ (to

nearest whole number) In thousands of

dollars:

μ = \$45,000 σ

= \$47,000

The mean is now much closer to the most probable value.

And the standard deviation is a little smaller (showing that the values are more central.)

1.14 Randomprocess:

In real-life applications, we are often interested in multiple observations of random values over a period of time. For example, suppose that you are observing the stock price of a company over the next few months. In particular, let S(t) be the stock price at time t∈ [0, ∞). Here, we assumet=0 refers to current time. Figure below shows a possible outcome of this random experiment from time t=0 to time t=1.

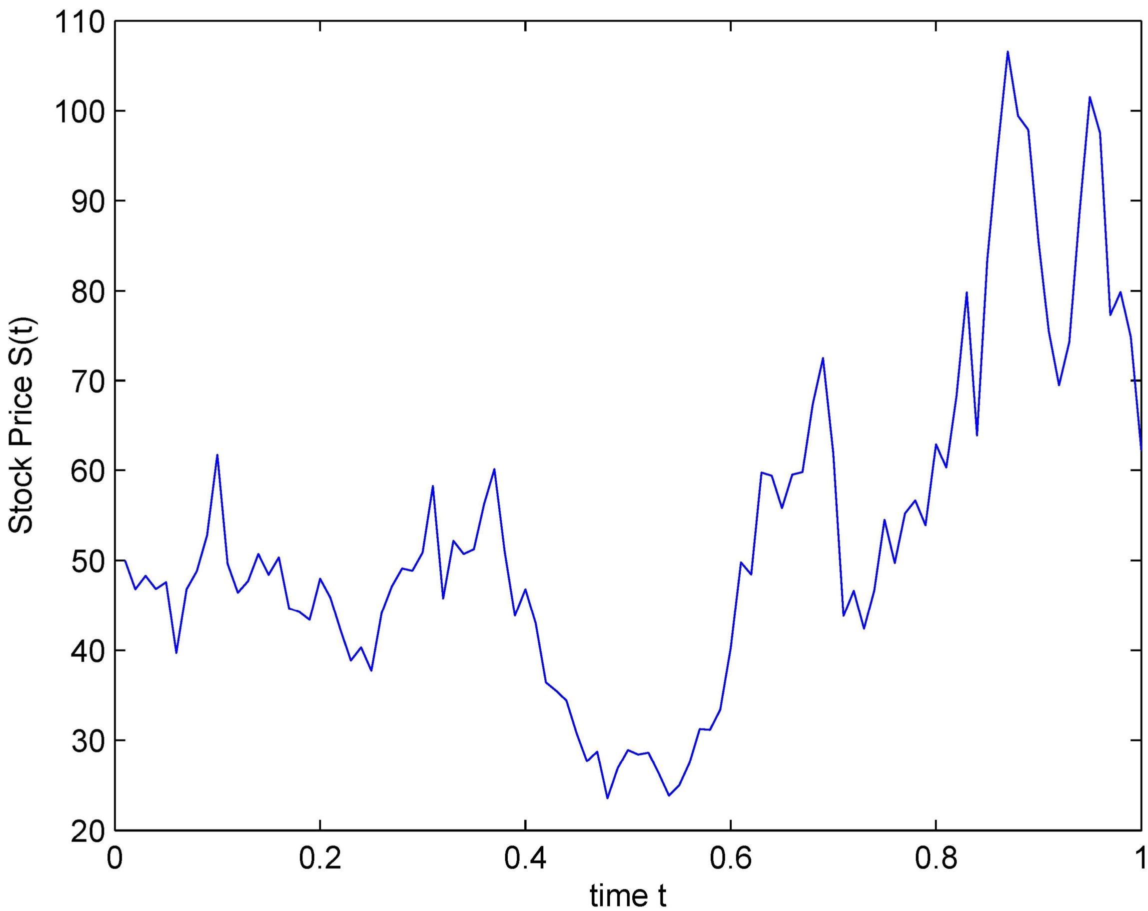


Figure 1: A possible realization of values of a stock observed as a function of time. Here, S(t) is an example of a random process

Note that at any fixed time t1∈ [0,∞), S(t1) is a random variable. Based on your knowledge of finance and the historical data, you might be able to provide a PDF for S(t1). If you choose another time t2∈ [0,∞), you obtain another random variableS(t2) that could potentially have a different PDF. When we consider

the values of S(t) for t∈ [0, ∞) collectively, we say S(t) is a random process or a stochastic process. We may show this process by{S(t), t∈ [0, ∞)}.

Therefore, a random process is a collection of random variables usually indexed by time (or sometimes by space)

A continuous-time random process is a random process {X(t),t□J}, where Jis an interval on the real line such as [−1,1], [0,∞),(−∞,∞), etc.

A discrete-time random process (or a random sequence) is a random process {X(n)=X_n,n□J}, where J is a countable set such as N or Z.

Example -1

You have 1000 dollars to put in an account with interest rate R, compounded annually. That is, if X_n is the value of the account at year n, thenX_n=1000$(1+R)^n$, for n=0,1,2,□.

The value of R is a random variable that is determined when you put the money in the bank, but it does not not change after that. In particular, assume that R□Uniform(0.04,0.05).

a) Find all possible sample functions for the random process {X_n, n=0,1, 2,...}.
b) Find the expected value of your account at year three. That is, find E[X_3].

Solution

a. Here, the randomness in X_n comes from the random variable R. As soon as you know R, you know the entire sequence X_n for $n = 0, 1, 2, \cdots$. In particular, if $R = r$, then

$$X_n = 1000(1+r)^n, \quad \text{for all } n \in \{0, 1, 2, \cdots\}.$$

Thus, here sample functions are of the form $f(n) = 1000(1+r)^n$, $n = 0, 1, 2, \cdots$, where $r \in [0.04, 0.05]$. For any $r \in [0.04, 0.05]$, you obtain a sample function for the random process X_n.

b. The random variable X_3 is given by

$$X_3 = 1000(1+R)^3.$$

If you let $Y = 1 + R$, then $Y \sim Uniform(1.04, 1.05)$, so

$$f_Y(y) = \begin{cases} 100 & 1.04 \leq y \leq 1.05 \\ 0 & \text{otherwise} \end{cases}$$

To obtain $E[X_3]$, we can write

$$\begin{aligned} E[X_3] &= 1000E[Y^3] \\ &= 1000 \int_{1.04}^{1.05} 100y^3 \quad dy \quad (\text{by LOTUS}) \\ &= \frac{10^5}{4}\left[y^4\right]_{1.04}^{1.05} \\ &= \frac{10^5}{4}\left[(1.05)^4 - (1.04)^4\right] \\ &\approx 1,141.2 \end{aligned}$$

Example

Let $\{X(t), t \in [0, \infty)\}$ be defined as

$$X(t) = A + Bt, \quad \text{for all } t \in [0, \infty),$$

where A and B are independent normal $N(1,1)$ random variables.

a. Find all possible sample functions for this random process.
b. Define the random variable $Y = X(1)$. Find the PDF of Y.
c. Let also $Z = X(2)$. Find $E[YZ]$.

Solution

a. Here, we note that the randomness in $X(t)$ comes from the two random variables A and B. The random variable A can take any real value $a \in \mathbb{R}$. The random variable B can also take any real value $b \in \mathbb{R}$. As soon as we know the values of A and B, the entire process $X(t)$ is known. In particular, if $A = a$ and $B = b$, then

$$X(t) = a + bt, \quad \text{for all } t \in [0, \infty).$$

Thus, here, sample functions are of the form $f(t) = a + bt$, $t \geq 0$, where $a, b \in \mathbb{R}$. For any $a, b \in \mathbb{R}$ you obtain a sample function for the random process $X(t)$.

b. We have

$$Y = X(1) = A + B.$$

Since A and B are independent $N(1,1)$ random variables, $Y = A + B$ is also normal with

$$\begin{aligned} EY &= E[A+B] \\ &= E[A] + E[B] \\ &= 1 + 1 \\ &= 2, \end{aligned}$$

$$\begin{aligned} \text{Var}(Y) &= \text{Var}(A+B) \\ &= \text{Var}(A) + \text{Var}(B) \quad \text{(since } A \text{ and } B \text{ are independent)} \\ &= 1 + 1 \\ &= 2. \end{aligned}$$

Thus, we conclude that $Y \sim N(2,2)$:

$$f_Y(y) = \frac{1}{\sqrt{4\pi}} e^{-\frac{(y-2)^2}{4}}.$$

c. We have

$$\begin{aligned} E[YZ] &= E[(A+B)(A+2B)] \\ &= E[A^2 + 3AB + 2B^2] \\ &= E[A^2] + 3E[AB] + 2E[B^2] \\ &= 2 + 3E[A]E[B] + 2 \cdot 2 \quad \text{(since } A \text{ and } B \text{ are independent)} \\ &= 9. \end{aligned}$$

1.15 Stationaryand ergodic processes:

A stationary process is a stochastic process whose joint probability distribution does not change when shifted in time or space. For a stationary process, mean and variance, if they exist, do not change over time or position. Stationarity is a key concept in time series analysis as it allows powerful techniques for modeling and forecasting to be developed. Time series is a set of data ordered in time usually recorded at regular time intervals of time. In probability theory a time series is a collection of random variable $\{X(t), t \geq 0\}$ indexed by time. One of the main features of time series is the interdependency of observation over time. This interdependency needs to be accounted in the time series data modeling to improve temporal behavior and forecast of future moment. So stationarity is used as a tool in time series analysis when raw data is often transformed to become stationary.

Definition 1. Mean function: It is defined as $m(t) = E(X(t))$, which may or may not be dependent on t.
Definition 2. Second Order Process: A stochastic process is called a second order process if it’s second order moment is finite for all t i.e., $E(X^2(t)) < \infty$.

Definition 3. Covariance function: It is denoted by C(s,t) given by $C(s, t) = cov(X(s), X(t)) = E(X(s)X(t)) - E(X(s))E(X(t))$

A Stochastic process has to be a second order process for covariance function to exist. The Covariance function satisfies the following properties:

1. $C(s, t) = C(t, s) \; \forall \; t, s \in T$
2. Using Schwarz inequality $C(s, t) \leq \sqrt{[C(s, s)C(t, t)]}$
3. It is non negative definite i.e, for a set of real numbers $a_1, a_2, a_3, \ldots, a_n$ and

$$Q = \sum_{i}\sum_{j} a_i a_j C(t_i, t_j) = E\left(\sum_{i} a_i X(t_i)\right)^2 \geq 0$$

4. Sum and product also covariance functions.

Definition 4. Auto-Correlation function: It is denoted by R(s, t), given by:

$$R(s, t) = \frac{E[X(s)X(t)] - E[X(s)]E[X(t)]}{\sqrt{Var\, X(s)}\sqrt{Var\, X(t)}}$$

Now assuming, R(s, t) depends only on $|t-s|$, $E(X(t)) = \mu$ and $Var(X(t)) = \sigma^2$ we get,

$$R(\tau) = \frac{(E[X(t)] - \mu)(E[X(t + \tau)] - \mu)}{\sigma^2}$$

Definition5. Independent Increments: If for every $t_1 < t_2 < \ldots < t_n$ $X(t_2) - X(t_1), X(t_3) - X(t_2), \ldots, X(t_n) - X(t_{n-1})$ are mutually independent random variables ∀ n.

Definition6. Ergodic Property: It is the time average of a function along a realization or sample exist almost everywhere and is related to the space average. That means whenever the stochastic process is ergodic the time average is same for all almost initial points i.e., the process evolved for a longer time forgets its initial state.

Definition7. Strict Sense Stationary: If for arbitrary □1 < □2 < . . . < □□ the joint distribution of the random vectors {X(t1), X(t2), . . . , X(tn)} and $\{X(t_{1+h}), X(t_{2+h}), \ldots, X(t_{n+h})\}$ is same ∀h then the stochastic process {X(t), t □ T} is said to be strict sense stationary of order n. If the above definition holds for every integer n then the process is called a strict sense stationary Process.

Definition8. Wide Sense Stationary Process A Stochastic process is said to be wide sense stationary or weakly stationary or covariance stationary if the following properties hold- • m(t) = E[X(t)] is independent of t. • $E[X^2(t)] < \infty$. • c(s, t) is a function of the time difference |t − s| only i.e., c(s, t) = f(t − s) ∀ t, s. Also neither a strict sense stationary process imply the wide sense stationary nor wide sense stationary imply the strict sense stationary process.

1.16 Correlationcoefficient

The correlation coefficient of two variables in a data set equals to their covariance divided by the product of their individual standard deviations. It is a normalized measurement of how the two are linearly related.

Covariance

Covariance is a measure of how much two random variables vary together. It's similar to variance, but where variance tells you how a single variable varies, co variance tells you how two variables vary together.

Cov(X,Y) = Σ E((X-μ)E(Y-ν)) / n-1 where:

X is a random variable

E(X) = μ is the expected value (the mean) of the random variable X and

E(Y) = ν is the expected value (the mean) of the random variable Y n =

the number of items in the data set

Calculate covariance for the following data set:

x: 2.1, 2.5, 3.6, 4.0 (mean = 3.1) y:

8, 10, 12, 14 (mean = 11)

Substitute the values into the formula and solve:

Cov(X,Y) = ΣE((X-μ)(Y-ν)) / n-1

= (2.1-3.1)(8-11)+(2.5-3.1)(10-11)+(3.6-3.1)(12-11)+(4.0-3.1)(14-11) /(4-1)

= (-1)(-3) + (-0.6)(-1)+(.5)(1)+(0.9)(3) / 3

= 3 + 0.6 + .5 + 2.7 / 3

= 6.8/3

= 2.267

The result is positive, meaning that the variables are positively related.

Note on dividing by n or n-1:

When dealing with samples, there are n-1 terms that have the freedom to vary (see: Degrees of Freedom).

If you are finding the covariance of just two random variables, just divide by n.

1.17 Autocorrelation function:

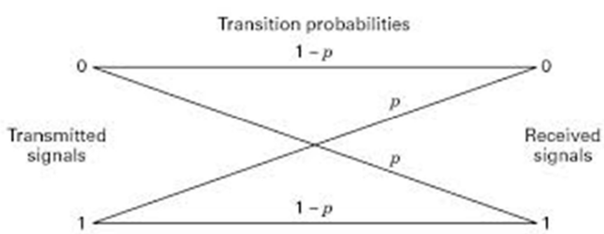
Let us consider two random variables $X_{ti} = X(t_i), i = 1, 2$

The correlation between X_{t1} & X_{t2} is measured by their joint moment:

$$E[X_{t1}X_{t2}] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_{t1}.x_{t2}\, p(x_{t1}, x_{t2})\, dx_{t1}dx_{t2} \qquad 2.8.2$$

This joint moment is also known as the autocorrelation function, $\Phi(t_1,t_2)$ of the stochastic process X(t). In general, $\Phi(t_1,t_2)$ is dependent on t_1 and t_2.

1.18 Binary Symmetric Channel



A binary symmetric channel (or BSC) is a common communications channel model used in coding theory and information theory.

In this model, a transmitter wishes to send a bit (a zero or a one), and the receiver receives a bit.

It is assumed that the bit is usually transmitted correctly, but that it will be "flipped" with a small probability (the "crossover probability").

This channel is used frequently in information theory because it is one of the simplest channels to analyze.

A binary symmetric channel with crossover probability p denoted by, is a channel with binary input and binary output and probability of error p; that is, if X is the transmitted random variable and Y the received variable, then the channel is characterized by the conditional probabilities

$\Pr(Y = 0 \mid X = 0) = 1 - p$ Pr(

$Y = 0 \mid X = 1) = p$

$\Pr(Y = 1 \mid X = 0) = p$

$\Pr(Y = 1 \mid X = 1) = 1 - p$

It is assumed that $0 \leq p \leq 1/2$. If $p > 1/2$, then the receiver can swap the output (interpret 1 when it sees 0, and vice versa) and obtain an equivalent channel with crossover probability $1 - p \leq 1/2$.

This channel is often used by theorists because it is one of the simplest noisy channels to analyze. Many problems in communication theory can be reduced to a BSC.

Conversely, being able to transmit effectively over the BSC can give rise to solutions for more complicated channels.

Module-II

Signal Vector Representation:

Introduction

Consider the most basic form of a digital communication system depicted in fig 2.1. A message source emits one symbol every T seconds, with the symbols belonging to an alphabet of M symbols denoted by □ , □ , … … , □ .Consider, for example, in any event, the a priori probabilities □ , □ , … … . , □ specify the nessage source output.In the absence of prior information, it is customary to assume that the M symbols of the alphabet are equally likely. Then may express the probability that symbol □ is emitted by the source as

□ = □(□)

= _ □□□ □ = 1,2, … , □ (2.1)

The transmitter takes the massage source output □ and codes it into a distinct signal □ (t) suitable for transmission over the channel. The signal □ (t) occupies the full duration T allotted to symbol □ . Most importance,□ (t) is a real-valued energy signal, as shown by

$$E_i = \int_0^T s_i^2(t)\, dt, \qquad i = 1, 2, \ldots, M \tag{2.2}$$

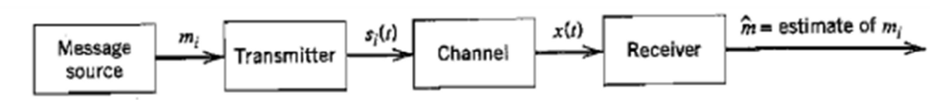


FIG 2.1 Block diagram of a generic digital communication system.

The channel is assumed to have two characteristics:

1.The channel is linear, with a bandwidth that is wide enough to accommodate the transmission of signal □ (t) with negligible or no distortion.

2. The channel noise, w(t),is the simple function of a zero-mean white Gaussian noise process. The reasons for this second assumption are that it makes receiver calculations tractable, and it is a reasonable description of the type of noise present in many practical communication systems.

We refer to such a channel as an additive white Gaussian noise (AWGN) channel. Accordingly, we may express the received signal x(t) as

$$x(t) = s_i(t) + w(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases} \tag{2.3}$$

And thus model the channel as in fig 2.2.

The receiver has the task of observing the receiver x(t) for a duration of T seconds and making a best estimate of the transmitted signal □ (□) or,equivalently,the symbol □ .The requirement is therefore to design the receiver so as to minimize the average probality of symbol error, define as

$$P_e = \sum_{i=1}^{M} p_i\, P(\hat{m} \neq m_i \,|m_i) \tag{2.4}$$

Where □ is the transmitted symbol, m^ is the estimate produced by the receiver, and (p(m^≠□ |□) is the conditional error probability given the □ symbol was sent. The resulting receiver is said to be optimum the minimum probability of error sense.

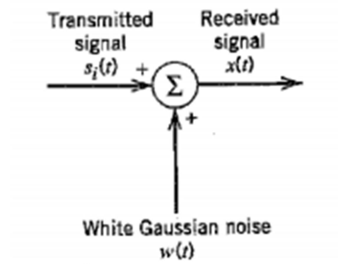


FIG 2.2 Additive white Gaussian noise (AWGN) model of a channel

Geometric Representation of signals

The essence of geometric representation of signal is to represent any set of M energy signals {□ (□)} as linear combinations of N orthonormal basis functions, where □ ≤ □. That is to say, given a set of realvalued energy signals □ (□), □ (□), … … . , □ (□),each of duration T seconds,we write

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases} \tag{2.5}$$

Where the coefficients of the expansion are defined by

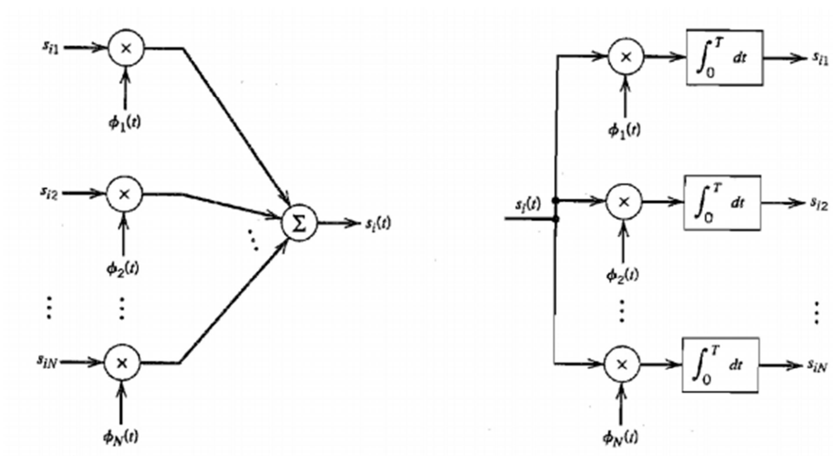
$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\,dt, \qquad \begin{cases} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{cases} \tag{2.6}$$

The rea –valued basis $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ functions are orthonormal, by which we mean

$$\int_0^T \phi_i(t)\phi_j(t)\,dt = \delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \ne j \end{cases} \tag{2.7}$$

Where $ is the kronecker delta. The first condition of equation (2.7) states that each basis function is normalized to have to have unit energy. The second condition states that the basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are orthogonal with respect to each other over the interval 0 ≤ t ≤ T.

- Given the N elements of the vectors □ operating as input,we may use the scheme shown in fig 5.3a togenerate the signa □ (□),which follows



(a) (b)

FIG 2.3 (a) Synthesizer for generating the signal □ (t). (b) Analyzer for generating the set of signal vectors {□ }.

- Conversely, given the signals □ (t),i=1,2,……,M,operating as input,we may use the scheme shown in fig 5.3(b).This second scheme consists of a bank of N product-integrators or correlators with a common input,and with each one of them supplied with its own basis functon. The scheme of fig 5.3(b) may be viewed as an analyzer.

  Accordingly, we may state that each signal in the set { □ (t)} is completely determined by the vector of its coefficients

$$\mathbf{s}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, \qquad i = 1, 2, \ldots, M \tag{2.8}$$

The vector □ is called a signal vector.furthermore,if we conceptually extend our conventional notion of two and three-dimensional Euclidean spaces to an N-dimensional Euclidean space,we may visualize the set of signal vectors { □ |i=1,2,…..,M} as defining a corresponding set of M points in an N-dimensional Euclidean space is called the signal space.

This for of representation is illustrated in fig 2.4 for the case of a two-dimensional signal space with three signals, that is, N=2 and =3.

In an N-dimensional Euclidean space, we may define length of a signal vectors and angles between vectors. It is customary to denote the length of a signal vector □ by the symbol ||□ ||.The squaredlength of any signal vector □ is defined to be the inner product or dot product or dot product of □ with itself, as shown by

$$\begin{aligned} \| \mathbf{s}_i \|^2 &= \mathbf{s}_i^T \mathbf{s}_i \\ &= \sum_{j=1}^{N} s_{ij}^2, \qquad i = 1, 2, \ldots, M \end{aligned} \tag{2.9}$$

Where □ is the jth element of □ , and the superscript T denotes matrix transposition.

There is an interesting relationship between the energy content of a signal and its representation as a vector. By definition, the energy of a signal □ (□) of duration T seconds is

$$E_i = \int_0^T s_i^2(t)\, dt \tag{2.10}$$

Therefore, substituting Equation (2.5) into (2.10), we get

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}\phi_k(t)\right]dt$$

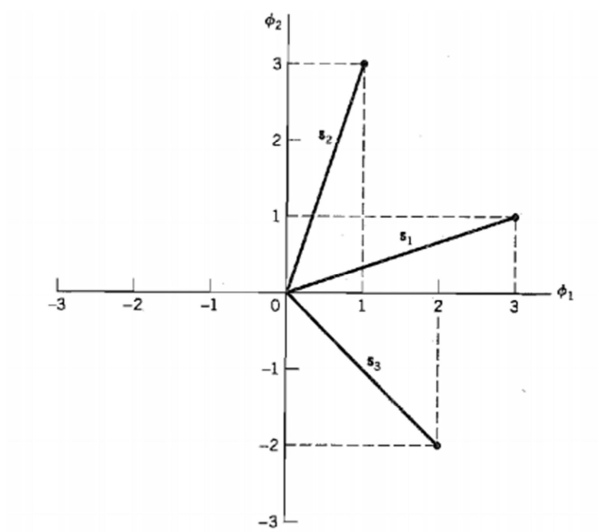


FIG 2.4 illustrating the geometric representation of signals for the case when N=2 and M=3.

Interchanging the order of summation and integration, and then rearranging terms, we get

$$E_i = \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}s_{ik}\int_0^T \phi_j(t)\phi_k(t)dt \tag{2.11}$$

But since theΦ (□) from an orthonormal set,in accordance with the two conditions of Equation (2.7),we find that Equation (2.11) reduces simply to

$$\begin{aligned} E_i &= \sum_{j=1}^{N} s_{ij}^2 \\ &= \| \mathbf{s}_i \|^2 \end{aligned} \tag{2.12}$$

Thus equations (2.9) and (2.12) show that the energy of a signal □ (t) is equal to the squared length of the signal vector □ (□) representing it.

$$\int_0^T s_i(t)s_k(t)\, dt = \mathbf{s}_i^T\mathbf{s}_k \tag{2.13}$$

Yet another useful relation involving the vector representations of the signals □ (t) and □ (□) is decribed by

$$\| \mathbf{s}_i - \mathbf{s}_k \|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2$$
$$= \int_0^T (s_i(t) - s_k(t))^2 dt \tag{2.14}$$

Where‖□ − □ ‖ is the Euclidean distance,□ , between the points represented by the signal vectors □ □□□ □ .

By definition, the cosine of the angleΦ is equal to the inner product of these two vectors divided by the product of their individual norms, as shown by

$$\cos \theta_{ik} = \frac{\mathbf{s}_i^T \mathbf{s}_k}{\| \mathbf{s}_i \| \; \| \mathbf{s}_k \|} \tag{2.15}$$

The two vectors □ □□□ □ are thus orthogonal or perpendicular to each other if their inner product □ □ is zero, in which case Φ = 90 degrees; this condition is intuitively satisfying.

□ EXAMPLE 2.1 Schwarz Inequality

Consider any pair of energy signal □ (□) and □ (□). The Schwarz inequality states that

$$\left(\int_{-\infty}^{\infty} s_1(t) s_2(t) dt \right)^2 \leq \left(\int_{-\infty}^{\infty} s_1^2(t) dt \right) \left(\int_{-\infty}^{\infty} s_2^2(t) dt \right) \tag{2.16}$$

The equality holds if and only if □ (□) = □□ (□), where c is any constant.

To prove this important inequality, let □ (□) □□□ □ (□) be express in terms of the pair of orthonormal basis functions Φ (□) □□□ Φ (□) as follows :

$$s_1(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t)$$
$$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t)$$

Where Φ (□) □□□ Φ (□) satisfy the orthonormality condition over the entire time interval (-∞,∞):

$$\int_{-\infty}^{\infty} \phi_i(t)\phi_j(t) dt = \delta_{ij} = \begin{cases} 1 & \text{for } j = i \\ 0 & \text{otherwise} \end{cases}$$

On this basis, we may represent the signals □ (□)and □ (□) by the following respective pair of vectors, as illustrated in fig 2.5:

$$\mathbf{s}_1 = \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix}$$
$$\mathbf{s}_2 = \begin{bmatrix} s_{21} \\ s_{22} \end{bmatrix}$$

From Fig 2.5 we readily that angle Φ subtended between the vectors □ □□□ □ is

$$\cos\theta = \frac{\mathbf{s}_1^T\mathbf{s}_2}{\|\mathbf{s}_1\|\,\|\mathbf{s}_2\|} = \frac{\int_{-\infty}^{\infty} s_1(t)s_2(t)dt}{\left(\int_{-\infty}^{\infty} s_1^2(t)dt\right)^{1/2}\left(\int_{-\infty}^{\infty} s_2^2(t)dt\right)^{1/2}} \quad (2.17)$$

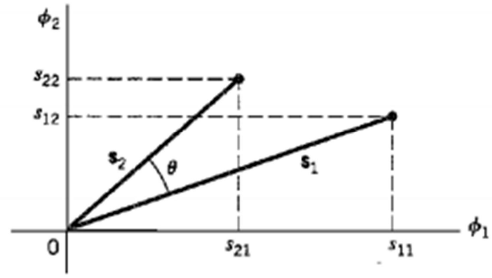


FIG 2.5 Vector representations of signals □ (□) □□□ □ (t) , providing the background picture for proving the Schwarz inequality.

The proofof the Schwarz inequality, as presented here, applies to real-valued signals. It may be readily extended to complex-valued signals, which case Equation (5.16) is reformulated as

$$\left|\int_{-\infty}^{\infty} s_1(t)s_2^*(t)dt\right|^2 \le \left(\int_{-\infty}^{\infty} |s_1(t)|^2 dt\right)\left(\int_{-\infty}^{\infty} |s_2(t)|^2 dt\right) \quad (2.18)$$

Where the equality holds if and only if □ (□) = □□ (□),where c is a constant; see problem 5.9.It is the complex of the Schwarz inequality that was used in chapter 4 to derive the matched filter.

□ GRAM –SCHMIDT ORTHOGONALIZATION PROCEDURE

Having demonstrated the elegance of the geometric representation of energy signals, how do we justify it in mathematical terms? To proceed with the formulation of this procedure, suppose we have a set of M energy signals denoted by □ (□), □ (□), … … …. , □ (□).starting with □ (□) chosen from this set arbitrarily, the first basis function is define by

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \quad (2.19)$$

Where □ is the energy of the signal □ (□).The, clearly, we have

$$s_1(t) = \sqrt{E_1}\phi_1(t) = s_{11}\phi_1(t) \quad (2.20)$$

Where the coefficient □ = √□ and Φ (□) has unit energy, as required.

Next, using the signal□ (□), we define the coefficient □ as

$$s_{21} = \int_0^T s_2(t)\phi_1(t)dt \quad (2.21)$$

We may thus introduce a new intermediate function

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \tag{2.22}$$

Which is orthogonal to Φ (□) over the interval 0 ≤ □ ≤ □ by virtue of Equation (2.21) and the fact that that the basis function Φ (□) has unit energy. Now, we are ready to define the second basis function as

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)dt}} \tag{2.23}$$

Substituting Equation (2.22) and (2.23) and simplifying, we get the desired result

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \tag{2.24}$$

Where □ is the energy of the signal □ (□). It is clear from Equation (2.23) that

$$\int_0^T \phi_2^2(t)dt = 1$$

And from Equation (2.24) that

$$\int_0^T \phi_1(t)\phi_2(t)dt = 0$$

That is to say, Φ (□) □□□ Φ (□) from an orthonormal pair, as required.
Continuing in this fashion, we may in general define

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\phi_j(t) \tag{2.25}$$

Where the coefficients □ are themselves defined by

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \qquad j = 1, 2, \ldots, i-1 \tag{2.26}$$

Equation (2.22) is a special case of Equation (2.25) with I =2. Note also that for I = 1, the function □ (□) □□□□□□□ □□ □ (□)
Given the □ (□), we may now define the set of basis functions

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)dt}}, \qquad i = 1, 2, \ldots, N$$

(2.27) Which form an orthonormal set. The dimension N is less than or equal to the number of given signals, M, depending on one of two possibilities:

- The signals □ (□), □ (□), ……… , □ (□) form a linearly independent set, in which case N= M.
- The signals □ (□), □ (□), ……… , □ (□) are not linearly independent, in which case N < M, and the intermediate function □ (□) is zero for I > N.

2.3 Conversion of the continuous AWGN Channel into a vector channel

Suppose that the input to the bank of N product integrators or correlates in Fig 2.3b is not the transmitted signal □ (□) but rather the received signal x(t) defined in accordance with the idealized AWGN channel of Fig 2.2. That is to say,

$$x(t) = s_i(t) + w(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases} \tag{2.28}$$

Where w(t) is a sample function of a white Gaussian noise process W(t) of zero mean and power spectral density □ /2. Correspondingly, we find that the output of correlate j, say, is the sample value of a random variable□ , as shown by

$$\begin{aligned} x_j &= \int_0^T x(t)\phi_j(t)dt \\ &= s_{ij} + w_j, \qquad j = 1, 2, \ldots, N \end{aligned} \tag{2.29}$$

The first component, □ , is a deterministic quantity contributed by the transmitted signal □ (□); it is define by

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt \tag{2.30}$$

The second component, □ , is the sample value of a random variable □ that arises because of the presence of the channel noise □(□); it is define by

$$w_j = \int_0^T w(t)\phi_j(t)dt \tag{2.31}$$

Consider next a new random process □′(t) whose sample function x'(t) is related to the received signal x(t) as follows:

$$x'(t) = x(t) - \sum_{j=1}^{N} x_j\phi_j(t) \tag{2.32}$$

Substituting Equations (2.28) and (2.29) into (2.32), and then using the expansion of Equation (2.5), we get

$$\begin{aligned} x'(t) &= s_i(t) + w(t) - \sum_{j=1}^{N} (s_{ij} + w_j)\phi_j(t) \\ &= w(t) - \sum_{j=1}^{N} w_j\phi_j(t) \\ &= w'(t) \end{aligned} \tag{2.33}$$

The sample function x'(t) therefore depends solely on the channel noise w(t). on the basis of Equations (2.32) and (2.33), we may thus express the received signal as

$$\begin{aligned} x(t) &= \sum_{j=1}^{N} x_j\phi_j(t) + x'(t) \\ &= \sum_{j=1}^{N} x_j\phi_j(t) + w'(t) \end{aligned} \tag{2.34}$$

□ STATISTICAL CHARACTERIZATION OF THE CORRELATOR OUTPUTS

We now wish to develop a statistical charactarization of set of □correlator outputs.

Let X(t) denotes the random process, a sample function of which is represented by the received signal x(t). correspondingly let X_j denotes the random variable whose sample value is represented by the correlator output $x_{i,j}$ =1,2,…,N. according to the AWGN model of fig 2.2, the random process X(t) is a gaussian process. It follows therefore that X_j is a gaussian random variable for all j. Hence X_j is charactarized completely my its mean and variance which are determined next.

Let □ denote the random variable represented by the sample value □ produced by the jth correlator in response to the white Gaussian noise component w(t).The random variable □ has zero mean,because the noise process W(t) represented by w(t) in the AWGN model of Fig 2.2 has zero mean by definition,the mean of □ depends only on□ ,as shown by

$$\begin{aligned}\mu_{X_j} &= E[X_j] \\ &= E[s_{ij} + W_j] \\ &= s_{ij} + E[W_j] \\ &= s_{ij}\end{aligned}$$

To find the variance of □ we note that

$$\begin{aligned}\sigma_{X_j}^2 &= \mathrm{var}[X_j] \\ &= E[(X_j - s_{ij})^2] \\ &= E[W_j^2]\end{aligned}$$

Where the last line follows from Equation (2.29) with □ □□□ □ replaced by
□ □□□ □ , respectively.According to Equation (2.31), the random variable □ is defined by

$$W_j = \int_0^T W(t)\phi_j(t)dt$$

We may therefore expand Equation (2.36) as follows:

$$\begin{aligned}\sigma_{X_j}^2 &= E\left[\int_0^T W(t)\phi_j(t)dt \int_0^T W(u)\phi_j(u)du\right] \\ &= E\left[\int_0^T \int_0^T \phi_j(t)\phi_j(u)W(t)W(u)dtdu\right]\end{aligned} \tag{2.37}$$

Interchanging the order of integration and expectation:

$$\begin{aligned}\sigma_{X_j}^2 &= \int_0^T \int_0^T \phi_j(t)\phi_j(u)E[W(t)W(u)]dtdu \\ &= \int_0^T \int_0^T \phi_j(t)\phi_j(u)R_W(t, u)dtdu\end{aligned} \tag{2.38}$$

Where $R_w(t, u)$ is the autocorrelation function of noise process W (t). Since the noise is stationary, $R_w(t, u)$ depends only on the time difference t-u. Furthermore since the noise W(t) is white with constant power spectral density $N_o/2$, we may express:

$$R_W(t, u) = \frac{N_0}{2}\,\delta(t-u) \tag{2.39}$$

Therefore substituting equations (2.39) into (2.38) and then using the shifting property of delta function $\delta(t)$, we get

$$\begin{aligned}\sigma^2_{X_j} &= \frac{N_0}{2}\int_0^T\int_0^T \phi_j(t)\phi_j(u)\ \delta(t-u)\,dt\,du \\ &= \frac{N_0}{2}\int_0^T \phi_j^2(t)\,dt\end{aligned} \tag{2.40}$$

Since the $\phi_j(t)$ have the unit energy, by definition we finally get the simple result,

$$\sigma^2_{X_j} = \frac{N_0}{2} \qquad \text{for all } j \tag{2.41}$$

This important result shows all the correlator outputs.

Moreover since the $\phi_j(t)$ from the orthogonal set we find that the X_j are mutually uncorrelated, as shown:

$$\begin{aligned}\operatorname{cov}[X_jX_k] &= E[(X_j-\mu_{X_j})(X_k-\mu_{X_k})] \\ &= E[(X_j-s_{ij})(X_k-s_{ik})] \\ &= E[W_jW_k] \\ &= E\left[\int_0^T W(t)\phi_j(t)\,dt \int_0^T W(u)\phi_k(u)\,du\right] \\ &= \int_0^T\int_0^T \phi_j(t)\phi_k(u)R_W(t,u)\,dt\,du \\ &= \frac{N_0}{2}\int_0^T\int_0^T \phi_j(t)\phi_k(u)\ \delta(t-u)\,dt\,du \\ &= \frac{N_0}{2}\int_0^T \phi_j(t)\phi_k(t)\,dt \\ &= 0, \qquad j \neq k\end{aligned} \tag{2.42}$$

Since □ are Gaussian random variables, eq (2.42) implies that they are also statistically independent. Define the vectors of N random variables

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} \tag{2.43}$$

Whose elements are independent Gaussian random variables with mean value equal to S_{ij} and variance equal to $N_o/2$. Since the elements of the vector X are statistically independent, we may express the conditional probability density function of the vector X, given that the $s_i(t)$ or correspondingly the symbol m_i was transmitted,

$$f_{\mathbf{X}}(\mathbf{x}|m_i) = \prod_{j=1}^{N} f_{X_j}(x_j|m_i), \qquad i = 1, 2, \ldots, M \tag{2.44}$$

The vector x is called the observation vector, correspondingly x_j is called an observable element.

Any channel that satisfies above eq is called memory less channel.

Since each □ is a Gaussian random variable with mean □ and variance □ /2, we have

$$f_{X_j}(x_j|m_i) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_j - s_{ij})^2\right], \qquad \begin{matrix} j = 1, 2, \ldots, N \\ i = 1, 2, \ldots, M \end{matrix} \tag{2.45}$$

Therefore, substituting Equation (2.44) yiels

$$f_{\mathbf{X}}(\mathbf{x}|m_i) = (\pi N_0)^{-N/2} \exp\left[-\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2\right], \qquad i = 1, 2, \ldots, M \tag{2.46}$$

It is clear now that the element of the random vector X completely characterize the summation term ∑ Xj □$_j$(t) whose sample value is represented by the first term in eq (2.34). However there remains a noise term w'(t) in this equation which depends only on the channel noise w(t). since the noise process W(t) represented by w(t) is Gaussian with zero mean, it follows that the noise process W'(t) represented by a sample function w'(t) is also a zero mean Gaussian process. Finally we note that any random variable W'(t_k) say derived from the noise process W'(t) by sampling it at time t_k is in fact statistically independent of the set of random variables {X_j); that is to say:

$$E[X_j W'(t_k)] = 0, \qquad \begin{cases} j = 1, 2, \ldots, N \\ 0 \le t_k \le T \end{cases} \tag{2.47}$$

Above eq states that the random variable W' (t_k) is irrelevant to the decision s to which particular signal was actually transmitted. In other word collector outputs determined by the received signal x(t) are the only data that are useful for the decision making process and hence represents sufficient statistics for the problem at hand.

We may now summarize the results presented in the section by formulating the theorem of irrelevance:

Insofar as signal detection in additive white Gaussian noise is concerned only the projections of the noise onto the basic functions of the signal set {□ (□)} affects the sufficient statistics of the detection problem; the reminder of the noise is irrelevant.

As a corollary on this theorem, we may state that the AWGN channel of figure 2.2. is equivalent to a Ndimensional vector channel described by the observation vector x= s_i + w, i=1,2,….,M

Where the dimension □ is the number of basic functions involved in formulating the signal vector s_i. the individual components of the signal vectors s_is and noise vector w are defined by eq (2.6) and (2.13) respectively. The theorem of irrelevance and its corollary are indeed basic to the understanding of the signal detection problem as described next.

2.4 Likelihood functions:

The conditional probability density functions $f_x(x|m_i)$, □ = 1,2, . . , □ are the very characterization of an AWGN channel. Their derivation leads to a functional dependence on the observation vector k, given the transmitted message symbol m_i however at the receiver we have the exact opposite situations: we are given the observation vector x and the requirement is to estimate the message symbol m_i that is responsible for the generating x. to emphasize this latter viewpoint, we introduce the idea of a likelihood function, denoted by □(□) and defined by

$$L(m_i) = f_{\mathbf{X}}(\mathbf{x}|m_i), \qquad i = 1, 2, \ldots, M \tag{2.49}$$

It is an important however to recognize that although the $L(m_i)$ and $f_x(x|m_i)$ have exactly the same mathematical form, their individual meanings are different.

In practice, we find it m ore convenient to work e=with the log-likelihood functions denoted by $l(m_i)$ and defined by

$$l(m_i) = \log L(m_i), \qquad i = 1, 2, \ldots, M \tag{2.50}$$

The log-likelihood functions bear a one-to-one relationship to the likelihood functions for two reasons:

1. By definition a probability density function is always nonnegative. It follows therefore that the likelihood function is likewise a nonnegative quantity.
2. The logarithmic functions are monotonically increasing function of irs argument.

The use of equation (2.46) in (2.50) yields the log-likelihood functions for an AWGN channel as

$$l(m_i) = -\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2, \qquad i = 1, 2, \ldots, M \tag{2.51}$$

Where we have ignored the constant term $-(N/2)\log(\pi N_o)$ as it bears no relation whatsoever to the message symbol m_i. note that the s_{ij}, j=1,2,……,.N are in the elements of the signal vector s_i representing the message symbol m_i. with equation (2.51) at our disposal, we are now ready to address the basic receiver design problem.

2.5 Coherent detection of signals in noise:

Maximum likelihood decoding

Suppose that in each time slot of duration T seconds, one of the m possible signals $s_1(t)$, $s_2(t)$,…..,$s_M(t)$ is transmitted with equal probability, 1/M. For geometric signal representation, the signal $s_i(t)$, i=1,2,…..,M, is applied to a bank of correlators, with a common input and supplied with an appropriate set of N orthonormal basic function. The resulting correlator outputs define the signal vectors s_i. since the knowledge of the signal vector $s_i$is as good as knowing the transmitted signal $s_i(t)$ itself, and vice versa we may represent $s_i(t)$ by a point in a Euclidean space of dimensions N≤M. we refer to this point as the transmitted signal point or message point. The set of message points corresponding to the set of transmitted signals $\{s_i(t)\}^M_{i=1}$ is called a signal constellation.

However the representation of the received signal x(t) is complicated by the presence of additive noise w(t). we note that when the received signal x(t) is applied to the bank of N correlator outputs define the observation vector x. from equation (5.48), the vector x differs from the signal vector s_i by the noise vector w whose orientation is completely random. The noise vector w is completely characterized by the noise w(t); the converse of this statement, however is not true. The noise vector w represents that portions of the noise w(t) that will interfere with the detection process; the remaining portion of this noise denoted by w'(t), is tuned out by the bank of correlators.

Now based on the observation vector x, we may represent the received signal x(t) by a point in the same euclidean space used to represent the transmitted signal. We refer to this second point as the received signal point. The received signal point wanders about the message point in a completely random fashion in the sense that it may lie anywhere inside a Gaussian-distributed "cloud" centered on the message point. This is illustrated in fig 5.7a for the case of a three dimensional signal space. For a particular realization of the noise vector w the relationship between the observation vector x and the signal vector $s_i$is as illustrated in the figure 2.7b.

We are now ready to state the signal detection problem:

Given the observation vector x, perform a mapping from x to an estimate $\hat{m}$ of the transmitted symbol, m_i, in a way that would minimize the probability of error in the decision-making process.
Suppose that, given the observation vector x, we make the decision $\hat{m} = m_i$. The probability of error in this decision, which we denote $P_e(m_i|x)$, is simply

$$P_e(m_i|x) = P(m_i \text{ not sent} \mid x)$$

$$= 1 - P(m_i \text{ sent} \mid x) \qquad (2.52)$$

The decision making criterion is to minimize the probability of error in mapping each given observation vector x into a decision. On the basis of equation (2.52), we may therefore state the optimum decision rule:

$$\text{Set } \hat{m} = m_i \text{ if}$$
$$P(m_i \text{ sent} \mid \mathbf{x}) \geq P(m_k \text{ sent} \mid \mathbf{x}) \qquad \text{for all } k \neq i \qquad (2.53)$$

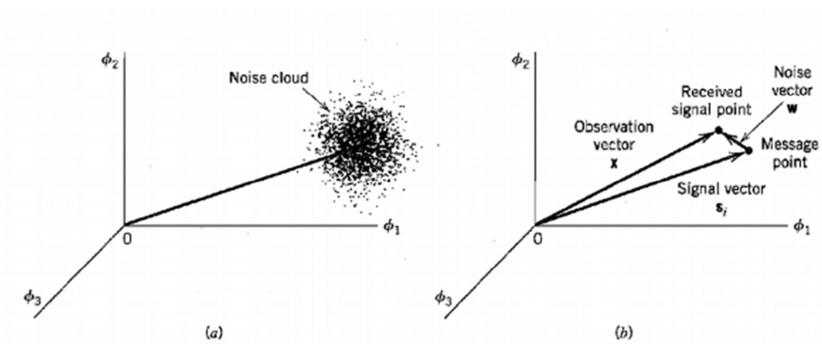


Figure 2.7-Illustrating the effect of noise perturbation depicted in (a) on the location of the received signal point (b)

Where k=1,2,….,M. this decision rule is referred to as the maximum a posteriori probability (MAP) rule.

The condition of equation (2.53) may be expressed more explicitly in terms of the priori probabilities of the transmitted signals and in terms of the likelihood functions. Using bay's rule in equation (2.53), and for the moment ignoring possible ties in the decision-making process, we may restate the MAP rule as follows:

$$\text{Set } \hat{m} = m_i \text{ if}$$
$$\frac{p_k f_{\mathbf{X}}(\mathbf{x}|m_k)}{f_{\mathbf{X}}(\mathbf{x})} \text{ is maximum for } k = i \tag{2.54}$$

Where p_k is the priori probability of transmitting symbol m_k, $f_x(x|m_k)$ is the conditional probability density function of the random observation vector X given the transmission of symbol m_k and $f_x(x)$ is the unconditional probability density function of X. in the equation (2.54) we may note the following:

- The denominator term $f_x(x)$ is independent of the transmitted symbol.
- The a priory probability $p_k – p_i$ when all the source symbols are transmitted with equal probability
- The conditional probability density function $f_x(x\ m_k)$bears a one-to-one relationship to the loglikelihood function $l(m_k)$.

Accordingly we may restate the decision rule of equation (2.54) in terms of $l(m_k)$ simply as follows:

$$\text{Set } \hat{m} = m_i \text{ if}$$
$$l(m_k) \text{ is maximum for } k = i \tag{2.55}$$

The decision rule is referred to as the maximum likelihood rule, and the device for its implementation is correspondingly referred to as the maximum likelihood decoder. According to the equation (2.55), a maximum likelihood decoder computes the log-likelihood functions as metrics for all the m possible message symbols, compares them, and then decides in favor of the maximum. Thus the maximum likelihood decoder differs from the maximum a posteriori decoder in that it assumes equally likely message symbols.

It is useful to have a graphical interpretation of the maximum likelihood decision rule.

The maximum likelihood decision rule of equation (2.55) with the channel noise □(□) being additive as the only restriction imposed on it. We next specialize this rule for the case when □(□) is both white and Gaussian.

From the log-likelihood function defined in equation (2.51) for an AWGN channel we note that $l(m_k)$ attains its maximum value when the summation term

$$\sum_{j=1}^{N} (x_j - s_{kj})^2$$

Is minimized by the choice k=i. accordingly, we may formulate the maximum likelihood decision rule for an AWGN channel as

Observation vector x lies in region □ if

$$\sum_{j=1}^{N} (x_j - s_{kj})^2 \text{ is minimum for } k = i \tag{2.57}$$

Next, we note from our earlier discussion that

$$\sum_{j=1}^{N} (x_j - s_{kj})^2 = \| \mathbf{x} - \mathbf{s}_k \|^2$$

(2.59)

Where $\| x - s_k |$ is the Euclidean distance between the received signal point and message point, represented by the vectors x and s_k, respectively. Accordingly we may restate the decision rule of equation (2.56) as follows:

Observation vector x lies in region □ if the Euclidean distance ‖ x –□ ‖ is minimum for k= I (2.59)

Equation (2.58) states that the maximum likelihood decision rule is simply to choose the message point closet to the received signal point, which is intuitively satisfying.

In the practice, the need for squarer's in the decision rule of equation (2.58) is avoided by recognizing that

$$\sum_{j=1}^{N} (x_j - s_{kj})^2 = \sum_{j=1}^{N} x_j^2 - 2 \sum_{j=1}^{N} x_j s_{kj} + \sum_{j=1}^{N} s_{kj}^2$$

(2.60)

The first summation term of this expansion is independent of the index k and may therefore be ignored. The second summation term is the inner product of the observation vector x and signal vector s_k. the third summation term is the energy of the transmitted signal $s_k(t)$. Accordingly we may formulate a decision rule equivalent to that of equation (2.58) as follows;

Observation vector x lies in region □ if

$$\sum_{j=1}^{N} x_j s_{kj} - \frac{1}{2} E_k \text{ is maximum for } k = i$$

(2.61)

Where $E_k$is the energy of the transmitted signal $s_k(t)$:

$$E_k = \sum_{j=1}^{N} s_{kj}^2$$

(2.62)

From the equation (2.60) we deduce that for an AWGN channel, the decision regions are regions of the N-dimensional observation space Z, bounded by linear

[(□ − 1)- dimensional hyper plane] boundaries. Figure 2.8 shows the example of decision regions for

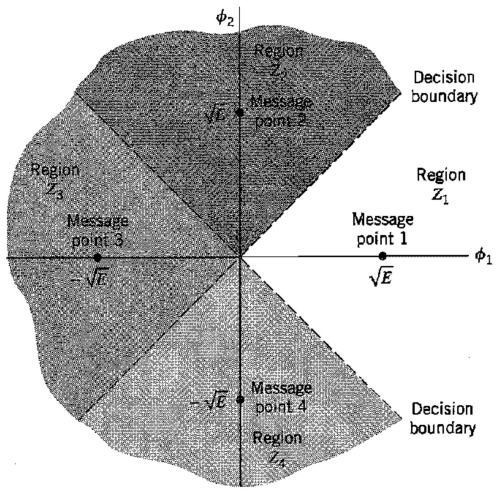


Fig. 2.8 illustrating the partitioning of the observation space into decision regions for the case when N= 2 and M= 4; it is assumed that the M transmitted symbols are equally likely.

M=4 signals and N=2 dimensions, assuming that the signals are transmitted with equal energy, E, and equal probability.

Sample Questions:

1. Figure 1. displays the waveforms of four signals □ (□), □ (□), □ (□) and □ (□).
Using the Gram-Schmidt orthogonalization procedure, find an orthonormal basis function for this set of signals

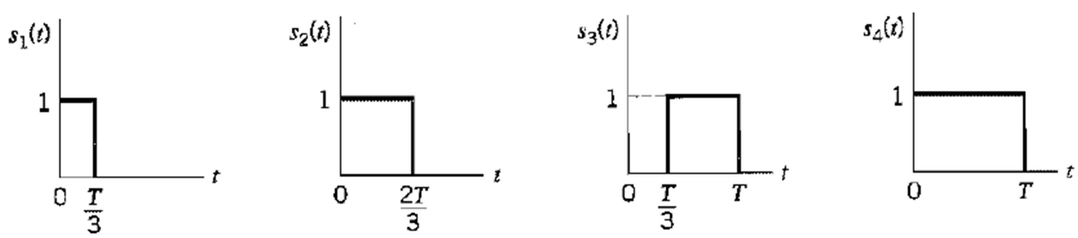


Figure 1

2) A pair of signals □ (□) and □ (□) have a common duration T, show that

(a) ∫ □ (□)□ (□)□□ = □□□ □□

(b)∫ □ (□) − □ (□) □□ = ‖□ − □ ‖

Where □ and □ denote the vector representations of the signals □ (□) and □ (□) respectively.

3) Prove the Gram-Schmidt orthogonalization procedure.

4) How is orthogonality of two signals defined? Explain the term ‘norm of the signal’? What is its physical significance?

Module-III

Sampling theorem and Pulse Modulation:

Introduction:

Sampling theorem states that in order to reconstruct the original signal from the samples the signal has to be sampled at a rate greater than twice the highest frequency component of the message signal.

$$f_s \geq 2 f_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.1$$

Where fs is the sampling rate, fm is the highest frequency component in the message signal

Sampling Period: The sampling Period or sampling time is the inverse of the sampling frequency giving the information about the time interval between two consecutive samples

$$T_s = 1/f_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...3.2$$

Nyquist rate: it is the minimum rate of sampling that s required for a signal which is bandlimited to f_m Hz.

$$f_s = 2 f_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...3.3$$

Proof of sampling theorem:

Consider a continuous time signal x(t). The spectrum of x(t) is a band limited to f_m Hz i.e. the spectrum of x(t) is zero for $|\omega|>\omega_m$.

Sampling of input signal x(t) can be obtained by multiplying x(t) with an impulse train δ(t) of period T_s. The output of multiplier is a discrete signal called sampled signal which is represented with y(t) in the following diagrams:

Mathematically the process of sampling can be explained by the following equations.

Sampled signal

$$y(t)=x(t).\delta(t)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots3.4$$

The trigonometric Fourier series representation of δ(t) is given by

$$\delta(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots3.5$$

by solving we get

$$\delta(t) = \frac{1}{Ts} + \sum_{n=1}^{\infty} \left(\frac{2}{Ts} \cos n\omega_s t\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots3.6$$

Substitute δ(t) in equation 1. $y(t)=x(t).\delta(t)$

$$\therefore y(t) = x(t)\left[\frac{1}{Ts} + \sum_{n=1}^{\infty}\frac{2}{Ts}(\cos n\omega_s t)\right] \quad \text{...3.7}$$

$$\therefore y(t) = \frac{1}{Ts}\left[x(t) + 2\sum_{n=1}^{\infty} x(t)(\cos n\omega_s t)\right] \quad \text{...3.8}$$

$$\therefore y(t) = \frac{1}{Ts}\left[x(t) + 2x(t).\cos\omega_s t + 2x(t).\cos\omega_s t + 2x(t).\cos 3\omega_s t......\right] \quad \text{............................3.9}$$

Taking Fourier transform on the both side

$$\therefore Y(\omega) = \frac{1}{Ts}\left[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s)......\right] \quad \text{............3.10}$$

$$\therefore Y(\omega) = \frac{1}{Ts}\sum X(\omega - n\omega_s)$$

where n= 0,±1,±2,.. ....3.11 The spectrum of the sampled signal is observed.

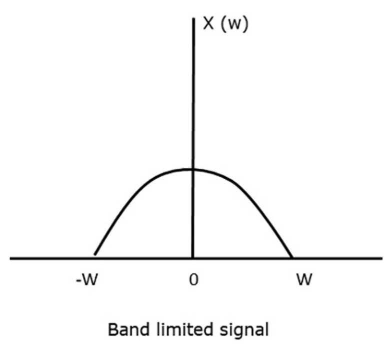


Fig. 3.1: Band limited signal

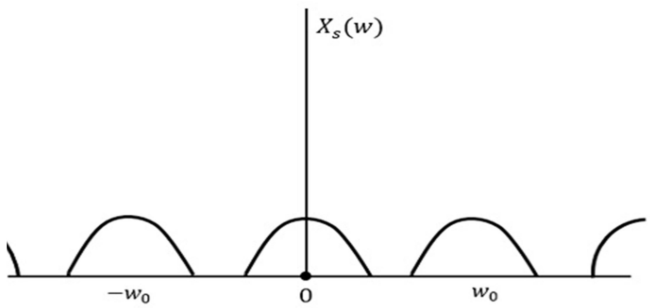


Fig 3.2: when the sampling rate is greater than twice the highest frequency $f_s > 2f_m$

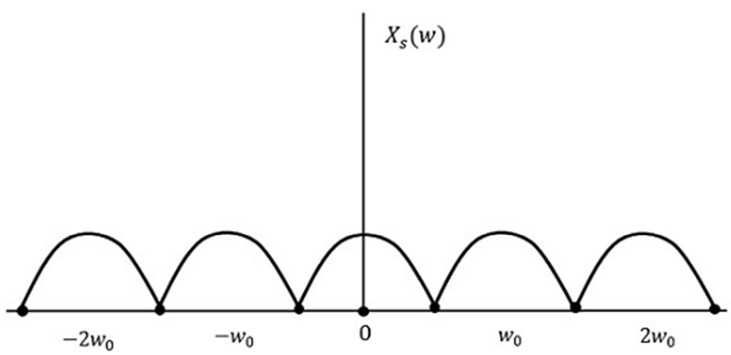


Fig 3.3: when the sampling rate is equal to twice the highest frequency $f_s = 2f_m$

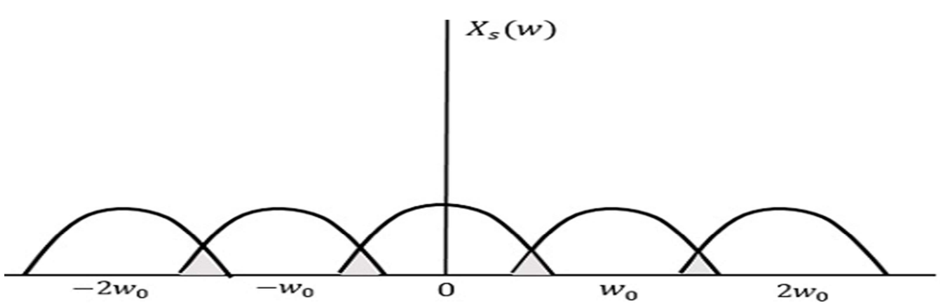


Fig 3.4: when the sampling rate is less than twice the highest frequency $f_s < 2f_m$

The over-lapping of information takes place when the sampling rate is less than twice the highest frequency $f_s < 2f_m$ which leads to mixing up and loss of information. This unwanted phenomenon of overlapping is called as Aliasing.

Aliasing effect

From the previous section it is seen that by sampling a Band-limited signal at a rate greater than twice the bandwidth of the signal, it is possible to reconstruct the original signal. But when the sampling rate is less than twice the bandwidth of the band-limited signal, different translated versions of the original spectrum overlap in the spectrum of the sampled signal. This effect is called aliasing.

If we reconstruct this signal using a low-pass filter, we might get a signal completely different from the original signal.

Pulse Modulation

Pulse modulation consists essentially of sampling analog information signals and then converting those samples into discrete pulses and transporting the pulses from a source to a destination over a physical transmission medium.

The four predominant methods of pulse modulation:

1) Pulse width modulation (PWM)

2) Pulse position modulation (PPM)

3) pulse amplitude modulation (PAM)

4) pulse code modulation (PCM).

Pulse Width Modulation

PWM is sometimes called pulse duration modulation (PDM) or pulse length modulation (PLM), as the width (active portion of the duty cycle) of a constant amplitude pulse is varied proportional to the amplitude of the analog signal at the time the signal is sampled.

The maximum analog signal amplitude produces the widest pulse, and the minimum analog signal amplitude produces the narrowest pulse. Note, however, that all pulses have the same amplitude.

Pulse Position Modulation

With PPM, the position of a constant-width pulse within a prescribed time slot is varied according to the amplitude of the sample of the analog signal. The higher the amplitude of the sample, the farther to the right the pulse is positioned within the prescribed time slot. The highest amplitude sample produces a pulse to the far right, and the lowest amplitude sample produces a pulse to the far left.

Pulse Amplitude Modulation

With PAM, the amplitude of a constant width, constant-position pulse is varied according to the amplitude of the sample of the analog signal. The amplitude of a pulse coincides with the amplitude of the analog signal.

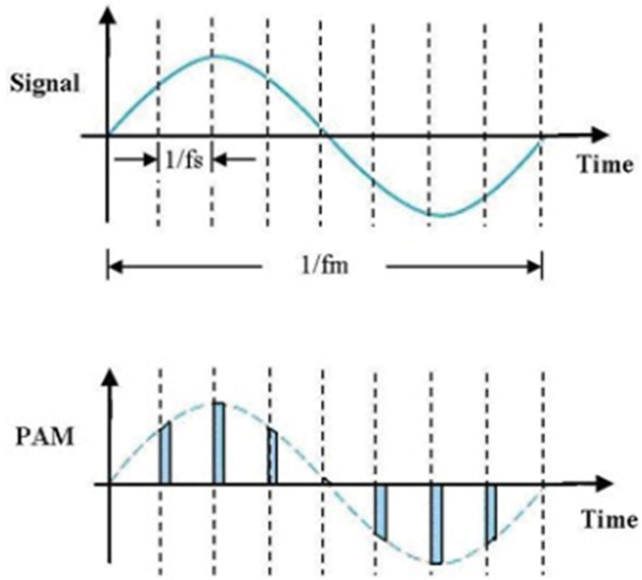


Fig 3.5: PAM waveform

PAM is a technique where a continuous in time and continuous in amplitude waveform is converted to a discrete in time and continuous in amplitude waveform. With PAM, the amplitude of a pulse changes with respect to the analog signal. The amplitude of a pulse coincides with the amplitude of the analog signal.

Application of PAM:

Ethernet:Ethernet communication standard are an example of PAM usage. In particular, 100BASE-T4 and BroadR-Reach Ethernet standard, use three-level PAM modulation (PAM-3), 1000BASE-T Gigabit Ethernet uses five-level PAM-5 modulation

Digital television: The North American Advanced Television Systems Committee standards for digital television uses a form of PAM to broadcast the data that makes up the television signal. This system, known as 8VSB, is based on a three-level PAM like 100BASE-TX.

Electronic driver Pulse-amplitude modulation has also been developed for the control of light-emitting diodes (LEDs), especially for lighting applications. Telecommunication systems: Applied in some electronic switching systems.

Sample & Hold Circuit

Sample & Hold Circuit is used to sample the given input signal and to hold the sampled value. It samples an analog signal for a short interval of time in the range of 1 to 10µS and to hold on its last sampled value until the input signal is sampled again. The holding period may be from a few milliseconds to several seconds. When the pulse is high signal is sampled and when the pulse is low signal value is holded. Thus the circuit has two modes of operation depending upon the logic level of S/H command signal.

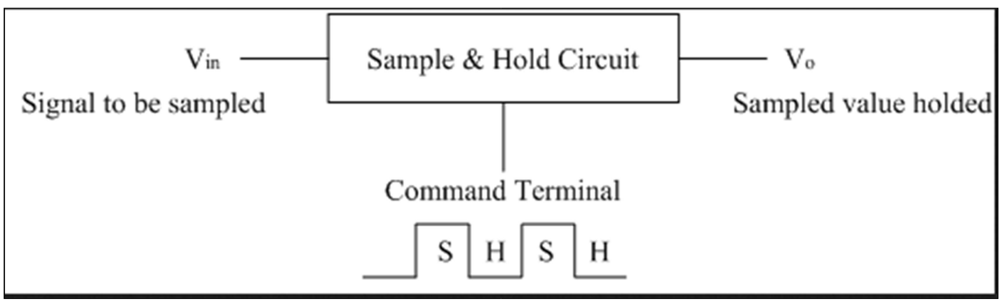


Fig 3.6: Sample and Hold

Sample and Hold Circuit:

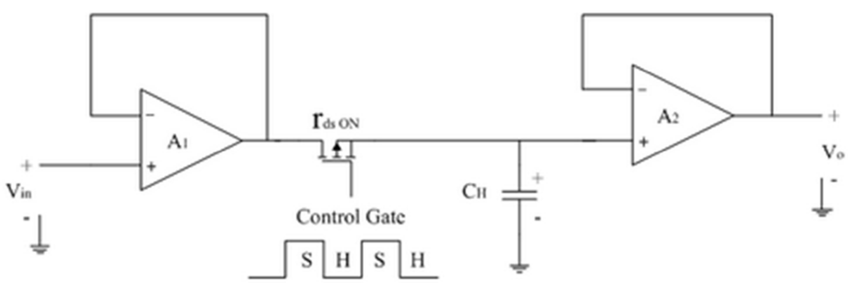


Fig 3.7: Sample and Hold Circuit:

Working Principle: Two Amplifier A1 and A2 are used which are voltage follower circuits. Switching ON/OFF is performed by FET which is controlled by the S/H pulses. At Vin signal to be sampled is applied. Input impedance of A1 is very high so input voltage source is not loaded. While sampling output of A1 is same as Vin. When S/H pulse is applied FET switches ON and starts conducting. Resistance between drain and source i.e. r_{dsON} is very small. For voltage follower, A1 and A2 have 100% feedback (β=1).

Therefore output impedance of A1 and A2 is very small. Now capacitor C starts charging through r_{dsON} and output impedance of A1.

Charging Time Constant = $r_{dsON} \times r_{out} \times C$

As r_{dsON} and r_{out} are very small, capacitor C charges through very quickly to Vin (i.e. capacitor tracks the input signal). At the end FET is off, so almost acts as open circuit. So capacitor isolates from previous circuits and it holds the charge of last sampled value. As input impedance of A2 is very large, capacitor discharging time is very high, so it almost holds the charge. Also Gain of A2 is unity.

Therefore
V_{out} = Charge on capacitor

As r_{out} of A2 is very small, we can take V_{out} across any value of RL
Performance parameters of S/H circuit:

The performance of an ordinary S/H circuit can be characterized by Vio, Gain error, nonlinearity etc. Consider the following figure to define some of the important parameters of S/H amplifier.

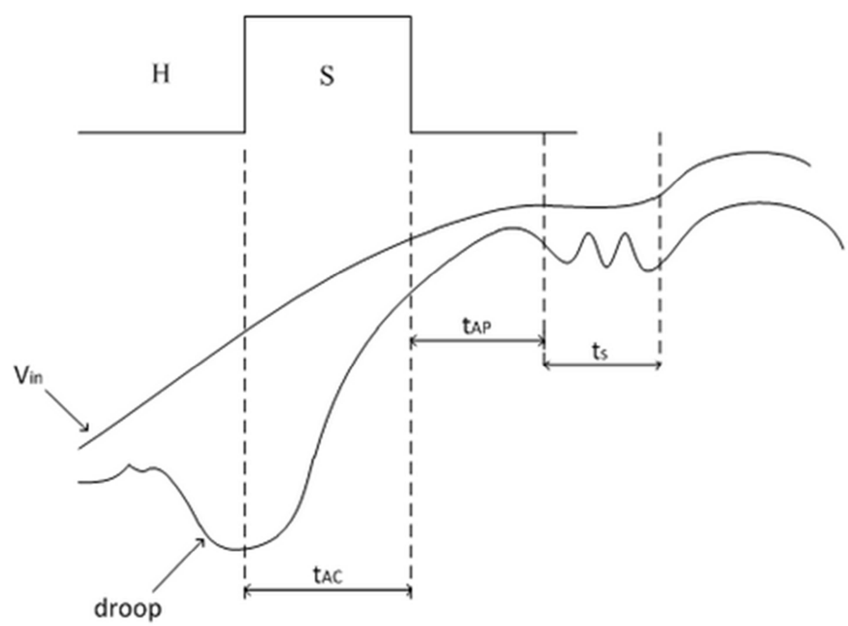


Fig 3.8: Performance parameters of S/H circuit:

1. Acquisition Time (t_{AC}): It is the time required for the holding capacitor CH to charge upto a level close to the input voltage during sampling. It depends on three factors namely RC time constant, Maximum output current of op-amp and slew rate of op-amp.
2. Aperture Time (t_{AP}): Ideally as soon as the hold command is given to S/H circuit, the circuit should stop following any changes taking place in the input and hold the latest sampled value. But practically, the S/H circuit will follow the changes in input voltage for a short period of time, even

after receiving the hold command. This period is called as aperture time. It is due to the propagation delays of the driver and the switch.

3. Aperture Uncertainty (Δt_{AP}):It is the variation in the aperture time from sample to sample.

4. Hold mode settling time (t_s): After the application of hold command, it takes a certain amount of time for Vo to settle within a specified error band such as 1%, 0.1%, 0.01%.

5. Hold Step: At the time of switching from sample to hold or hold to sample mode, there is an unwanted transfer of charge between the switch driver and holding capacitor CH. This changes the capacitor voltage and hence output voltage. These changes in output voltage are referred as hold step, pedestal error or sample and hold offset.

6. Feed through: In the hold mode, because of stray capacitances across switch there is a small amount of ac coupling between Vo and Vin. This ac coupling causes output voltage to vary with variation in the input voltage. This is referred as feed through.

7. Voltage Droop: The leakage current causes voltage of the capacitor to drop down. This is referred as droop.

This sample and hold circuit is readily built in IC form is available (monolithic) and are comparatively inexpensive. For this IC user has to connect only a single capacitor externally.
National Semiconductor ICs LM 198/298/398.

Interlacing And Multiplexing Of Samples

The main objective a digital communication system is to maximize the bandwidth utilisation and the increase the data rate over the channel. For this purpose if we multiplex the different signals over the same channel. We can time disown multiplex different signals over the sampling interval of a sampled signal. In a sampled signal the sampled arrive at a time interval of Ts seconds so during this time interval the no signal is being transmitted over the channel. In a PAM TDM system the analog signal is sampled at a rate greater than twice the band limit of the signal. The samples are generated at a time interval of $T_{s.}$ So samples of other source can be time division multiplexed in this time interval.

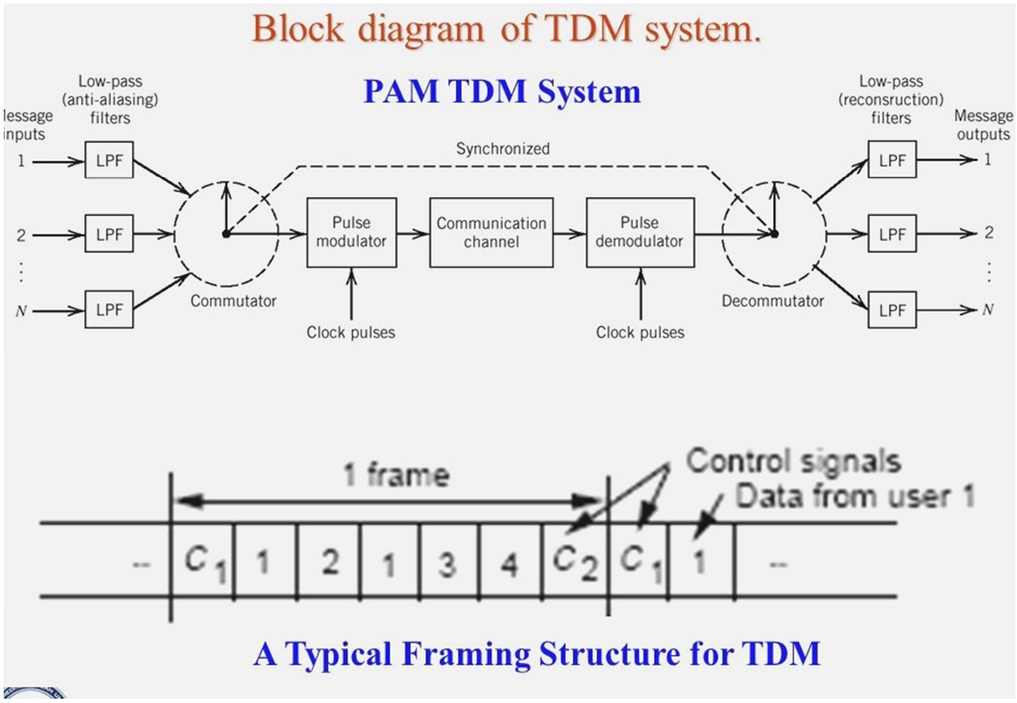


Fig 3.9: Time Division Multiplexing

Working Principle: In this block diagram N number of sources are Time Division Multiplexed. This is done the commutator present which scans all the sources n the time interval of Ts seconds. If the sampling rate is 8Khz then the sampling time Ts is equal to 125µs. So the commutator should scan all the sources within the 125µs. This Ts is divided equally among the N sources.

The commutator and decommutator are synchronised electronic switches which rotate at a speed of 2fm rotations/ second. When commutator is connected to source 1 to take the sample of source 1 the decommutator is connected to the outlet 1 which further is connected to the low pass filter where the samples are reconstructed to form the analog signal 1. As the next sample of source 1 appears after Ts second. When commutator is connected to source 2 to take the sample of source 2 the decommutator is connected to the outlet 2 which further is connected to the low pass filter where the samples are reconstructed to form the analog signal 2.

Pulse Code Modulation (PCM)

Pulse Code Modulation: In a PCM system the analog signal is converted o the digital signal by the three process of Sampling, Quantization and Coding.

Sampling: By the process of sampling a signal which is continuous in time and amplitude is converted to a discrete in time and continuous amplitude signal. For sampling the sampling theorem has to be followed where the sampling rate has to be more than twice the highest frequency component of the message signal.

$$f_s \geq 2 f_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

Quantization: By the process of quantization a discrete in time and continuous amplitude signal is converted to a discrete in time and discrete in amplitude signal.

Coding: In addition, PCM involves another step, namely, conversion of quantized amplitudes into a sequence of simpler pulse patterns (usually binary), generally called as code words.(The word code in pulse code modulation refers to the fact that every quantized sample is converted to an R-bit code word.)

Folded Binary Code

With quantization, the total voltage range is subdivided into a smaller number of sub-ranges.

Eg. The PCM code shown in table below is a three-bit sign- magnitude code with eight possible combinations (four positive and four negative). The leftmost bit is the sign bit (1 = + and 0 = -), and the two rightmost bits represent magnitude. This type of code is called a folded binary code because the codes on the bottom half of the table is a mirror image of the codes on the top half, except for the sign bit.

Three-Bit PCM Code

(8 Sub ranges)

Sign	Magnitude		Decimal value	Quantization range
1	1	1	+3	+2.5 V to +3.5 V
1	1	0	+2	+1.5 V to +2.5 V
1	0	1	+1	+0.5 V to +1.5 V
1	0	0	+0	0 V to +0.5 V
0	0	0	-0	0 V to -0.5 V
0	0	1	-1	-0.5 V to -1.5 V
0	1	0	-2	-1.5 V to -2.5 V
0	1	1	-3	-2.5 V to -3.5 V

PCM system Block Diagram:

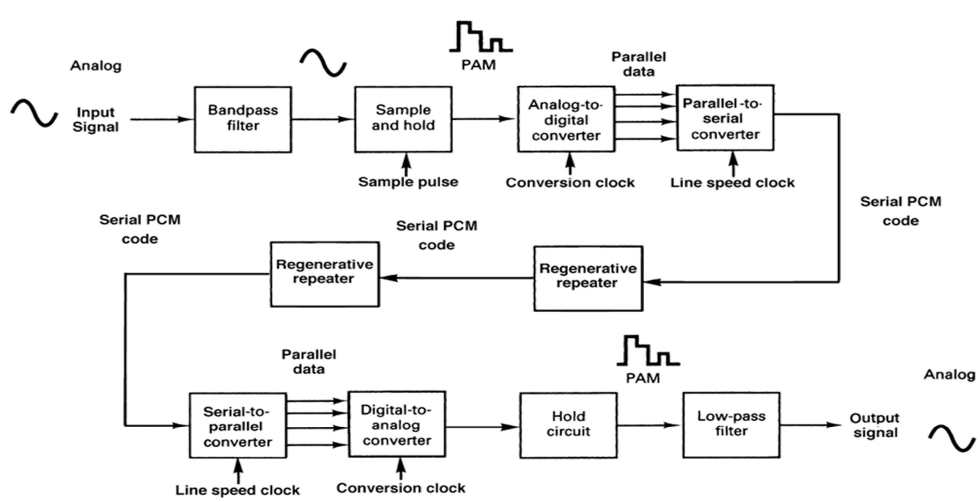


Fig.3.10.: The block diagram of a single-channel, simplex (one-way only) PCM system.

Transmitter:

The band pass filter limits the frequency of the analog input signal to the standard voice-band frequency range of 300 Hz to 3000 Hz.

The sample- and- hold circuit periodically samples the analog input signal and converts those samples to a multilevel PAM signal.

The analog-to-digital converter (ADC) converts the PAM samples to parallel PCM codes, which are converted to serial binary data in the parallel-to-serial converter and then outputted onto the transmission linear serial digital pulses.

The transmission line repeaters are placed at prescribed distances to regenerate the digital pulses.

Receiver:

In the receiver, the serial-to-parallel converter converts serial pulses received from the transmission line to parallel PCM codes.

The digital-to-analog converter (DAC) converts the parallel PCM codes to multilevel PAM signals. The hold circuit is basically a low pass filter that converts the PAM signals back to its original analog form.

Quantization

Quantization is the process of converting an infinite number of possibilities to a finite number of conditions. Analog signals contain an infinite number of amplitude possibilities.

The process by which a discrete in time and continuous amplitude signal (analog sample) is converted to a discrete in time and discrete in amplitude signal s called Quantization. Soin quantization it rounds off the value to one of its closest permissible value on it.

Uniform quantization:

The quantization process can be illustrated graphically as fig below. Let us consider a signal whose range is from V_L to V_H

The quantization Process has a two-fold effect:

1. The peak-to-peak range of the input sample values is subdivided into a finite set of decision levels or decision thresholds that are aligned with the risers of the staircase, the difference between two adjacent discrete values, Δ, is called the step size of the quantizer.

Step size $\Delta = (V_H - V_L)/M$ where V_L to V_H is the range of the signal to be quantized

2. The output is assigned a discrete value selected from a finite set of representation levels that are aligned with the treads of the staircase. A quantizer is memory less in that the quantizer output is determined only by the value of a corresponding input sample, independently of earlier analog samples applied to the input.

The signal g(t) denotes the input of the quantizer, and the ĝ(t) represents the output. As can be seen from the figure, the quantization process implies that a straight line relation between the input and output of a linear continuous system is replaced by a staircase characteristic. . M is the number of quantization levels. These quantized levels are (voltage level) has one code assigned to it except zero volts, magnitude difference between adjacent steps is called the quantization interval or step size . The magnitude of a step size is also called the resolution. The smaller the magnitude of a step size, the better (smaller) the resolution and the more accurately the quantized signal will resemble the original analog sample.

The error signal is the difference between the input and the quantizer output.

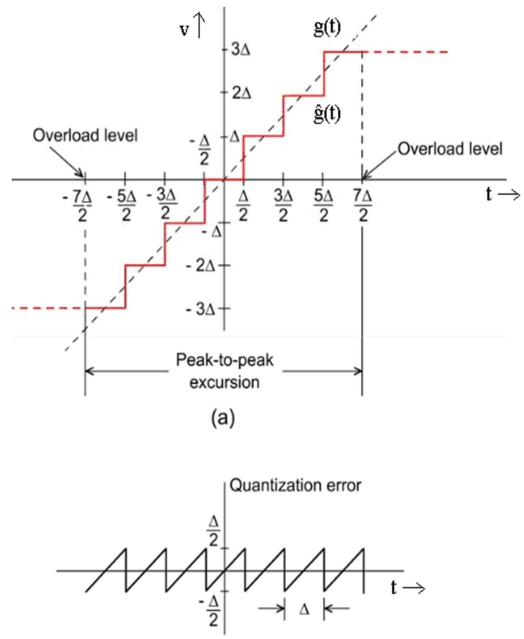


Fig.3.11: Qantization

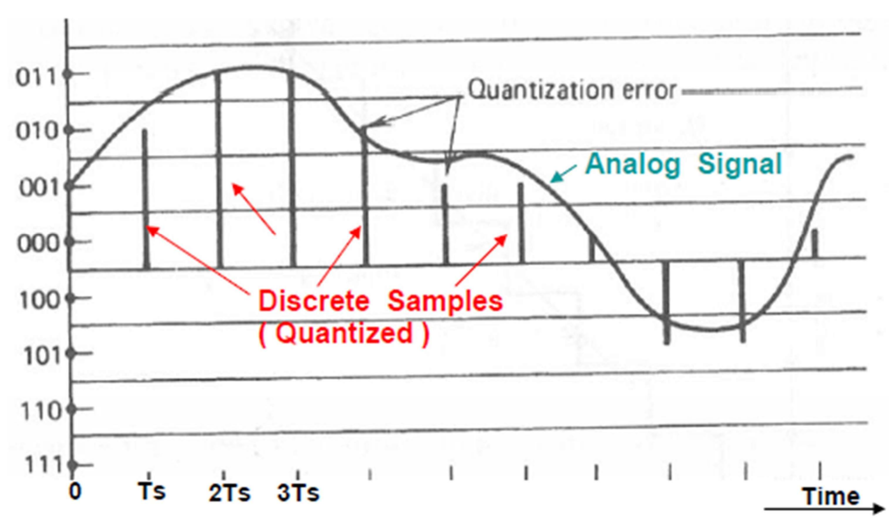


Fig. 3.12: Qantization

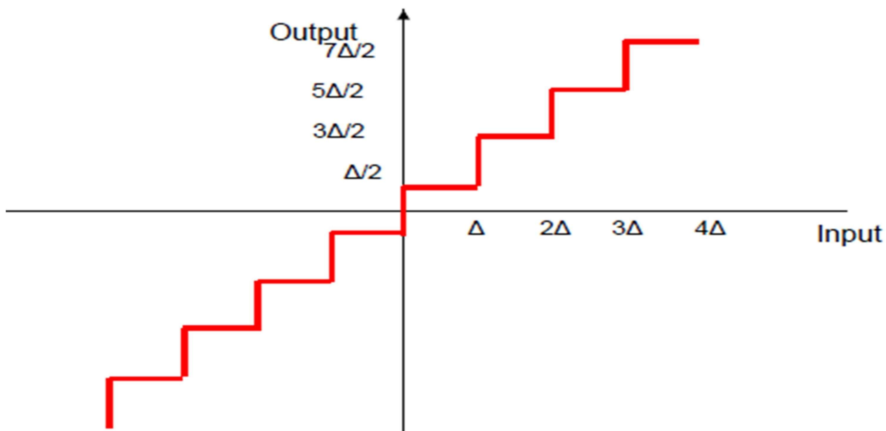
In uniform type, the quantization levels are uniformly spaced, whereas in nonuniform type the spacing between the levels will be unequal and mostly the relation is logarithmic.

Types of Uniform Quantizers: (based on I/P - O/P Characteristics) 1. Mid-Rise type Quantizer

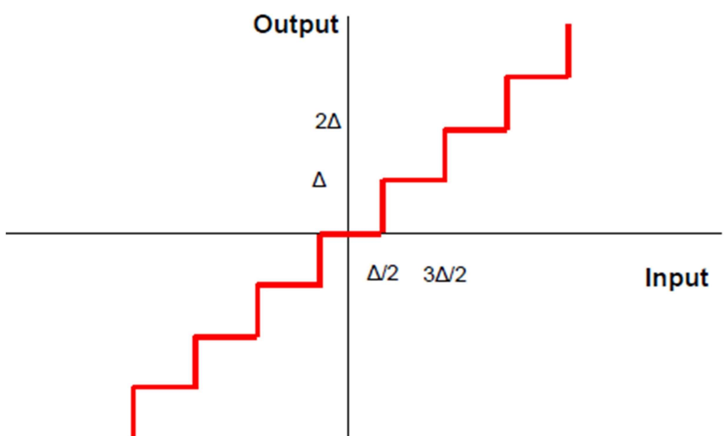
2. Mid-Tread type Quantizer

In the stair case like graph, the origin lies the middle of the tread portion in Mid –Tread type where as the origin lies in the middle of the rise portion in the Mid-Rise type.
Mid – tread type: Quantization levels – odd number.

Mid – Rise type: Quantization levels – even number.

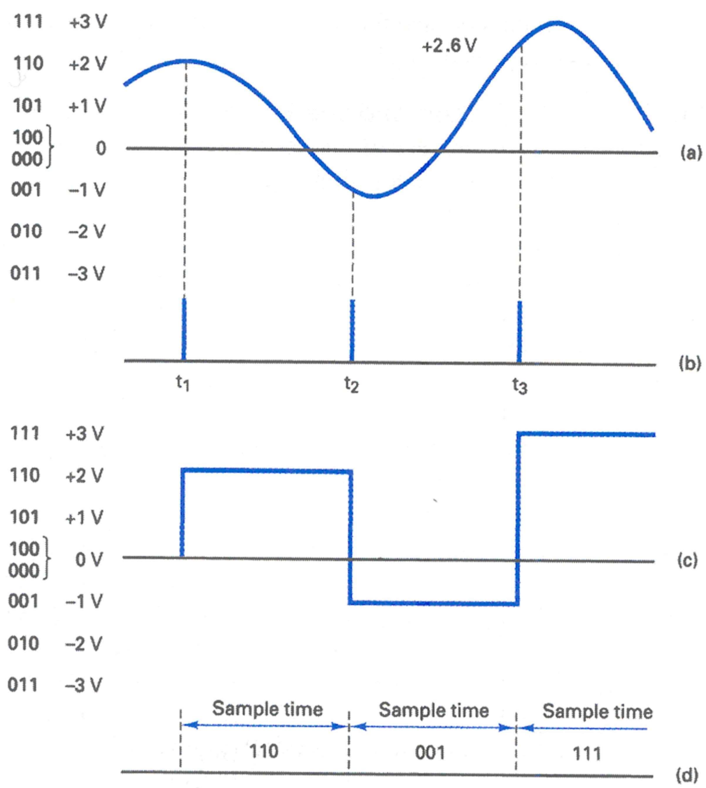


Midrise Type Quantizer



Mid-Tread type Quantizer

Fig: 3.13 Mid Tread type Quantizer



(a) Analog input signal; (b) sample pulse; (c) PAM signal; (d) PCM code

Fig 3.15: PCM Quantization error:The quantization error occurs because the original signal in a particular step is approximated to a single voltage level hence resulting in quantization error.

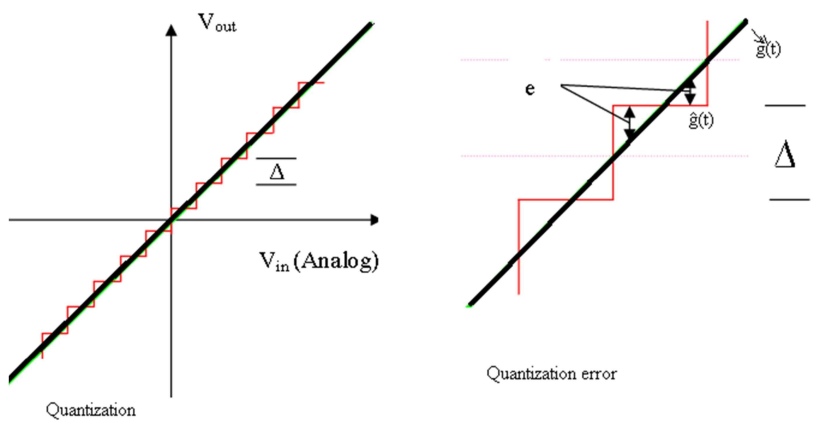


Fig: 3.16 Quantization error

Quantization error is uniformly distributed. Integrates to 1

Probability density function (PDF) Uniformly distributed from

–Δ /2 ... + Δ /2 provided that: Not Gaussian!

$$\sigma_{QNoise}^{2} = E(e^2) = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta} e^2 de = \frac{1}{\Delta}\frac{e^3}{3}\Bigg|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{1}{\Delta}\left[\frac{\left(\frac{\Delta}{2}\right)^3}{3} - \frac{\left(-\frac{\Delta}{2}\right)^3}{3}\right] = \frac{1}{\Delta}\left[\frac{\Delta^3}{24} - \frac{-\Delta^3}{24}\right] = \frac{\Delta^2}{12}$$

………………..3.13

$$\sigma_{QNoise}^{2} = \text{Quantization Noise Power} = \frac{\Delta^2}{12}$$

Δ ………………….…3.14

Signal to Quantization Noise Ratio:

RMS value for a full scale sinusoidal input is

$$V_{MaxSignal\_rms} = \frac{\left(\frac{2^N}{2}\right)}{\sqrt{2}}\Delta$$

………………….…3.15

$$\text{Max SNR} = 20\log\left(\frac{\frac{\left(\frac{2^N}{2}\right)}{\sqrt{2}}\Delta}{\frac{\Delta}{\sqrt{12}}}\right) = 20\log\left(\frac{\sqrt{6}}{2}2^N\right) = 20\log\left(\frac{\sqrt{6}}{2}\right) + 20N\log(2)$$

………3.16

= 1.76+6.02N dB ………………………………………………………………..3.17

Companding

Companding is the process of compressing and then expanding high amplitude analog signals are compressed prior to transmission and then expanded in the receiver. Higher amplitude analog signals are compressed and

Dynamic range is improved. Early PCM systems used analog companding, where as modern systems use digital companding.

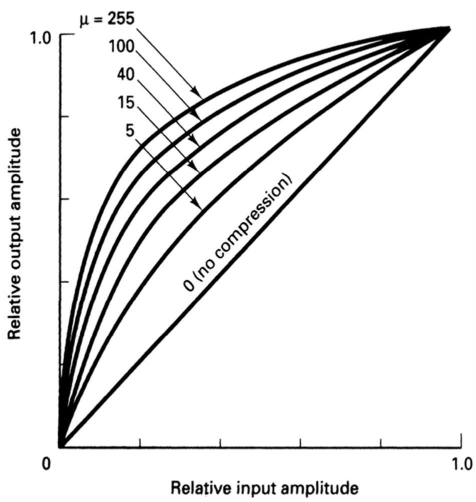
PCM system with analog companding.

In the transmitter, the dynamic range of the analog signal is compressed, and then converted o a linear PCM code. In the receiver, the PCM code is converted to a PAM signal, filtered, and then expanded back to its original dynamic range. There are two methods of analog companding currently being used that closely approximate a logarithmic function and are often called log-PCM codes.

The two methods are 1) μ-law and

2) A-law

μ-law companding



$$V_{out} = \frac{V_{max} \ln\left(1 + \mu V_{in}/V_{max}\right)}{\ln\left(1 + \mu\right)}$$

Fig.3.17: μ-law companding

where Vmax = maximum uncompressed analog input amplitude(volts)

Vin = amplitude of the input signal at a particular instant of time (volts)

μ = parameter used to define the amount of compression (unit less)

Vout = compressed output amplitude (volts)

A-law companding

A-law is superior to μ-law in terms of small-signal quality.

The compression characteristic is given by

where y = Vout x = Vin / Vmax

$$y = \begin{cases} \dfrac{A|x|}{1+\log A}, & 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{1+\log(A|x|)}{1+\log A}, & \dfrac{1}{A} \le |x| \le 1 \end{cases}$$

Delta Modulation

Delta modulation uses a single-bit DPCM code to achieve digital transmission of analog signals. In PCM, each code is a binary representation of both the sign and the magnitude of a particular sample. Therefore, multiple-bit codes are required to represent the many values that the sample can be. Whereas in delta modulation, a coded representation of the sample, only a single bit is transmitted, which simply indicates whether that sample is larger or smaller than the previous sample.

The Delta Modulator comprises of a 1-bit quantizer and a delay circuit along with two summer circuits. Following is the block diagram of a delta modulator.

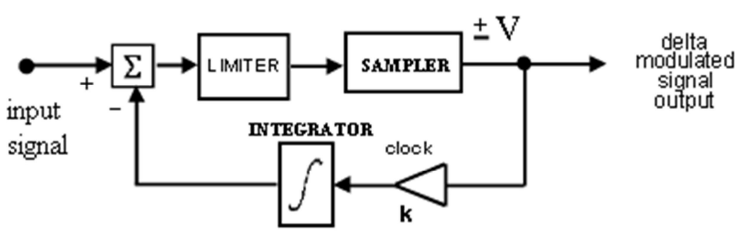


Fig 3.18: Delta Modulator

The operation of a delta modulator is to periodically sample the input message, to make a comparison of the current sample with that preceding it, and to output a single bit which indicates the sign of the difference between the two samples. This in principle would require a sample-and-hold type circuit.If the current sample is smaller than the previous sample, a logic 0 is transmitted. If the current sample is larger than the previous sample, a logic 1 is transmitted.

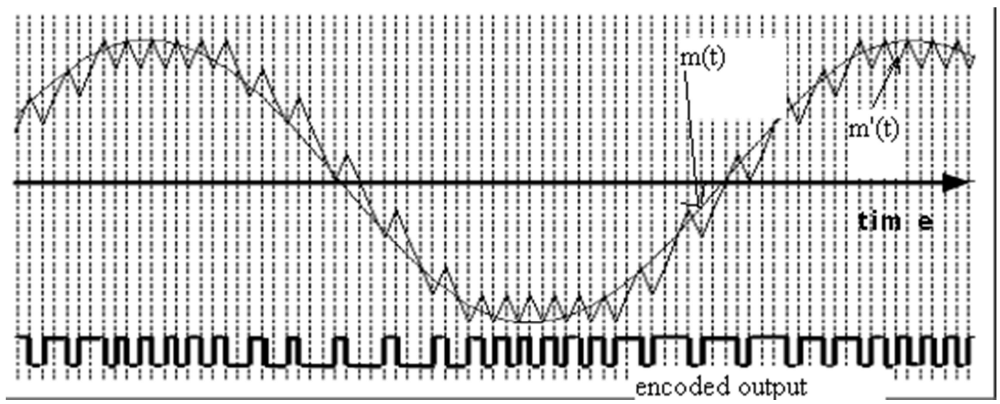


Fig 3.19: Delta Modulator Output Waveforms

DM also suffers from a few limitations such as the following:

Slope overload and granular error:

If the input signal amplitude changes fast, the step-by-step accumulation process may not catch up with the rate of change

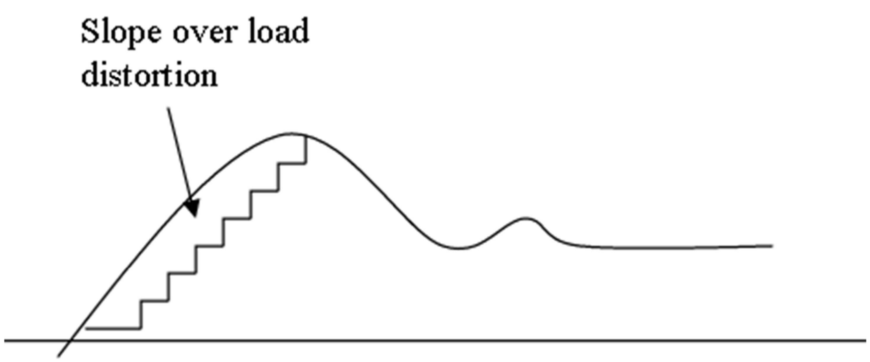


Fig 3.20: Slope Overload Error

Granular noise: If the step-size is made arbitrarily large to avoid slope-overload distortion, it may lead to 'granular noise'. Imagine that the input speech signal is fluctuating but very close to zero over limited time duration. This may happen due to pauses between sentences or else. During such moments, our delta modulator is likely to produce a fairly long sequence of 101010...., reflecting that the accumulator output is close but alternating around the input signal.

Condition for avoiding slope overload: we may observe that if an input signal x(t) changes more than half of the step size (i.e. by 's') within a sampling interval, there will be slope-overload distortion. So, the desired limiting condition on the input signal x(t) for avoiding slope-overloading is,

$$\frac{dx(t)}{dt}\Big|_{max} \le \frac{s}{T_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.13$$

Adaptive Delta Modulation:

A larger step-size is needed in the steep slope of modulating signal and a smaller step size is needed where the message has a small slope. The minute details get missed in the process. So, it would be better if we can control the adjustment of step-size, according to our requirement in order to obtain the sampling in a desired fashion. This is the concept of Adaptive Delta Modulation.

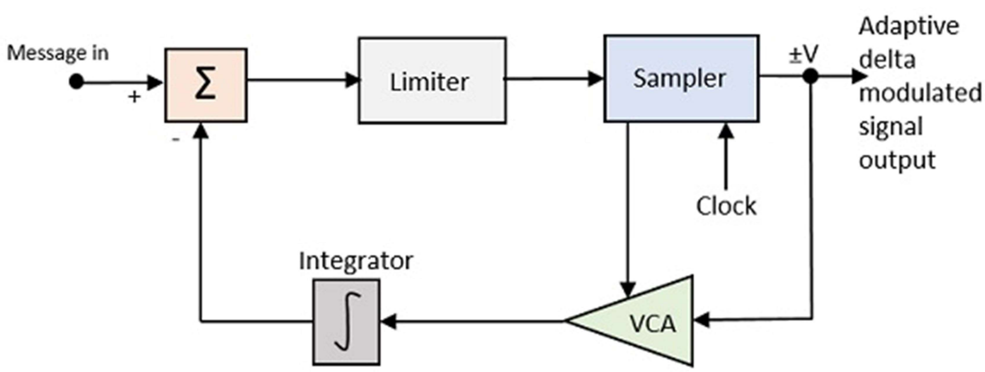


Fig 3.21: ADM Transmitter

The gain of the voltage controlled amplifier is adjusted by the output signal from the sampler. The amplifier gain determines the step-size and both are proportional.

ADM quantizes the difference between the value of the current sample and the predicted value of the next sample. It uses a variable step height to predict the next values, for the faithful reproduction of the fast varying values.

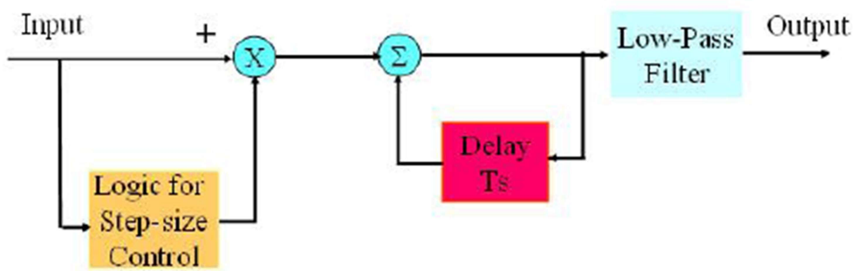
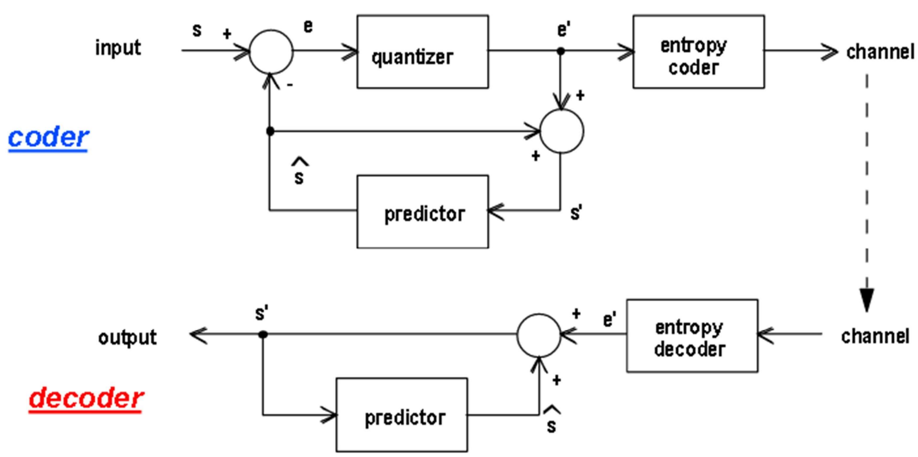


Fig 3.22: ADM Receiver

Differential PCM

With Differential Pulse Code Modulation (DPCM), the difference in the amplitude of two successive samples is transmitted rather than the actual sample. Because the range of sample differences is typically less than the range of individual samples, fewer bits are required for DPCM than conventional PCM.



3.23: Differential PCM

Multiple Choice Questions:

1) Which of the modulation technique is digital in nature.

a) PAM b) PPM c) DM d) None of these.

2) Quantisation noise occurs in

a) PAM b)PWM c) DM d) none of these

3) The standard data rate of a PCM voice channel is

a) 8kbps b) 8bps c) 16kbps d) 64kbps

4) PAM signal can be detected using the

a) LPF b) BPF c) HPF d) bandstop filter

5) The combination of compressor and expander is called as……..

6) The non uniformquantisation leads to

a) reduction in transmission bandwidth

b) increase in max SNR

c) increase in SNR for low level signals

d) simplification of quantisation process

7) Eye pattern is used to study

a) Bit Error Rate

b) Error Vector Magnitude

c) Quantization Noise

e) Inter Symbol Interference.

8) Alternate Mark Inversion (AMI) signalling is also known as
a) Bipolar signalling b) Manchester signalling
c) Polar signalling d) Unipolar signalling.

9) Nyquist Rate for sampling a signal represented by x(t) = sine(200t) +sine(400t)

a) 200 b) 400 c) 300 d) 250

10) Companding is used

a. to overcome quantizing noise in PCM

b. in PCM transmitters, to allow amplitude limited in the receivers

c. to protect small signals in PCM from quantizing distortion

d. in PCM receivers, to overcome impulse noise

11) Nyquist Rate for sampling a signal represented by x(t) = sine(200t)sine(400t)

a) 200 b) 400 c) 300 d) 600

Sample Questions

1) Explain Quantisation Error. How does the quantisation error depend on the step Size?
2) State the sampling theorem. Explain how two PAM signals can be Time division multiplexed.
3) Explain companding. State the need for companding.
4) State and prove the sampling theorem
5) Can we state that Delta Modulation is a 1 bit DPCM technique. Justify

Module-IV

Digital Data Transmission

Match Filter

Receiver Model:

In our simple model, the signal □ (□), denoting one the two possible received signals □ (□) and □ (□) is processed through a filter and then sampled at time□ . The received signal is corrupted by noise which also passes through the filter and corrupts the sample value which is thus □ = □ˆ(□) + □(□).

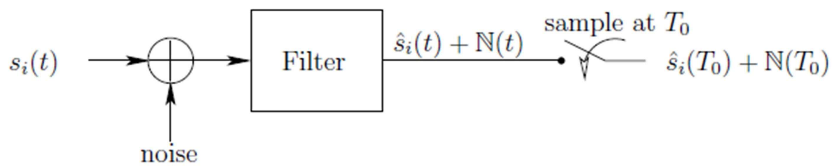


Fig. 4.1: Block diagram of a baseband receiver

The pdf of □(□) does not depend on whether □ (□) or □ (□) is received, that is, the noise is assumed to be independent of the received signal. A good model for □(□) is a zero-mean Gaussian random variable with variance □ . The min-max decision rule is the same as the maximum-likelihood decision rule is the same as the minimum-error-probability decision rule (since we are assuming equally likely signals) and consists of deciding that a 0 or a 1 was transmitted according as |□ − □ˆ(□)|is smaller than or larger than |□ − □ˆ(□)|. Equivalently, assuming that□ˆ(□) > □ˆ(□), the receiver compares the sample value □ to a threshold □ =

ˆ( ) ˆ( )

___________ and decides that a 0 or a 1 was transmitted according as □ is larger than or smaller

ˆ( )ˆ( ) than □. The error probability □ achieved by this decision rule is □ __________ = □(□□□) where SNR (signal-to-noise ratio) is the value of the argument of the □(.) function. Note that□ decreases rapidly as SNR increases. It is also worth noting that there are many different definitions for SNR that are in common use, and thus, care must be taken in comparing systems from different designers since they be defining SNR differently. However, regardless of the exact definition of SNR, □ decreases as SNR increases in a properly designed system, that is, everyone is in agreement that increasing SNR is a good thing to do.

The noise in the system is almost always referred to as channel noise though most of it actually arises in the front end of the receiver. The random motion of electrons in the electrical conductors comprising the front end of the receiver creates small time-varying voltages - referred to as thermal noise voltages - that are on the order of a few micro-volts or so. Based on experimental evidence, the thermal noise is modelled as a stationary Gaussian random process. It suffices to note that if we sample the noise at the receiver input at any time instant, then an reasonable model for this noise sample is a zero-mean Gaussian random variable whose variance is the same regardless of the choice of time instant. The reader may then wonder why it is necessary to use a filter as shown in the above figure of the receiver model. Why not just sample at the receiver input itself and make a decision based on that sample value? After all, the sample will be □ (□)(instead of□ˆ(□)) plus a Gaussian noise variable. Unfortunately, in many instances, the noise voltages can be of the same order of magnitude as (or even considerably larger than) the voltages created by the received signals, especially when the transmitter is far away, or is restricted in transmitter power. Thus, the error probability can be unacceptably large if we were to make a decision based on a sample taken at the receiver input. But, can filtering ameliorate this situation? In almost all instances, it can. Informally speaking, the noise at the receiver input is broadband noise in comparison to which the received signals are narrowband signals.1 Thus, even a simple band-pass filter which passes the signals□ (□) and □ (□) (and the in-band noise) unchanged while eliminating the out of-band noise will reduce the noise variance considerably, and thereby reduce the error probability achieved. The use of a filter can be beneficial even when the received signals are strong enough that sampling at the receiver input gives acceptably small error probability. In such a case, the filter can be used not as a device for improving the error probability from acceptably small to fantastically small, but rather as a costeffective way of achieving the same acceptably small error probability while increasing the spatial distance over which the communication system can operate, or reducing the transmitter power which in turn can have additional side benefits such as a reduction in the size and weight of the transmitter, an increase in battery life, etc. Finally, for those still unconvinced of the utility of filtering before sampling, consider that it is universal practice to amplify the receiver input before any sampling is done, and for reasons of power efficiency and ease of implementation, amplifiers also act as band-pass filters. The reason for amplification is that it is much easier to design a circuit that triggers when its input exceeds 0.25V (where the threshold may vary ±0:01V (say) due to manufacturing process variations or circuit component tolerances) than it is to design a circuit that triggers at a threshold of 0:25 ±0:01 □□. Note that since □ is determined by the ratio of signal level to noise level, and the amplifier increases the signal level and the noise level by the same factor, amplification by itself does not change the error probability—but amplification does make implementation of the sampler, A/D converter, threshold device etc. a lot easier. In summary, digital communication receivers amplify and filter the received signals (plus noise) before sampling and making a decision as to which bit was transmitted. Since the analysis of error probability is unaffected by the amplifier gain, we do not include amplifier gain explicitly in our analysis, though we do incorporate the filtering in the amplifier(s)[2] into the filter shown in the figure above. One other consequence of amplification is important, and simplifies our analysis. Remember that thermal noise is present in all the electrical conductors in the amplifier/filter combination and not just in those in the front end of the receiver. However, because of the amplification that the thermal noise present in the front end of the receiver undergoes as it passes through the amplifier/filter combination, this amplified noise from the front end is the predominant noise present in the output of the amplifier/filter combination, completely swamping out the few micro-volts contribution to the noise from the electrical conductors in the output impedance of the amplifier/filter. In other words, for all practical purposes, the noise □(□) shown in the figure above can be taken to be the result of filtering the noise process in the front end of the receiver. Notice that such an assumption would not be valid in the absence of amplification since the thermal noise generated at the filter output would be comparable in magnitude to that passing through the filter and appearing at its output. Finally, following convention, we blame it all on the channel and say that the noise □(□) at the filter output is the result of filtering the Gaussian channel noise process. The channel itself is called a Gaussian noise channel.

Having justified the need for the filter shown in our model, let us consider what kind of filter we should use. For a Gaussian noise channel, the smallest error probability that can possibly be achieved with given received signals □ (□) and □ (□) is achievable by using a suitable linear filter, that is, a linear time-invariant system. Call this smallest achievable error probability □□. The optimum linear filter that achieves □□ is called a matched filter for signals □ (□) and□ (□). As the name implies, different signal sets have different matched filters (and achieve different minimum error probabilities). For given signals □ (□) and□ (□), the use of a linear filter not matched to them will result in a receiver with □ > □□. Notice also that the claim does not mean that a receiver with a nonlinear filter cannot achieve error probability□□: it might well do so, but so can the linear matched filter receiver achieve error probability□□ . What the claim does mean is that no receiver, whether using a linear filter or a nonlinear filter, can achieve an error probability smaller than □□ . That is the least error probability that we can achieve with signals □ (□) and□ (□), and we can achieve it with a linear (matched filter) receiver. Looking at receivers with nonlinear filters in the hope of getting error probability smaller than□□ is futile.

Restricting ourselves to linear filters, what can be said about the noise process at the filter output? Since the channel noise process is a zero-mean stationary Gaussian random process that passes through the filter, the noise process at the filter output is also a zero-mean stationary Gaussian random process. Thus, regardless of the choice of sampling instant□ , □(□)is a zeromean Gaussian random variable with fixed variance □ .

Line Coding

A line code is the code used for data transmission of a digital signal over a transmission line. This process of coding is chosen so as to avoid overlap and distortion of signal such as intersymbol interference.

Properties of Line Coding

Following are the properties of line coding −

- As the coding is done to make more bits transmit on a single signal, the bandwidth used is much reduced.
- For a given bandwidth, the power is efficiently used.
- The probability of error is much reduced.
- Error detection is done and the bipolar too has a correction capability.
- Power density is much favourable.
- The timing content is adequate.
- Long strings of 1s and 0s are avoided to maintain transparency.

Types of Line Coding

There are 3 types of Line Coding

- Unipolar
- Polar
- Bi-polar

Unipolar Signaling

Unipolar signaling is also called as On-Off Keying or simply OOK.

The presence of pulse represents a 1 and the absence of pulse represents a 0.

There are two variations in Unipolar signaling −

- Non Return to Zero (NRZ)
- Return to Zero (RZ)

Unipolar Non-Return to Zero (NRZ)

In this type of unipolar signaling, a High in data is represented by a positive pulse called as Mark, which has a duration T_0 equal to the symbol bit duration. A Low in data input has no pulse.

The following figure clearly depicts this.

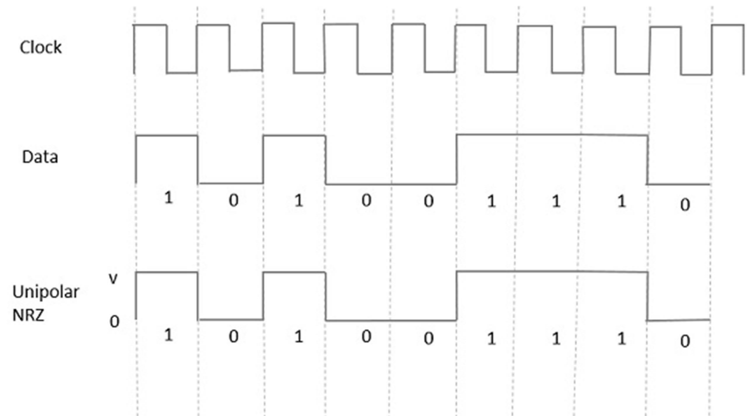


Fig. 4.2(a): Unipolar-NRZ format

Advantages

The advantages of Unipolar NRZ are − □

It is simple.

- A lesser bandwidth is required.

Disadvantages

The disadvantages of Unipolar NRZ are − □

No error correction done.

- Presence of low frequency components may cause the signal droop.
- No clock is present.
- Loss of synchronization is likely to occur (especially for long strings of 1s and 0s).

Unipolar Return to Zero (RZ)

In this type of unipolar signaling, a High in data, though represented by a Mark pulse, its duration T_0 is less than the symbol bit duration. Half of the bit duration remains high but it immediately returns to zero and shows the absence of pulse during the remaining half of the bit duration.

It is clearly understood with the help of the following figure.

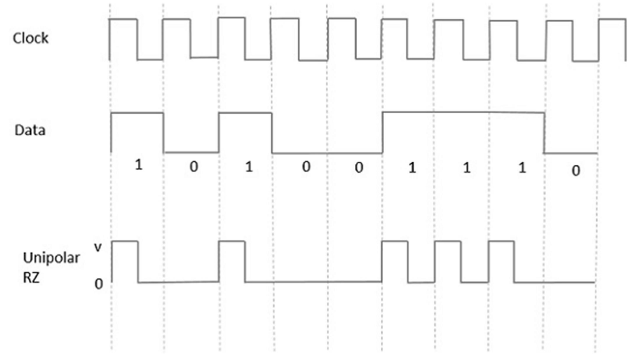


Fig. 4.2(b): Unipolar-RZ format

Advantages

The advantages of Unipolar RZ are −

- It is simple.
- The spectral line present at the symbol rate can be used as a clock.

Disadvantages

The disadvantages of Unipolar RZ are −

- No error correction.
- Occupies twice the bandwidth as unipolar NRZ.
- The signal droop is caused at the places where signal is non-zero at 0 Hz.

Polar Signaling

There are two methods of Polar Signaling. They are −

- Polar NRZ
- Polar RZ

Polar NRZ

In this type of Polar signaling, a High in data is represented by a positive pulse, while a Low in data is represented by a negative pulse. The following figure depicts this well.

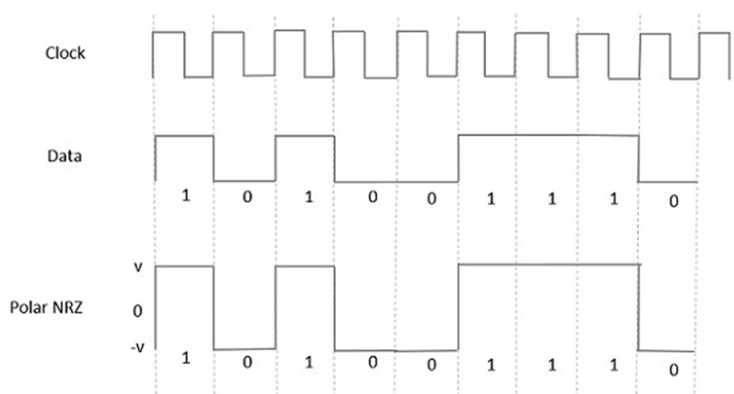


Fig. 4.3(a): Polar-NRZ format

Advantages

The advantages of Polar NRZ are −

It is simple.

- No low-frequency components are present.

Disadvantages

The disadvantages of Polar NRZ are −

No error correction.

- No clock is present.
- The signal droop is caused at the places where the signal is non-zero at 0 Hz.

Polar RZ

In this type of Polar signalling, a High in data, though represented by a Mark pulse, its duration T_0 is less than the symbol bit duration. Half of the bit duration remains high but it immediately returns to zero and shows the absence of pulse during the remaining half of the bit duration.

However, for a Low input, a negative pulse represents the data, and the zero level remains same for the other half of the bit duration. The following figure depicts this clearly.

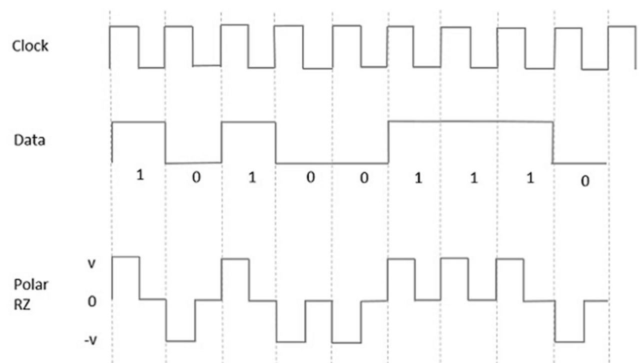


Fig. 4.3(b): Polar-RZ format

Advantages

The advantages of Polar RZ are −

- It is simple.
- No low-frequency components are present.

Disadvantages

The disadvantages of Polar RZ are −

No error correction.

- No clock is present.
- Occupies twice the bandwidth of Polar NRZ.
- The signal droop is caused at places where the signal is non-zero at 0 Hz.

Bipolar Signaling

This is an encoding technique which has three voltage levels namely +, - and 0. Such a signal is called as duo-binary signal.

An example of this type is Alternate Mark Inversion (AMI). For a 1, the voltage level gets a transition from + to – or from – to +, having alternate 1sto be of equal polarity. A 0 will have a zero voltage level.

Even in this method, we have two types.

- Bipolar NRZ
- Bipolar RZ

From the models so far discussed, we have learnt the difference between NRZ and RZ. It just goes in the same way here too. The following figure clearly depicts this.

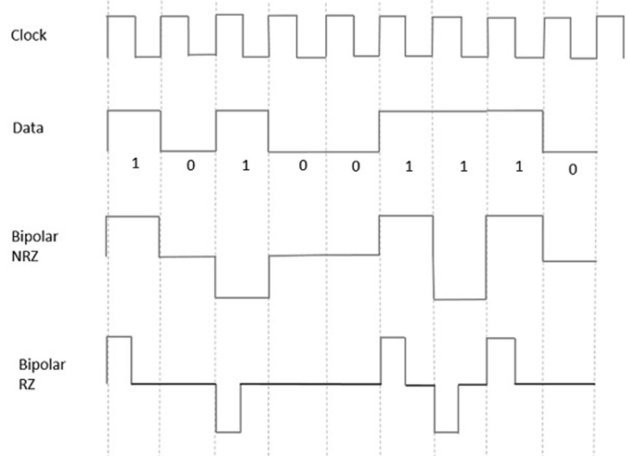


Fig. 4.4: Bipolar-RZ and Bipolar-NRZ format

The above figure has both the Bipolar NRZ and RZ waveforms. The pulse duration and symbol bit duration are equal in NRZ type, while the pulse duration is half of the symbol bit duration in RZ type.

Advantages

Following are the advantages − □

It is simple.

- No low-frequency components are present.
- Occupies low bandwidth than unipolar and polar NRZ schemes.
- This technique is suitable for transmission over AC coupled lines, as signal drooping doesn't occur here.
- A single error detection capability is present in this.

Disadvantages

Following are the disadvantages − □

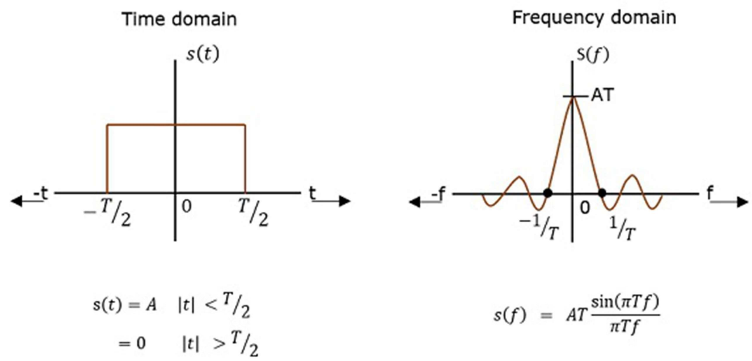
No clock is present.

- Long strings of data causes loss of synchronization.

Power Spectral Density

The function which describes how the power of a signal got distributed at various frequencies, in the frequency domain is called as Power Spectral Density (PSD).

PSD is the Fourier Transform of Auto-Correlation (Similarity between observations). It is in the form of a rectangular pulse.



PSD Derivation

According to the Einstein-Wiener-Khintchine theorem, if the auto correlation function or power spectral density of a random process is known, the other can be found exactly.

Hence, to derive the power spectral density, we shall use the time auto-correlation □ (□) of a power signal □(□) as shown below.

$$□ (□) = \frac{1}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} \lim □(□)\, □(□ + □)□□ \rightarrow$$

Since □(□)consists of impulses, □ (□) can be written as

$$□ (□) = \frac{1}{□} \quad □ \; □(□ - □□)$$

Where □ = lim → _∑ □ □

Getting to know that □ = □ for real signals, we have

$$□ (□) = \frac{1}{□} \; □ + 2 \quad □ \; \cos □□□$$

Since the pulse filter has the spectrum of (w)↔f(t), we have

$$□ (□) = |□(□)| \; □ (□)$$

$$= \frac{|□(□)|}{□} \quad □ \; □$$

$$= \frac{|□(□)|}{□} \quad □ \quad + 2 \quad □ \cos □□□$$

Hence, we get the equation for Power Spectral Density. Using this, we can find the PSD of various line codes.

Bi-phase Encoding

The signal level is checked twice for every bit time, both initially and in the middle. Hence, the clock rate is double the data transfer rate and thus the modulation rate is also doubled. The clock is taken from the signal itself. The bandwidth required for this coding is greater.

There are two types of Bi-phase Encoding.

- Bi-phase Manchester
- Differential Manchester

Bi-phase Manchester

In this type of coding, the transition is done at the middle of the bit-interval. The transition for the resultant pulse is from High to Low in the middle of the interval, for the input bit 1. While the transition is from Low to High for the input bit 0.

Differential Manchester

In this type of coding, there always occurs a transition in the middle of the bit interval. If there occurs a transition at the beginning of the bit interval, then the input bit is 0. If no transition occurs at the beginning of the bit interval, then the input bit is 1.

The following figure illustrates the waveforms of NRZ-L, NRZ-I, Bi-phase Manchester and Differential Manchester coding for different digital inputs.

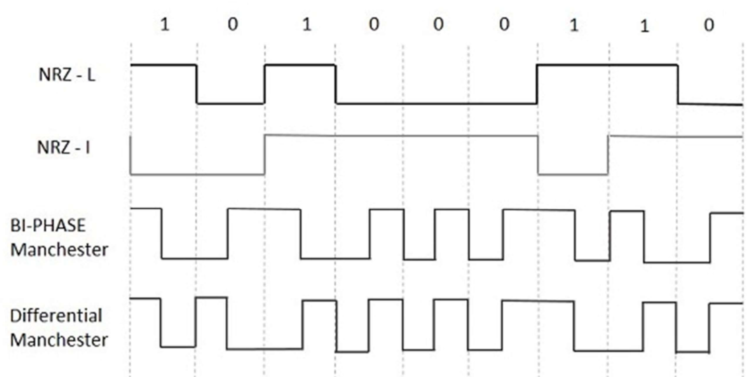


Fig. 4.5: Manchester coding format

After going through different types of coding techniques, we have an idea on how the data is prone to distortion and how the measures are taken to prevent it from getting affected so as to establish a reliable communication.

There is another important distortion which is most likely to occur, called as Inter-Symbol Interference (ISI).

Inter Symbol Interference

This is a form of distortion of a signal, in which one or more symbols interfere with subsequent signals, causing noise or delivering a poor output.

Causes of ISI

The main causes of ISI are − □

Multi-path Propagation

□ Non-linear frequency in channels

The ISI is unwanted and should be completely eliminated to get a clean output. The causes of ISI should also be resolved in order to lessen its effect.

To view ISI in a mathematical form present in the receiver output, we can consider the receiver output.

The receiving filter output □(□) is sampled at time □ = □□ (with i taking on integer values), yielding –

□(□) = □ □ □(□□ − □□) = □□ + □ □ □(□□ − □□)

In the above equation, the first term □□ is produced by the i^{th} transmitted bit.

The second term represents the residual effect of all other transmitted bits on the decoding of the i^{th} bit. This residual effect is called as Inter Symbol Interference.

In the absence of ISI, the output will be −

□(□) = □□

This equation shows that the i^{th} bit transmitted is correctly reproduced. However, the presence of ISI introduces bit errors and distortions in the output.

While designing the transmitter or a receiver, it is important that you minimize the effects of ISI, so as to receive the output with the least possible error rate.

Correlative Coding

So far, we've discussed that ISI is an unwanted phenomenon and degrades the signal. But the same ISI if used in a controlled manner, is possible to achieve a bit rate of 2W bits per second in a channel of bandwidth W Hertz. Such a scheme is called as Correlative Coding or Partial response signaling schemes.

Since the amount of ISI is known, it is easy to design the receiver according to the requirement so as to avoid the effect of ISI on the signal. The basic idea of correlative coding is achieved by considering an example of Duo-binary Signalling.

Duo-binary Signalling

The name duo-binary means doubling the binary system's transmission capability. To understand this, let us consider a binary input sequence $\{a_k\}$ consisting of uncorrelated binary digits each having a duration T_a seconds. In this, the signal 1 is represented by a +1 volt and the symbol 0 by a -1 volt. Therefore, the duo-binary coder output c_k is given as the sum of present binary digit a_k and the previous value a_{k-1} as shown in the following equation.

□ = □ + □

The above equation states that the input sequence of uncorrelated binary sequence $\{a_k\}$ is changed into a sequence of correlated three level pulses $\{c_k\}$. This correlation between the pulses may be understood as introducing ISI in the transmitted signal in an artificial manner.

Eye Pattern

An effective way to study the effects of ISI is the Eye Pattern. The name Eye Pattern was given from its resemblance to the human eye for binary waves. The interior region of the eye pattern is called the eye opening. The following figure shows the image of an eye-pattern.

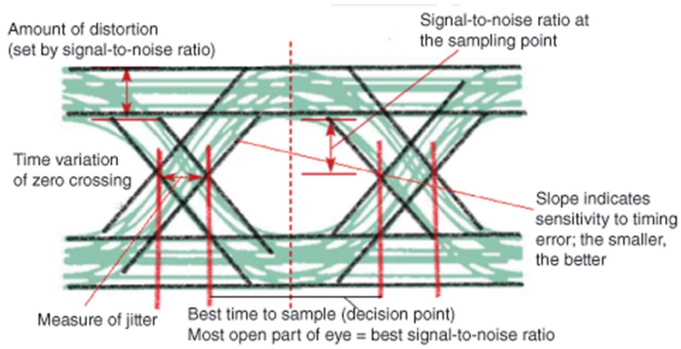


Fig. 4.6: Eye pattern

Jitter is the short-term variation of the instant of digital signal, from its ideal position, which may lead to data errors.

When the effect of ISI increases, traces from the upper portion to the lower portion of the eye opening increases and the eye gets completely closed, if ISI is very high.

An eye pattern provides the following information about a particular system.

- Actual eye patterns are used to estimate the bit error rate and the signal-to-noise ratio.
- The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI.
- The instant of time when the eye opening is wide, will be the preferred time for sampling.
- The rate of the closure of the eye, according to the sampling time, determines how sensitive the system is to the timing error.
- The height of the eye opening, at a specified sampling time, defines the margin over noise. Hence, the interpretation of eye pattern is an important consideration.

Equalization

For reliable communication to be established, we need to have a quality output. The transmission losses of the channel and other factors affecting the quality of the signal, have to be treated. The most occurring loss, as we have discussed, is the ISI.

To make the signal free from ISI, and to ensure a maximum signal to noise ratio, we need to implement a method called Equalization. The following figure shows an equalizer in the receiver portion of the communication system.

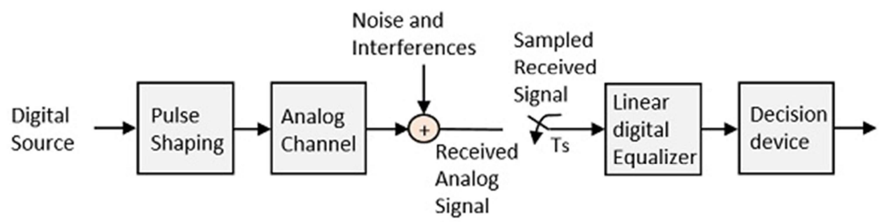


Fig. 4.7: Block diagram of an equalizer

The noise and interferences which are denoted in the figure, are likely to occur, during transmission. The regenerative repeater has an equalizer circuit, which compensates the transmission losses by shaping the circuit. The Equalizer is feasible to get implemented.

Error Probability and Figure-of-merit

The rate at which data can be communicated is called the data rate. The rate at which error occurs in the bits, while transmitting data is called the Bit Error Rate (BER).

The probability of the occurrence of BER is the Error Probability. The increase in Signal to Noise Ratio (SNR) decreases the BER, hence the Error Probability also gets decreased.

In an Analog receiver, the figure of merit at the detection process can be termed as the ratio of output SNR to the input SNR. A greater value of figure-of-merit will be an advantage.

Multiple choice questions (MCQs)

i) Eye pattern is used to study

a) Bit error rate b) Error vector magnitude
c) Quantization noise d) Inter Symbol Interference

(ii) Alternate Mark Inversion (AMI) signaling is known as

a) Bipolar signaling b) Polar signaling
c) Manchester signaling d) Unipolar signaling

(iii) A rectangular pulse of duration T is applied to matched filter. The output of the filter is a

a) Rectangular pulse of duration T b) Rectangular pulse of duration 2T
c) Triangular pulse d) Sine function

(iv) The spectral density of white noise is

a) Exponential b) Uniform c) Poisson d) Gaussian

(v) In manchester code, the symbol rate□ and data rate □ are related as:

(a) □ = 2 □ (b) □ = 2 □ (c) □ = □/2 (d) □ = □

(vi) Transversal equalizer uses tapped delay lines to

(a) reduce ISI (b) reduce BER
c) increase bit rate (d) increase band widths

Sample Questions:

1) What is an Integrate and Dump filter? Derive the expression of its Signal to noise Ratio.
2) Derive the probability of errors for Integrate and Dump filter.
3) What is an Optimum filter? Derive the probability of errors for an Optimum filter.
4) Derive the transfer function of an Optimum filter for digital reception with suitable assumptions.

5) What do you mean by match filter?
6) Prove that the SNR at the output of a matched filter is 8Es/□. Where Es is the signal energy and □/2 = Gn (f), for white gaussian noise. And hence deduce the transfer function of a matched filter.
7) A polar NRZ waveform has to be received with the help of a matched filter. Here a rectangular positive pulse represents binary one and a rectangular negative pulse represents binary zero. Determine the impulse response of the matched filter with proper sketch.
8) Explain the equivalence between the matched filter receiver and correlation receiver.
9) Why do we need to use the discrete PAM formats? Write the properties of Line Coding.
10) What is the difference between source coding and line coding?
11) Given the data stream 1110010100.Sketch the transmitted sequence of rectangular pulses for each of the following line codes: a) Unipolar NRZ, b) Unipolar RZ, c) Polar RZ, d) Polar NRZ e) Bipolar NRZ and f) Manchester coding.
12) Determine the bandwidth of bipolar format from the PSD. What are the advantages and disadvantages of bipolar signaling format? -Explain.
13) Sketch the PSD of PNRZ and PRZ and determine its bandwidth. What are the advantages and disadvantages of Polar signaling format? -Explain.
14) Write short note on High density bipolar signaling.
15) What is intersymbol interference(ISI)? 16) What is Nyquist criterion for zero ISI?
17) What are the limitations of Nyquist pulse? How is it solved using Raised Cosine pulse?
18) What is the roll of an equalizer?
19) A communication channel of bandwidth 75 kHz is required to transmit binary data at a rate of 0.1 Mbps using raised cosine pulses. Determine the roll-off factor.

What is Eye pattern? How is it generated in CRO? What information we get from it?

Module-V

Digital Modulation Techniques:

5.1 Digital Modulation:

Digital modulation used in modern communication systems is the process of impressing the data to be transmitted on the radio carrier. Most wireless transmissions today are digital, and with the limited spectrum available, the type of modulation is more critical than it has ever been.

The main goal of modulation today is to squeeze as much data into the least amount of spectrum possible. That objective, known as spectral efficiency, measures how quickly data can be transmitted in an assigned bandwidth. The unit of measurement is bits per second per Hz (b/s/Hz). Multiple techniques have emerged to achieve and improve spectral efficiency.

The most fundamental digital modulation techniques are based on keying:

1. PSK (phase-shift keying): a finite number of phases are used.
2. FSK (frequency-shift keying): a finite number of frequencies are used.
3. ASK (amplitude-shift keying): a finite number of amplitudes are used.

5.2 Coherent and non-coherent ASK, FSK and PSK:

Amplitude Shift Keying (ASK) is a type of Amplitude Modulation which represents the binary data in the form of variations in the amplitude of a signal.Any modulated signal has a high frequency carrier. The binary signal when ASK modulated, gives a zero value for Low input while it gives the carrier output for High input.The following figure represents ASK modulated waveform along with its input.

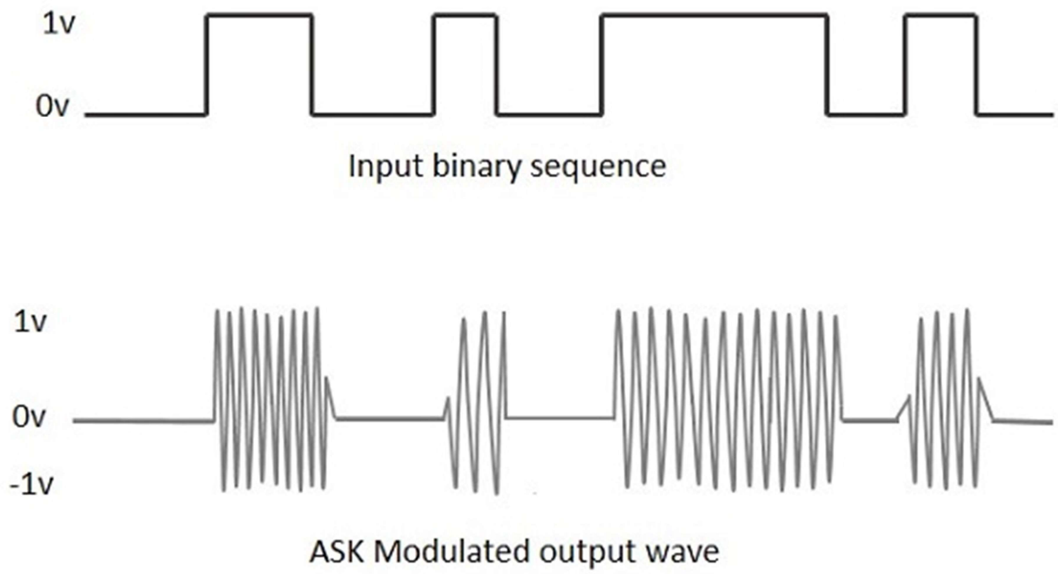


Figure 5.1:

To find the process of obtaining this ASK modulated wave, let us learn about the working of the ASK modulator.

ASK Modulator

The ASK modulator block diagram comprises of the carrier signal generator, the binary sequence from the message signal and the band-limited filter. Following is the block diagram of the ASK Modulator.

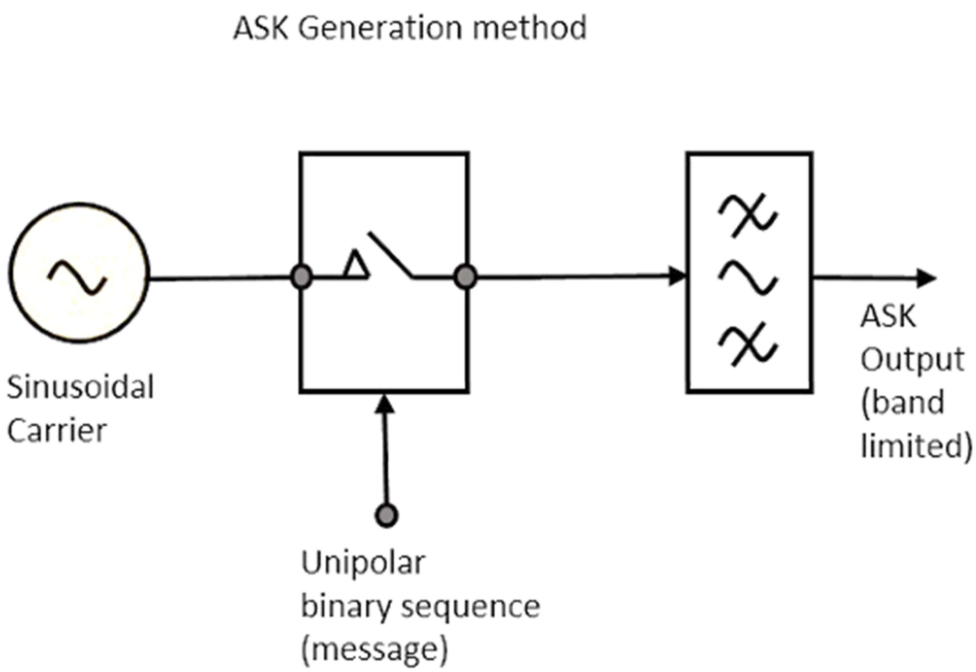


Figure 5.2

The carrier generator, sends a continuous high-frequency carrier. The binary sequence from the message signal makes the unipolar input to be either High or Low. The high signal closes the switch, allowing a carrier wave. Hence, the output will be the carrier signal at high input. When there is low input, the switch opens, allowing no voltage to appear. Hence, the output will be low.

The band-limiting filter, shapes the pulse depending upon the amplitude and phase characteristics of the band-limiting filter or the pulse-shaping filter.

ASK Demodulator

There are two types of ASK Demodulation techniques. They are −

Asynchronous ASK Demodulation/detection

Synchronous ASK Demodulation/detection

The clock frequency at the transmitter when matches with the clock frequency at the receiver, it is known as a Synchronous method, as the frequency gets synchronized. Otherwise, it is known as Asynchronous.

Asynchronous ASK Demodulator

The Asynchronous ASK detector consists of a half-wave rectifier, a low pass filter, and a comparator.

Following is the block diagram for the same.

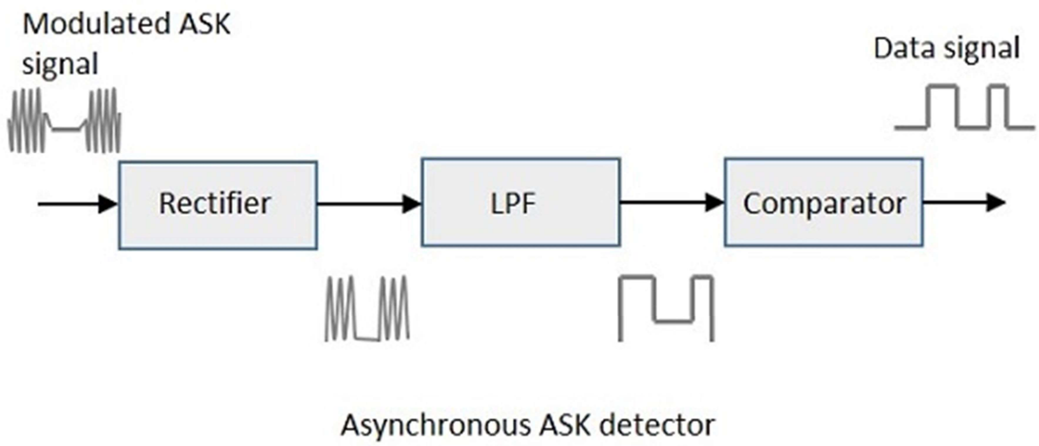


Figure 5.3

The modulated ASK signal is given to the half-wave rectifier, which delivers a positive half output. The low pass filter suppresses the higher frequencies and gives an envelope detected output from which the comparator delivers a digital output.

Synchronous ASK Demodulator

Synchronous ASK detector consists of a Square law detector, low pass filter, a comparator, and a voltage limiter. Following is the block diagram for the same.

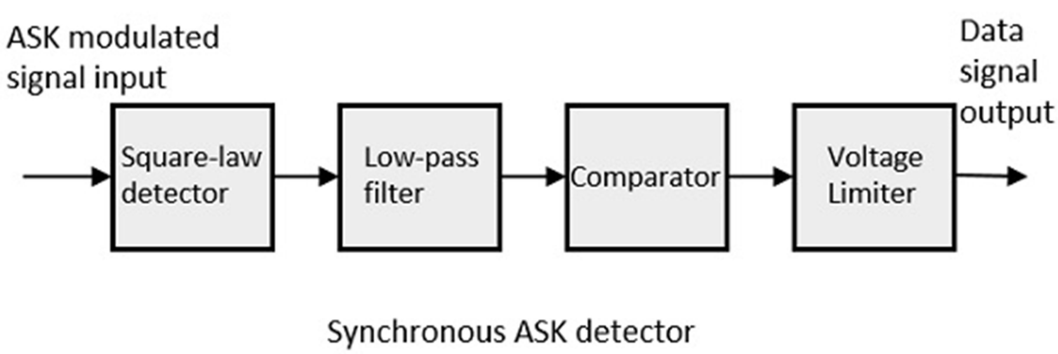


Figure 5.4

The ASK modulated input signal is given to the Square law detector. A square law detector is one whose output voltage is proportional to the square of the amplitude modulated input voltage. The low pass filter minimizes the higher frequencies. The comparator and the voltage limiter help to get a clean digital output.

Frequency Shift Keying (FSK) is the digital modulation technique in which the frequency of the carrier signal varies according to the digital signal changes. FSK is a scheme of frequency modulation.

The output of a FSK modulated wave is high in frequency for a binary High input and is low in frequency for a binary Low input. The binary 1s and 0s are called Mark and Space frequencies.

The following image is the diagrammatic representation of FSK modulated waveform along with its input.

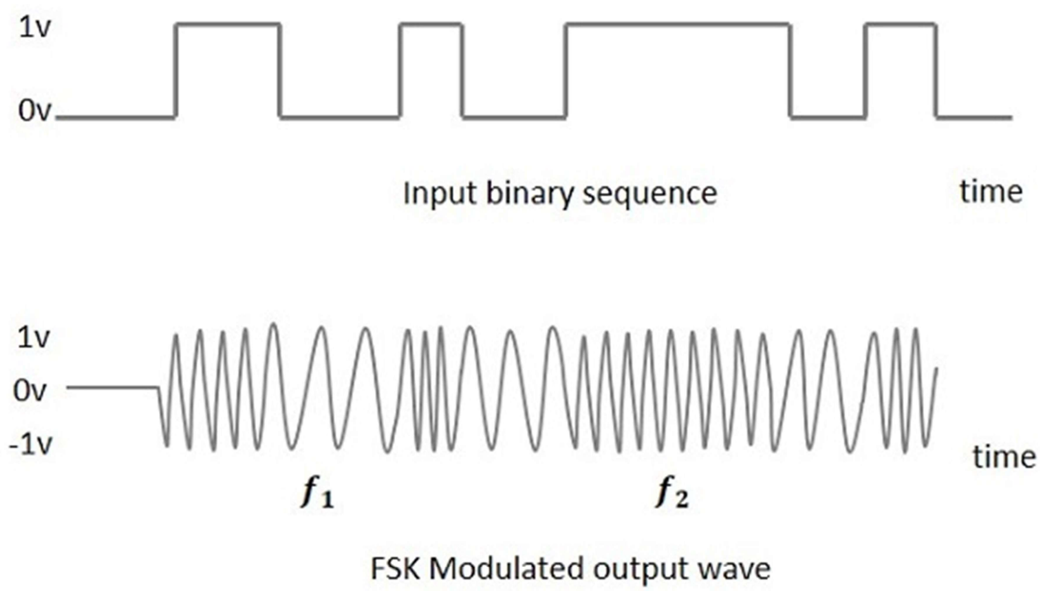


Figure 5.5

To find the process of obtaining this FSK modulated wave, let us know about the working of a FSK modulator.

FSK Modulator

The FSK modulator block diagram comprises of two oscillators with a clock and the input binary sequence. Following is its block diagram.

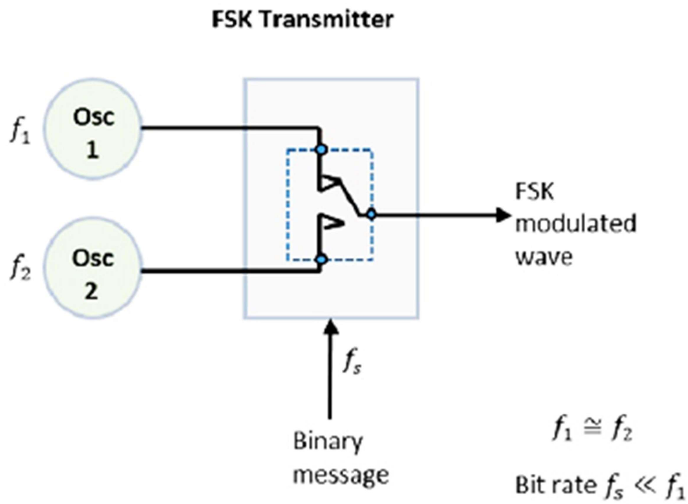


Figure 5.6

The two oscillators, producing a higher and a lower frequency signals, are connected to a switch along with an internal clock. To avoid the abrupt phase discontinuities of the output waveform during the transmission of the message, a clock is applied to both the oscillators, internally. The binary input sequence is applied to the transmitter so as to choose the frequencies according to the binary input.

FSK Demodulator

There are different methods for demodulating a FSK wave. The main methods of FSK detection are asynchronous detector and synchronous detector. The synchronous detector is a coherent one, while asynchronous detector is a non-coherent one.

Asynchronous FSK Detector

The block diagram of Asynchronous FSK detector consists of two band pass filters, two envelope detectors, and a decision circuit. Following is the diagrammatic representation.

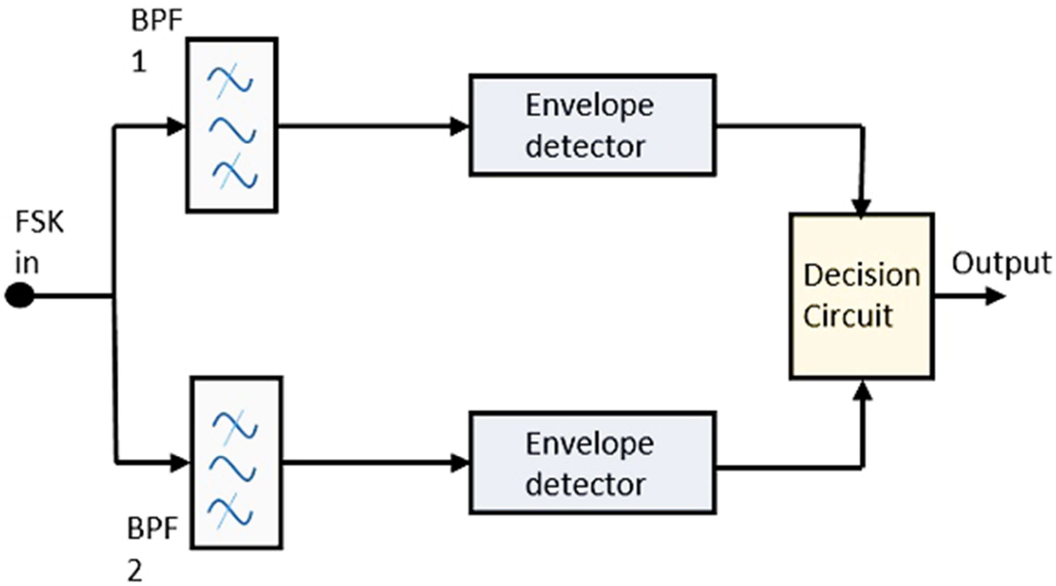


Figure 5.7

The FSK signal is passed through the two Band Pass Filters (BPFs), tuned to Space and Mark frequencies. The output from these two BPFs look like ASK signal, which is given to the envelope detector. The signal in each envelope detector is modulated asynchronously.

The decision circuit chooses which output is more likely and selects it from any one of the envelope detectors. It also re-shapes the waveform to a rectangular one.

Synchronous FSK Detector

The block diagram of Synchronous FSK detector consists of two mixers with local oscillator circuits, two band pass filters and a decision circuit. Following is the diagrammatic representation.

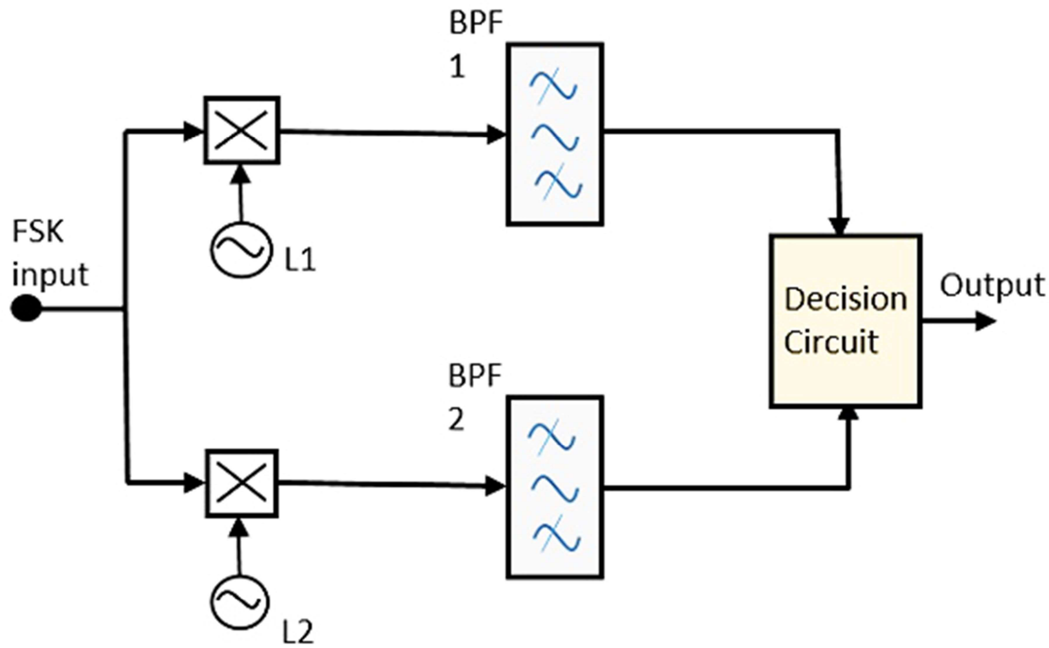


Figure 5.8

The FSK signal input is given to the two mixers with local oscillator circuits. These two are connected to two band pass filters. These combinations act as demodulators and the decision circuit chooses which output is more likely and selects it from any one of the detectors. The two signals have a minimum frequency separation.

For both of the demodulators, the bandwidth of each of them depends on their bit rate. This synchronous demodulator is a bit complex than asynchronous type demodulators.

5.3 Coherent Binary Phase Shift Keying (BPSK),

Phase Shift Keying (PSK) is the digital modulation technique in which the phase of the carrier signal is changed by varying the sine and cosine inputs at a particular time. PSK technique is widely used for wireless LANs, bio-metric, contactless operations, along with RFID and Bluetooth communications.

PSK is of two types, depending upon the phases the signal gets shifted. They are −

Binary Phase Shift Keying (BPSK)

This is also called as 2-phase PSK or Phase Reversal Keying. In this technique, the sine wave carrier takes two phase reversals such as 0° and 180°.

BPSK is basically a Double Side Band Suppressed Carrier (DSBSC) modulation scheme, for message being the digital information.

Quadrature Phase Shift Keying (QPSK)

This is the phase shift keying technique, in which the sine wave carrier takes four phase reversals such as 0°, 90°, 180°, and 270°.

If this kind of techniques are further extended, PSK can be done by eight or sixteen values also, depending upon the requirement.

BPSK Modulator

The block diagram of Binary Phase Shift Keying consists of the balance modulator which has the carrier sine wave as one input and the binary sequence as the other input. Following is the diagrammatic representation.

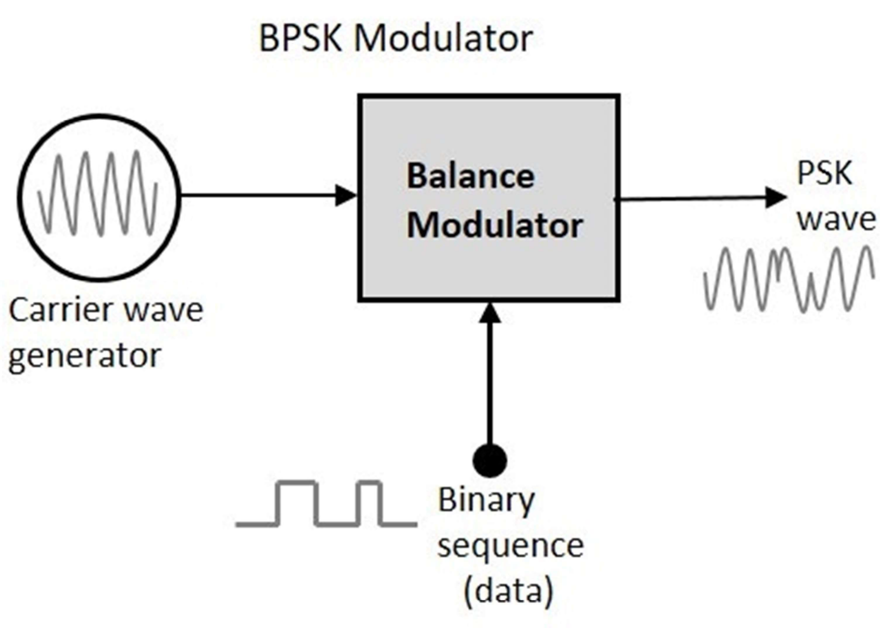


Figure 5.9

The modulation of BPSK is done using a balance modulator, which multiplies the two signals applied at the input. For a zero binary input, the phase will be 0° and for a high input, the phase reversal is of 180°.

Following is the diagrammatic representation of BPSK Modulated output wave along with its given input.

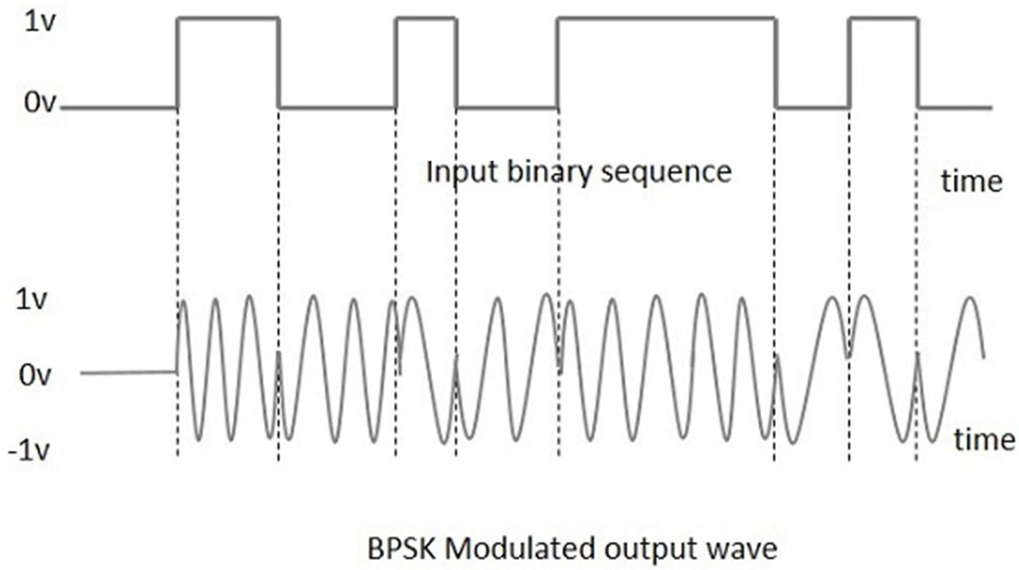


Figure 5.10

The output sine wave of the modulator will be the direct input carrier or the inverted (180° phase shifted) input carrier, which is a function of the data signal.

BPSK Demodulator

The block diagram of BPSK demodulator consists of a mixer with local oscillator circuit, a bandpass filter, a two-input detector circuit. The diagram is as follows.

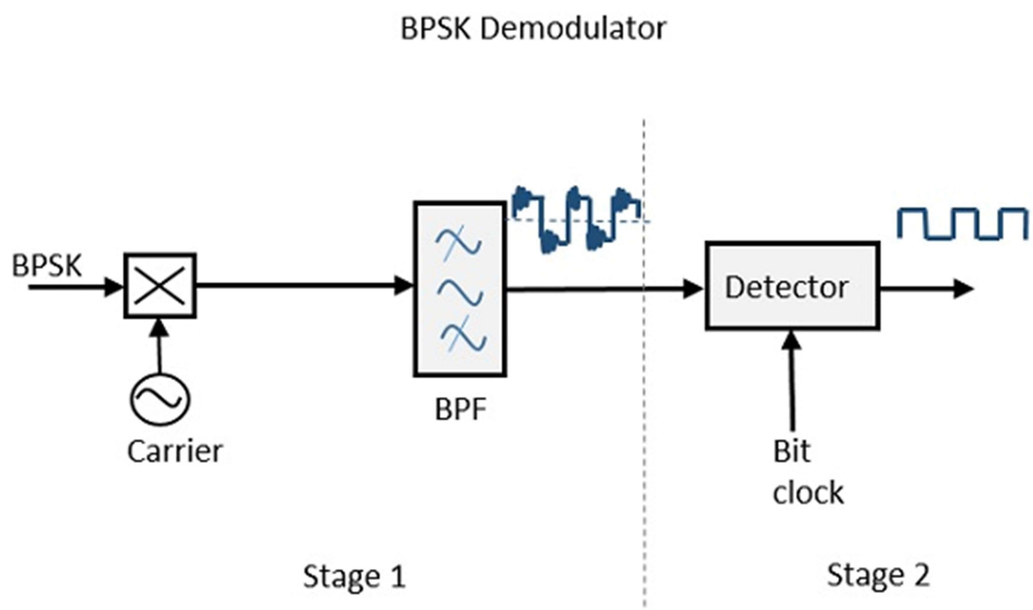


Figure 5.11

By recovering the band-limited message signal, with the help of the mixer circuit and the band pass filter, the first stage of demodulation gets completed. The base band signal which is band limited is obtained and this signal is used to regenerate the binary message bit stream.

In the next stage of demodulation, the bit clock rate is needed at the detector circuit to produce the original binary message signal. If the bit rate is a sub-multiple of the carrier frequency, then the bit clock regeneration is simplified. To make the circuit easily understandable, a decision-making circuit may also be inserted at the 2nd stage of detection.

5.4 DPSK and DEPSK

In Differential Phase Shift Keying (DPSK) the phase of the modulated signal is shifted relative to the previous signal element. No reference signal is considered here. The signal phase follows the high or low state of the previous element. This DPSK technique doesn't need a reference oscillator.

The following figure represents the model waveform of DPSK.

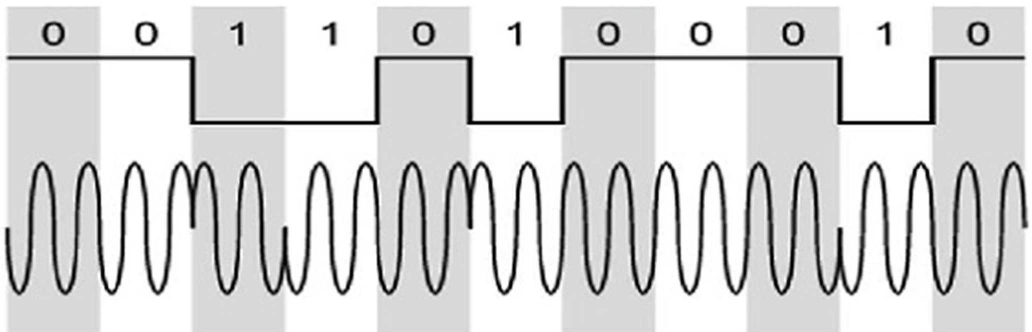


Figure 5.12

It is seen from the above figure that, if the data bit is Low i.e., 0, then the phase of the signal is not reversed, but continued as it was. If the data is a High i.e., 1, then the phase of the signal is reversed, as with NRZI, invert on 1 (a form of differential encoding).

If we observe the above waveform, we can say that the High state represents an M in the modulating signal and the Low state represents a W in the modulating signal.

DPSK Modulator

DPSK is a technique of BPSK, in which there is no reference phase signal. Here, the transmitted signal itself can be used as a reference signal. Following is the diagram of DPSK Modulator.

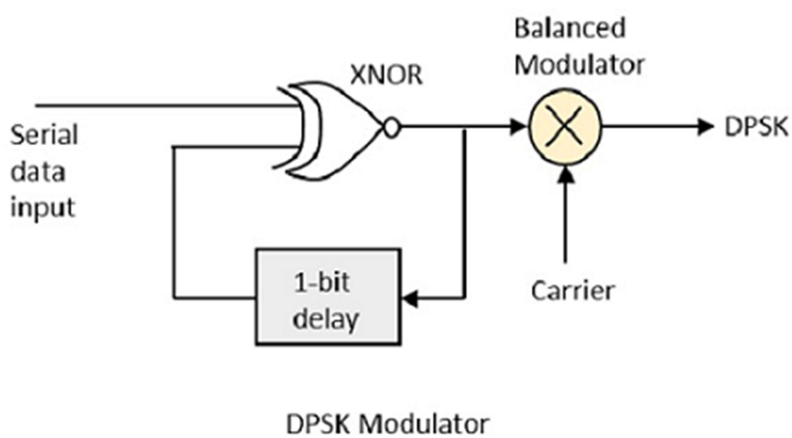


Figure 5.13

DPSK encodes two distinct signals, i.e., the carrier and the modulating signal with 180° phase shift each.

The serial data input is given to the XNOR gate and the output is again fed back to the other input through 1-bit delay. The output of the XNOR gate along with the carrier signal is given to the balance modulator, to produce the DPSK modulated signal.

DPSK Demodulator

In DPSK demodulator, the phase of the reversed bit is compared with the phase of the previous bit. Following is the block diagram of DPSK demodulator.

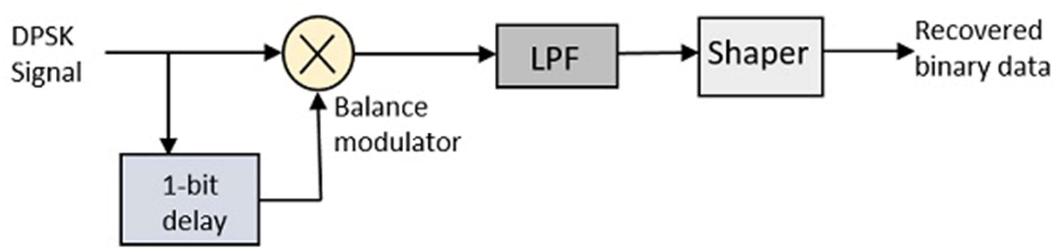


Figure 5.14

From the above figure, it is evident that the balance modulator is given the DPSK signal along with 1-bit delay input. That signal is made to confine to lower frequencies with the help of LPF. Then it is passed to a shaper circuit, which is a comparator or a Schmitt trigger circuit, to recover the original binary data as the output.

5.5 M-ary encoding

The word binary represents two bits. M represents a digit that corresponds to the number of conditions, levels, or combinations possible for a given number of binary variables.This is the type of digital modulation technique used for data transmission in which instead of one bit, two or more bits are transmitted at a time. As a single signal is used for multiple bit transmission, the channel bandwidth is reduced.

M-ary Equation

If a digital signal is given under four conditions, such as voltage levels, frequencies, phases, and amplitude, then M = 4.The number of bits necessary to produce a given number of conditions is expressed mathematically as

$N=\log_2 M$

Where

N is the number of bits necessary

M is the number of conditions, levels, or combinations possible with Nbits.

The above equation can be re-arranged as

$2^N = M$

For example, with two bits, $2^2 = 4$ conditions are possible.

Types of M-ary Techniques

In general, Multi-level (M-ary) modulation techniques are used in digital communications as the digital inputs with more than two modulation levels are allowed on the transmitter's input. Hence, these techniques are bandwidth efficient.

There are many M-ary modulation techniques. Some of these techniques, modulate one parameter of the carrier signal, such as amplitude, phase, and frequency.

M-ary ASK

This is called M-ary Amplitude Shift Keying (M-ASK) or M-ary Pulse Amplitude Modulation (PAM).

The amplitude of the carrier signal, takes on M different levels.

Representation of M-ary ASK

$S_m(t) = A_m \cos(2\pi f_c t)$

$A_m \in (2m-1-M)\Delta, m=1,2....M$ and $0 \leq t \leq T_s$ Some

prominent features of M-ary ASK are − This

method is also used in PAM.

Its implementation is simple.

M-ary ASK is susceptible to noise and distortion.

M-ary FSK

This is called as M-ary Frequency Shift Keying (M-ary FSK).

The frequency of the carrier signal, takes on M different levels.

Representation of M-ary FSK

$S_i(t) = \sqrt{\frac{2E_S}{\square}} \square\square\square \left(\frac{\pi}{\square}(n_c + i)t\right)$, $0 \leq t \leq T_{\square\square s}$ and $i = 1,2,3.....M$

Where f_c= for some fixed integer n.

Some prominent features of M-ary FSK are −

1. Not susceptible to noise as much as ASK.
2. The transmitted M number of signals are equal in energy and duration.
3. The signals are separated by 12Ts12Ts Hz making the signals orthogonal to each other.
4. Since M signals are orthogonal, there is no crowding in the signal space.
5. The bandwidth efficiency of M-ary FSK decreases and the power efficiency increases with the increase in M.

M-ary PSK

This is called as M-ary Phase Shift Keying (M-ary PSK).

The phase of the carrier signal, takes on M different levels.

Representation of M-ary PSK

$S_i(t)=\sqrt{\frac{2E}{}}$ □□□(□□□ + □□□), ,$0\le t\le T_s$ and i=1,2,3.....M

□□(□) = □□□/□ , i=1,2,3.....M

Some prominent features of M-ary PSK are −

1. The envelope is constant with more phase possibilities.
2. This method was used during the early days of space communication.
3. Better performance than ASK and FSK.
4. Minimal phase estimation error at the receiver.
5. The bandwidth efficiency of M-ary PSK decreases and the power efficiency increases with the increase in M.

So far, we have discussed different modulation techniques. The output of all these techniques is a binary sequence, represented as 1s and 0s. This binary or digital information has many types and forms, which are discussed further.

5.6 Quadrature Phase Shift Keying (QPSK)

The Quadrature Phase Shift Keying (QPSK) is a variation of BPSK, and it is also a Double Side Band Suppressed Carrier (DSBSC) modulation scheme, which sends two bits of digital information at a time, called as bigits.

Instead of the conversion of digital bits into a series of digital stream, it converts them into bit pairs. This decreases the data bit rate to half, which allows space for the other users.

QPSK Modulator

The QPSK Modulator uses a bit-splitter, two multipliers with local oscillator, a 2-bit serial to parallel converter, and a summer circuit. Following is the block diagram for the same.

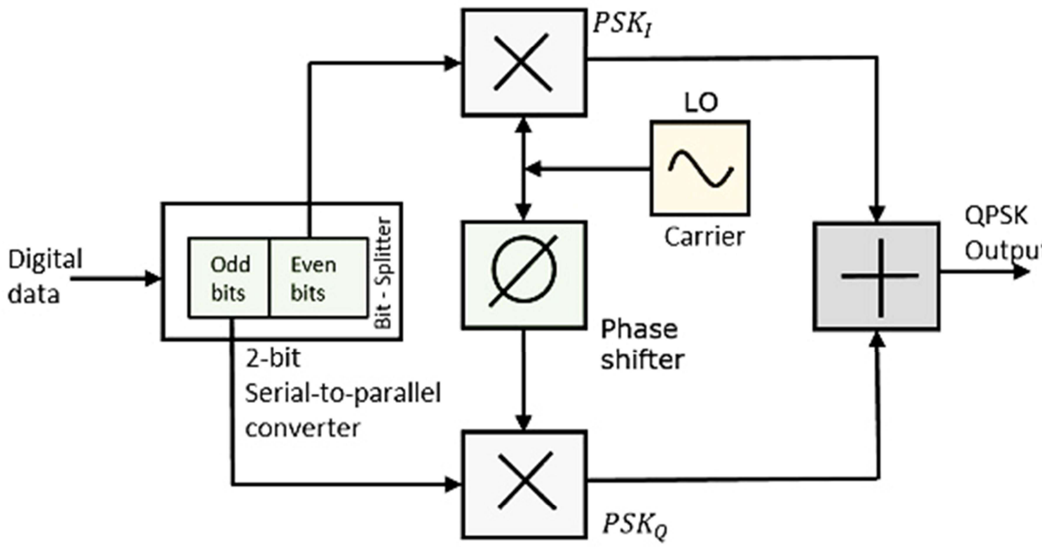


Figure 5.15

At the modulator's input, the message signal's even bits (i.e., 2nd bit, 4th bit, 6th bit, etc.) and odd bits (i.e., 1st bit, 3rd bit, 5th bit, etc.) are separated by the bits splitter and are multiplied with the same carrier to generate odd BPSK (called as PSKI) and even BPSK (called as PSKQ). The PSKQ signal is anyhow phase shifted by 90° before being modulated.

The QPSK waveform for two-bits input is as follows, which shows the modulated result for different instances of binary inputs.

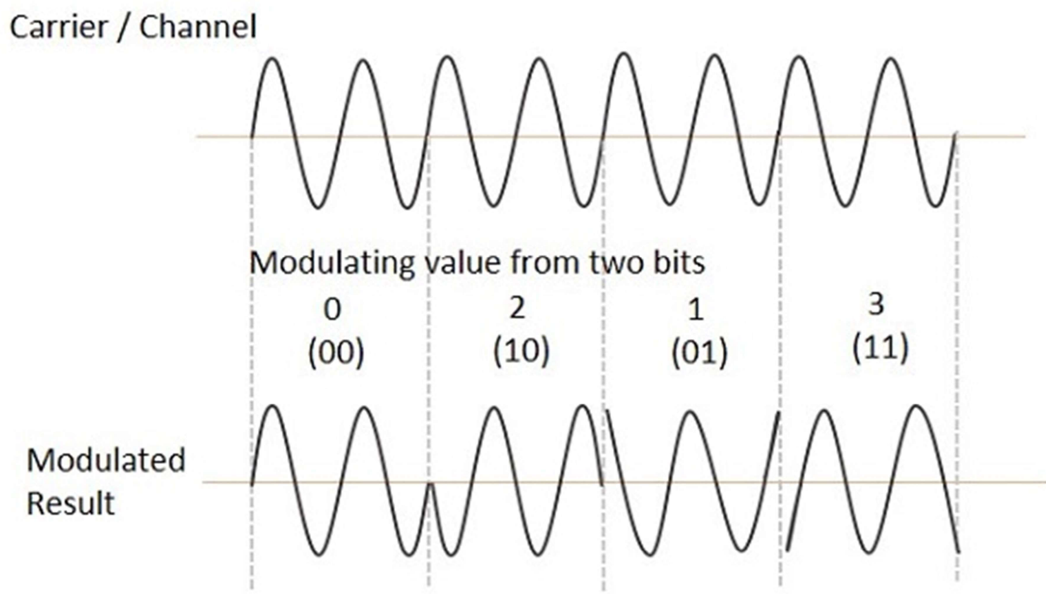


Figure 5.16

QPSK Demodulator

The QPSK Demodulator uses two product demodulator circuits with local oscillator, two band pass filters, two integrator circuits, and a 2-bit parallel to serial converter. Following is the diagram for the same.

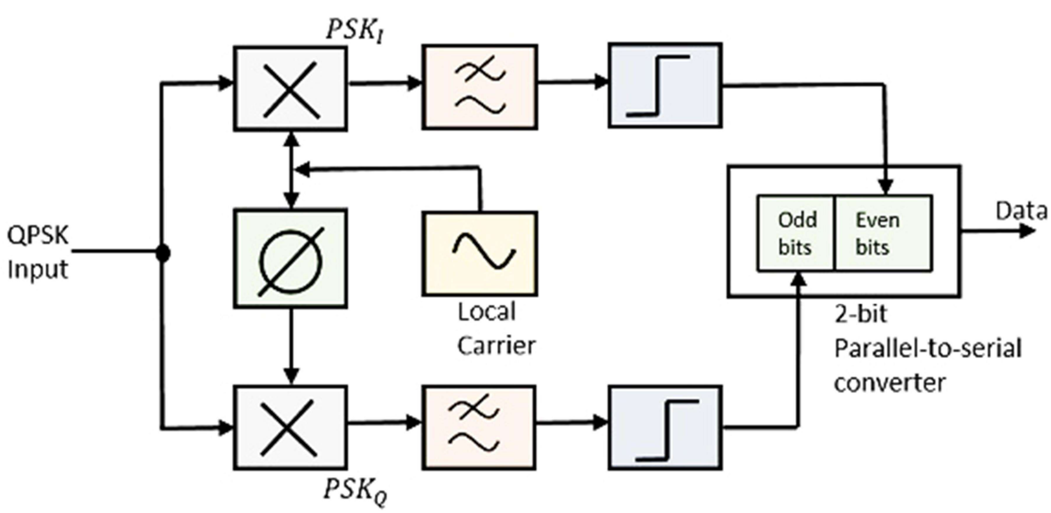


Figure 5.17

The two product detectors at the input of demodulator simultaneously demodulate the two BPSK signals. The pair of bits are recovered here from the original data. These signals after processing, are passed to the parallel to serial converter.

5.7 Minimum Shift Keying (MSK)

In digital modulation, minimum-shift keying (MSK) is a type of continuous-phase frequency-shift keying that was developed in the late 1950s and 1960s.[1] Similar to OQPSK, MSK is encoded with bits alternating between quadrature components, with the Q component delayed by half the symbol period.

However, instead of square pulses as OQPSK uses, MSK encodes each bit as a half sinusoid. This results in a constant-modulus signal (constant envelope signal), which reduces problems caused by non-linear distortion. In addition to being viewed as related to OQPSK, MSK can also be viewed as a continuous phase frequency shift keyed (CPFSK) signal with a frequency separation of one-half the bit rate.

In MSK the difference between the higher and lower frequency is identical to half the bit rate. Consequently, the waveforms used to represent a 0 and a 1 bit differ by exactly half a carrier period. Thus, the maximum frequency deviation is $\delta=2f_m = 0.25\ f_m$ where f_m is the maximum modulating frequency. As a result, the modulation index m is 0.5. This is the smallest FSK modulation index that can be chosen such that the waveforms for 0 and 1 are orthogonal. A variant of MSK called GMSK is used in the GSM mobile phone standard.

5.8 Gaussian Minimum Shift Keying (GMSK)

GMSK modulation is based on MSK, which is itself a form of continuous-phase frequency-shift keying. One of the problems with standard forms of PSK is that sidebands extend out from the carrier. To overcome this, MSK and its derivative GMSK can be used.

MSK and also GMSK modulation are what is known as a continuous phase scheme. Here there are no phase discontinuities because the frequency changes occur at the carrier zero crossing points. This arises as a result of the unique factor of MSK that the frequency difference between the logical one and logical zero states is always equal to half the data rate. This can be expressed in terms of the modulation index, and it is always equal to 0.5.

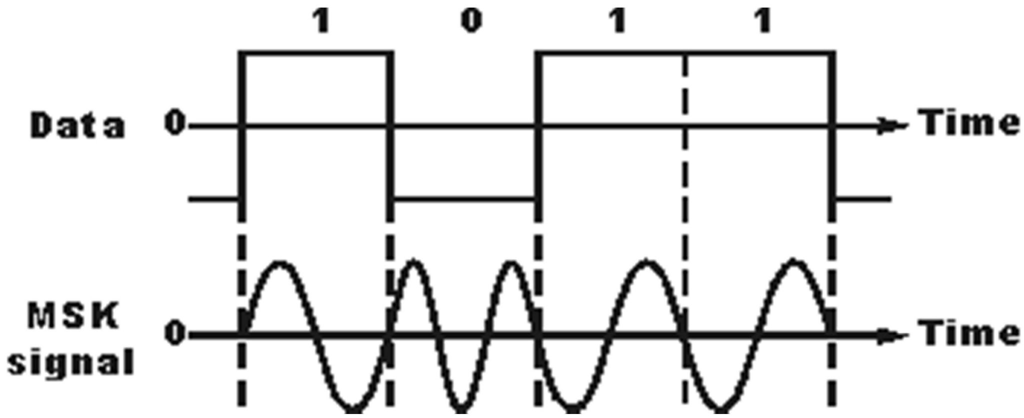


Figure 5.18

A plot of the spectrum of an MSK signal shows sidebands extending well beyond a bandwidth equal to the data rate. This can be reduced by passing the modulating signal through a low pass filter prior to applying it to the carrier. The requirements for the filter are that it should have a sharp cut-off, narrow bandwidth and its impulse response should show no overshoot. The ideal filter is known as a Gaussian filter which has a Gaussian shaped response to an impulse and no ringing. In this way the basic MSK signal is converted to GMSK modulation.

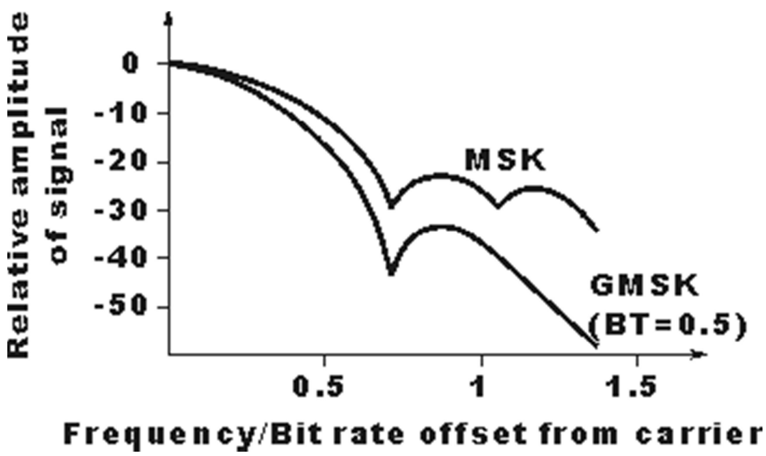


Figure 5.19: Spectral density of MSK and GMSK signals

5.9 Basic concept of OFDM

OFDM is a form of multicarrier modulation. An OFDM signal consists of a number of closely spaced modulated carriers. When modulation of any form - voice, data, etc. is applied to a carrier, then sidebands spread out either side. It is necessary for a receiver to be able to receive the whole signal to be able to successfully demodulate the data. As a result when signals are transmitted close to one another they must be

spaced so that the receiver can separate them using a filter and there must be a guard band between them. This is not the case with OFDM. Although the sidebands from each carrier overlap, they can still be received without the interference that might be expected because they are orthogonal to each another.
This is achieved by having the carrier spacing equal to the reciprocal of the symbol period.

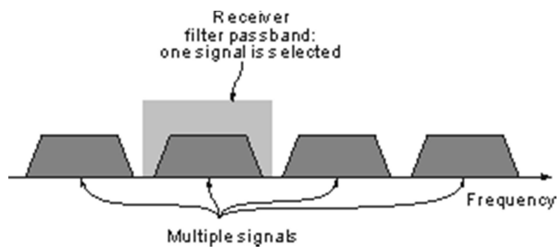


Figure 5.20: Traditional view of receiving signals carrying modulation

To see how OFDM works, it is necessary to look at the receiver. This acts as a bank of demodulators, translating each carrier down to DC. The resulting signal is integrated over the symbol period to regenerate the data from that carrier. The same demodulator also demodulates the other carriers. As the carrier spacing equal to the reciprocal of the symbol period means that they will have a whole number of cycles in the symbol period and their contribution will sum to zero - in other words there is no interference contribution.

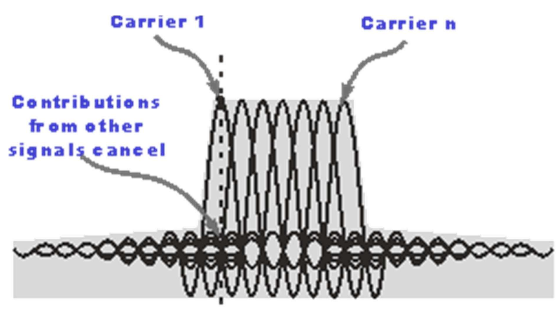


Figure 5.21: OFDM Spectrum

One requirement of the OFDM transmitting and receiving systems is that they must be linear. Any nonlinearity will cause interference between the carriers as a result of inter-modulation distortion. This will introduce unwanted signals that would cause interference and impair the orthogonality of the transmission.

In terms of the equipment to be used the high peak to average ratio of multi-carrier systems such as OFDM requires the RF final amplifier on the output of the transmitter to be able to handle the peaks whilst the average power is much lower and this leads to inefficiency. In some systems the peaks are limited. Although this introduces distortion that results in a higher level of data errors, the system can rely on the error correction to remove them.

OFDM advantages

OFDM has been used in many high data rate wireless systems because of the many advantages it provides.

Immunity to selective fading: One of the main advantages of OFDM is that is more resistant to frequency selective fading than single carrier systems because it divides the overall channel into multiple narrowband signals that are affected individually as flat fading sub-channels.

Resilience to interference: Interference appearing on a channel may be bandwidth limited and in this way will not affect all the sub-channels. This means that not all the data is lost.

Spectrum efficiency: Using close-spaced overlapping sub-carriers, a significant OFDM advantage is that it makes efficient use of the available spectrum.

Resilient to ISI: Another advantage of OFDM is that it is very resilient to inter-symbol and inter-frame interference. This results from the low data rate on each of the sub-channels.

Resilient to narrow-band effects: Using adequate channel coding and interleaving it is possible to recover symbols lost due to the frequency selectivity of the channel and narrow band interference. Not all the data is lost.

Simpler channel equalization: One of the issues with CDMA systems was the complexity of the channel equalization which had to be applied across the whole channel. An advantage of OFDM is that using multiple sub-channels, the channel equalization becomes much simpler.

OFDM disadvantages

Whilst OFDM has been widely used, there are still a few disadvantages to its use which need to be addressed when considering its use.

High peak to average power ratio: An OFDM signal has a noise like amplitude variation and has a relatively high large dynamic range, or peak to average power ratio. This impacts the RF amplifier efficiency as the amplifiers need to be linear and accommodate the large amplitude variations and these factors mean the amplifier cannot operate with a high efficiency level.

Sensitive to carrier offset and drift: Another disadvantage of OFDM is that is sensitive to carrier frequency offset and drift. Single carrier systems are less sensitive.

Module-VI

Performance issues for different digital modulation techniques

Eye pattern

The quality of digital transmission systems is determined using the bit error rate. Eye diagrams are a quick, visual means to quickly identify whether there are any signal integrity issues before moving on to more refined analysis. Due to noise, inter symbol interference degradation of quality occurs in each process modulation, transmission, and detection.

The eye pattern is experimental method that can, be displayed on an oscilloscope display in which a digital signal from a receiver is repetitively sampled and applied to the vertical input, while the data rate is used to trigger the horizontal sweep.

Eye pattern, also known as an eye diagram contains all the information concerning the degradation of quality.

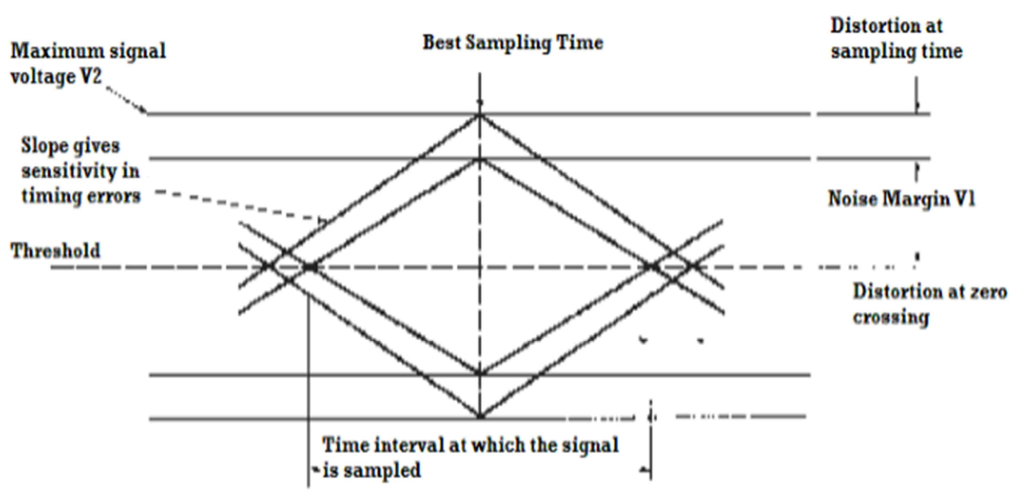


Fig 6.1: Eye Diagram

The interior region of eye pattern is called eye opening

The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI

- the optimum sampling time corresponds to the maximum eye opening

- the height of the eye opening at a specified sampling time is a measure of the additive channel noise.

The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.

Any non linear transmission distortion would reveal itself in an asymmetric or squinted eye. When the effected of ISI is excessive, traces from the upper portion of the eye pattern cross traces from lower portion with the result that the eye is completely closed.

E.g. For a data sequence 110010

Relative constellation error

A constellation diagram is a representation of a signal modulated by a digital modulationscheme as a two-dimensional xy-planescatter diagram in the complex plane at symbol sampling instants.

Error Vector Magnitude also called relative constellation error is a measure used to quantify the performance of a digital communication channel. in an digital communication channel would have all constellation points precisely at the ideal locations. Imperfections cause the actual constellation points to deviate from ideal, and EVM is a measure of how far the points are from those ideal locations. The root mean square EVM is a comprehensive measurement and is degraded by any imperfection in the RF channel.

Noise, distortion, spurious signals, and phase noise all degrade EVM, and therefore EVM provides a comprehensive measure of the quality of the radio receiver or transmitter for use in digital communications. Transmitter EVM can be measured by specialized equipment, which demodulates the received signal in a similar way to how a real radio demodulator does it. One of the stages in a typical phase-shift keying demodulation process produces a stream of I-Q points which can be used as a reasonably reliable estimate for the ideal transmitted signal in EVM calculation.

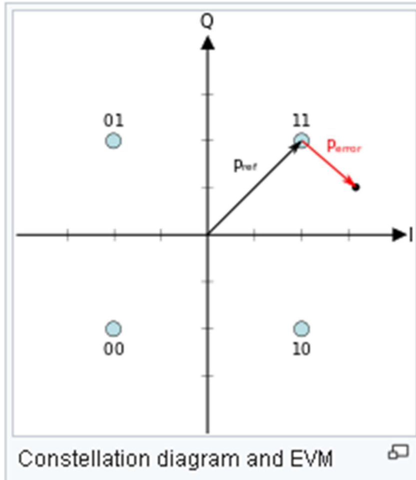


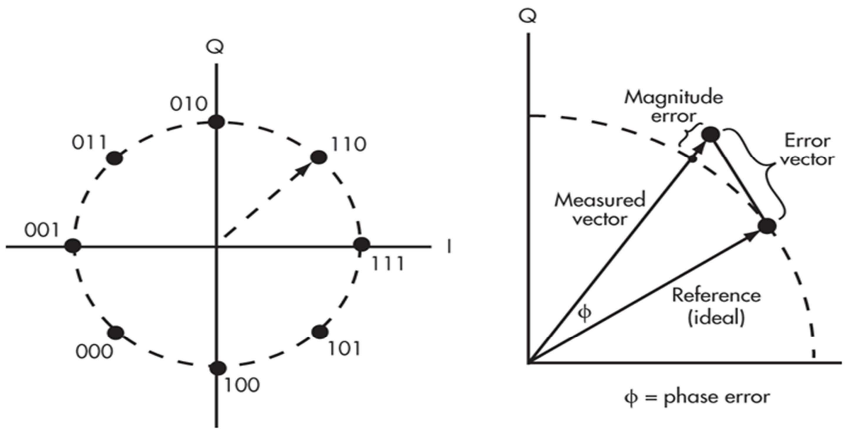
Fig 6.2 EVM

An error vector is a vector in the I-Q plane between the ideal constellation point and the point received by the receiver. In other words, it is the difference between actual received symbols and ideal symbols. The average amplitude of the error vector, normalized to peak signal amplitude, is the EVM. For the percentage format, root mean square (RMS) average is used

The error vector magnitude is equal to the ratio of the amplitude of the error vector to the root mean square (RMS) amplitude of the reference. It is defined in dB as:

EVM (dB) =10 $\log_{10}$ (P error/Preference)

where P_{error} is the RMS amplitude of the error vector. For single carrier modulations, $P_{reference}$ is, by convention, the amplitude of the outermost (highest power) point in the reference signal constellation. More recently, for multi-carrier modulations, $P_{reference}$ is defined as the reference constellation average power.



8 PSK constellation diagram **EVM**

Fg6.2 : EVM

EVM, as conventionally defined for single carrier modulations, is a ratio of a mean amplitude to a peak amplitude. Because the relationship between the peak and mean signal power is dependent on constellation geometry, different constellation types (e.g. 16-QAM and 64-QAM), subject to the same mean level of interference, will report different EVM values.

EVM, as defined for multi carrier modulations, is arguably the more satisfactory measurement because it is a ratio of two mean powers and is insensitive to the constellation geometry. In this form, EVM is closely related to Modulation error ratio, the ratio of mean signal power to mean error power.

A vector signal analyzer is an instrument that measures the magnitude and phase of the input signal at a single frequency within the IFbandwidth of the instrument. The primary use is to make in-channel measurements, such as error vector magnitude, code domain power, and spectral flatness, on known signals.

Questions:

1. What do you mean by Eye Diagram
2. Differentiate between the eye diagram and the EVM

MCQs

1) In uniform

quantization process

a. The step size

remains same

b. Step size varies according to the values of the input signal
c. The quantizer has linear characteristics
d. Both a and c are correct
ANSWER: (d) Both a and c are correct
2) The process of converting the analog sample into discrete form is called a.
Modulation
b. Multiplexing
c. Quantization
d. Sampling
ANSWER:(c) Quantization
3) The characteristics of compressor in µ-law companding are a.
Continuous in nature
b. Logarithmic in nature
c. Linear in nature
d. Discrete in nature
ANSWER: (a) Continuous in nature
4) The modulation techniques used to convert analog signal into digital signal are a.
Pulse code modulation
b. Delta modulation
c. Adaptive delta modulation
d. All of the above
ANSWER: (d) All of the above
5) The sequence of operations in which PCM is done is

a. Sampling, quantizing, encoding

b. Quantizing, encoding, sampling

c. Quantizing, sampling, encoding

d. None of the above

ANSWER:(a) Sampling, quantizing, encoding
6) In PCM, the parameter varied in accordance with the amplitude of the modulating signal is a. Amplitude
b. Frequency
c. Phase
d. None of the above
ANSWER: (d) None of the above 7)
One of the disadvantages of PCM is a.
It requires large bandwidth
b. Very high noise
c. Cannot be decoded easily

d. All of the above
ANSWER: (a) It requires large bandwidth
8) The expression for bandwidth BW of a PCM system, where v is the number of bits per sample and fm is the modulating frequency, is given by a. BW ≥ vfm
b. BW ≤ vfm
c. BW ≥ 2 vfm
d. BW ≥ 1/2 vfm
ANSWER: (a) BW ≥ vfm
9) The error probability of a PCM is
a. Calculated using noise and inter symbol interference
b. Gaussian noise + error component due to inter symbol interference
c. Calculated using power spectral density
d. All of the above
ANSWER: (d) All of the above
10) In Delta modulation,
a. One bit per sample is transmitted
b. All the coded bits used for sampling are transmitted
c. The step size is fixed
d. Both a and c are correct
ANSWER: (d) Both a and c are correct
11) In digital transmission, the modulation technique that requires minimum bandwidth is a. Delta modulation
b. PCM
c. DPCM
d. PAM
ANSWER: (a) Delta modulation
12) In Delta Modulation, the bit rate is a. N times the sampling frequency
b. N times the modulating frequency
c. N times the nyquist criteria
d. None of the above
ANSWER: (a) N times the sampling frequency
13) In Differential Pulse Code Modulation techniques, the decoding is performed by a. Accumulator
b. Sampler
c. PLL
d. Quantizer
ANSWER: (a) Accumulator
14) DPCM is a technique
a. To convert analog signal into digital signal
b. Where difference between successive samples of the analog signals are encoded into n-bit data streams
c. Where digital codes are the quantized values of the predicted value
d. All of the above
ANSWER: (d) All of the above
15) DPCM suffers from
a. Slope over load distortion
b. Quantization noise

c. Both a & b
d. None of the above ANSWER:(c) Both a & b 16) The noise that affects PCM a. Transmission noise
b. Quantizing noise
c. Transit noise
d. Both a and b are correct
ANSWER: (d) Both a and b are correct
17) The factors that cause quantizing error in delta modulation are a.
Slope overload distortion
b. Granular noise
c. White noise
d. Both a and b are correct
ANSWER:(d) Both a and b are correct
18) Granular noise occurs when a.
Step size is too small
b. Step size is too large
c. There is interference from the adjacent channel
d. Bandwidth is too large ANSWER: (b) Step size is too large
19) The crest factor of a waveform is given as – a.
2Peak value/ rms value
b. rms value / Peak value
c. Peak value/ rms value
d. Peak value/ 2rms value ANSWER: (c) Peak value/ rms value
20) The digital modulation technique in which the step size is varied according to the variation in the slope of the input is called a. Delta modulation
b. PCM
c. Adaptive delta modulation
d. PAM

ANSWER: (c) Adaptive delta modulation

21) The digital modulation scheme in which the step size is not fixed is

a. Delta modulation
b. Adaptive delta modulation
c. DPCM
d. PCM

ANSWER:(b) Adaptive delta modulation

22) In Adaptive Delta Modulation, the slope error reduces and

a. Quantization error decreases
b. Quantization error increases
c. Quantization error remains same
d. None of the above

ANSWER: (b) Quantization error increases

23) The number of voice channels that can be accommodated for transmission in T1 carrier system is

a. 24
b. 32
c. 56
d. 64

ANSWER: (a) 24

24) The maximum data transmission rate in T1 carrier system is

a. 2.6 megabits per second
b. 1000 megabits per second
c. 1.544 megabits per second
d. 5.6 megabits per second

ANSWER: (c) 1.544 megabits per second

25) T1 carrier system is used

a. For PCM voice transmission
b. For delta modulation
c. For frequency modulated signals
d. None of the above

ANSWER: (a) For PCM voice transmission

26) Matched filter may be optimally used only for

a. Gaussian noise
b. Transit time noise
c. Flicker
d. All of the above

ANSWER:(a) Gaussian noise

27) Characteristics of Matched filter are

a. Matched filter is used to maximize Signal to noise ratio even for non Gaussian noise
b. It gives the output as signal energy in the absence of noise
c. They are used for signal detection
d. All of the above

ANSWER: (d) All of the above

28) Matched filters may be used

a. To estimate the frequency of the received signal
b. In parameter estimation problems
c. To estimate the distance of the object
d. All of the above

ANSWER: (d) All of the above

29) The process of coding multiplexer output into electrical pulses or waveforms for transmission is called

a. Line coding
b. Amplitude modulation
c. FSK
d. Filtering

ANSWER:(a) Line coding

30) For a line code, the transmission bandwidth must be

a. Maximum possible
b. As small as possible
c. Depends on the signal
d. None of the above

ANSWER: (b) As small as possible

31) Regenerative repeaters are used for

a. Eliminating noise
b. Reconstruction of signals
c. Transmission over long distances
d. All of the above

ANSWER:(d) All of the above

32) Scrambling of data is

a. Removing long strings of 1's and 0's
b. Exchanging of data
c. Transmission of digital data
d. All of the above

ANSWER: (a) Removing long strings of 1's and 0's

33) In polar RZ format for coding, symbol '0' is represented by

a. Zero voltage
b. Negative voltage
c. Pulse is transmitted for half the duration
d. Both b and c are correct

ANSWER: (d) Both b and c are correct

34) In a uni-polar RZ format,

a. The waveform has zero value for symbol '0'
b. The waveform has A volts for symbol '1'
c. The waveform has positive and negative values for '1' and '0' symbol respectively d. Both a and b are correct

ANSWER: (d) Both a and b are correct

35) Polar coding is a technique in which

a. 1 is transmitted by a positive pulse and 0 is transmitted by negative pulse
b. 1 is transmitted by a positive pulse and 0 is transmitted by zero volts c. Both a & b
d. None of the above

ANSWER: (a) 1 is transmitted by a positive pulse and 0 is transmitted by negative pulse

36) The polarities in NRZ format use

a. Complete pulse duration
b. Half duration
c. Both positive as well as negative value
d. Each pulse is used for twice the duration

ANSWER: (a) Complete pulse duration

37) The format in which the positive half interval pulse is followed by a negative half interval pulse for transmission of '1' is

a. Polar NRZ format
b. Bipolar NRZ format
c. Manchester format
d. None of the above

ANSWER: (c) Manchester format

38) The maximum synchronizing capability in coding techniques is present in

a. Manchester format
b. Polar NRZ
c. Polar RZ
d. Polar quaternary NRZ

ANSWER: (a) Manchester format

39) The advantage of using Manchester format of coding is

a. Power saving
b. Polarity sense at the receiver
c. Noise immunity
d. None of the above

ANSWER: (a) Power saving

40) Alternate Mark Inversion (AMI) is also known as

a. Pseudo ternary coding
b. Manchester coding
c. Polar NRZ format
d. None of the above

ANSWER:(a) Pseudo ternary coding

41) In DPSK technique, the technique used to encode bits is

a. AMI
b. Differential code
c. Uni polar RZ format
d. Manchester format

ANSWER: (b)Differential code

42) The channel capacity according to Shannon's equation is

a. Maximum error free communication
b. Defined for optimum system
c. Information transmitted
d. All of the above

ANSWER: (d) All of the above

43) For a binary symmetric channel, the random bits are given as

a. Logic 1 given by probability P and logic 0 by (1-P)
b. Logic 1 given by probability 1-P and logic 0 by P
c. Logic 1 given by probability P2 and logic 0 by 1-P
d. Logic 1 given by probability P and logic 0 by (1-P)2

ANSWER: (a) Logic 1 given by probability P and logic 0 by (1-P)

44) The technique that may be used to increase average information per bit is
a. Shannon-Fano algorithm
b. ASK
c. FSK
d. Digital modulation techniques

ANSWER: (a) Shannon-Fano algorithm

45) Code rate r, k information bits and n as total bits, is defined as

a. r = k/n
b. k = n/r
c. r = k * n
d. n = r * k

ANSWER: (a) r = k/n

46) The information rate R for given average information H= 2.0 for analog signal band limited to B Hz is

a. 8 B bits/sec
b. 4 B bits/sec
c. 2 B bits/sec
d. 16 B bits/sec

ANSWER:(b) 4 B bits/sec

47) Information rate is defined as

a. Information per unit time
b. Average number of bits of information per second c. rH
d. All of the above

ANSWER: (d) All of the above

48) The mutual information

a. Is symmetric
b. Always non negative
c. Both a and b are correct
d. None of the above

ANSWER: (c) Both a and b are correct

49) The relation between entropy and mutual information is

a. I(X;Y) = H(X) – H(X/Y)
b. I(X;Y) = H(X/Y) – H(Y/X)
c. I(X;Y) = H(X) – H(Y)
d. I(X;Y) = H(Y) – H(X)

ANSWER:(a) I(X;Y) = H(X) – H(X/Y)

50) Entropy is

a. Average information per message
b. Information in a signal
c. Amplitude of signal
d. All of the above

ANSWER: (a) Average information per message

51) The memory less source refers to

a. No previous information
b. No message storage
c. Emitted message is independent of previous message
d. None of the above

ANSWER: (c) Emitted message is independent of previous message

52) The information I contained in a message with probability of occurrence is given by (k is constant)

a. I = k log21/P
b. I = k log2P
c. I = k log21/2P
d. I = k log21/P2

ANSWER:(a) I = k log21/P

53) The expected information contained in a message is called

a. Entropy
b. Efficiency
c. Coded signal
d. None of the above

ANSWER: (a) Entropy

54) Overhead bits are

a. Framing and synchronizing bits
b. Data due to noise
c. Encoded bits
d. None of the above
ANSWER: (a) Framing and synchronizing bits

55) ISI may be removed by using

a. Differential coding
b. Manchester coding
c. Polar NRZ
d. None of the above

ANSWER: (a) Differential coding

56) Timing jitter is

a. Change in amplitude
b. Change in frequency
c. Deviation in location of the pulses
d. All of the above

ANSWER: (c) Deviation in location of the pulses

57) Probability density function defines

a. Amplitudes of random noise
b. Density of signal
c. Probability of error
d. All of the above

ANSWER: (a) Amplitudes of random noise

58) Impulse noise is caused due to

a. Switching transients
b. Lightening strikes
c. Power line load switching
d. All of the above

ANSWER: (d) All of the above

59) In coherent detection of signals,

a. Local carrier is generated
b. Carrier of frequency and phase as same as transmitted carrier is generated
c. The carrier is in synchronization with modulated carrier
d. All of the above

ANSWER: (d) All of the above

60) Synchronization of signals is done using

a. Pilot clock
b. Extracting timing information from the received signal
c. Transmitter and receiver connected to master timing source
d. All of the above

ANSWER:(d) All of the above

61) Graphical representation of linear block code is known as

a. Pi graph
b. Matrix
c. Tanner graph
d. None of the above

ANSWER: (c) Tanner graph

62) A linear code

a. Sum of code words is also a code word
b. All-zero code word is a code word
c. Minimum hamming distance between two code words is equal to weight of any non zero code word d. All of the above

ANSWER: (d) All of the above

63) For decoding in convolution coding, in a code tree,

a. Diverge upward when a bit is 0 and diverge downward when the bit is 1
b. Diverge downward when a bit is 0 and diverge upward when the bit is 1
c. Diverge left when a bit is 0 and diverge right when the bit is 1
d. Diverge right when a bit is 0 and diverge left when the bit is 1

ANSWER: (a)Diverge upward when a bit is 0 and diverge downward when the bit is 1

64) The code in convolution coding is generated using

a. EX-OR logic
b. AND logic
c. OR logic
d. None of the above

ANSWER: (a) EX-OR logic

65) Interleaving process permits a burst of B bits, with l as consecutive code bits and t errors when
a. $B \leq 2tl$
b. $B \geq tl$
c. $B \leq tl/2$
d. $B \leq tl$

ANSWER: (d) $B \leq tl$

66) For a (7, 4) block code, 7 is the total number of bits and 4 is the number of

a. Information bits
b. Redundant bits
c. Total bits- information bits
d. None of the above

ANSWER: (a) Information bits

67) Parity bit coding may not be used for

a. Error in more than single bit
b. Which bit is in error
c. Both a & b
d. None of the above

ANSWER: (c) Both a & b

68) Parity check bit coding is used for

a. Error correction
b. Error detection
c. Error correction and detection
d. None of the above

ANSWER: (b) Error detection

69) For hamming distance dmin and t errors in the received word, the condition to be able to correct the errors is

a. $2t + 1 \leq dmin$
b. $2t + 2 \leq dmin$
c. $2t + 1 \leq 2dmin$
d. Both a and b

ANSWER: (d) Both a and b

70) For hamming distance dmin and number of errors D, the condition for receiving invalid codeword is

a. $D \leq dmin + 1$
b. $D \leq dmin - 1$
c. $D \leq 1 - dmin$
d. $D \leq dmin$

ANSWER:(b) $D \leq dmin - 1$

71) Run Length Encoding is used for

a. Reducing the repeated string of characters
b. Bit error correction
c. Correction of error in multiple bits
d. All of the above

ANSWER: (a) Reducing the repeated string of characters

72) The prefix code is also known as

a. Instantaneous code
b. Block code
c. Convolutional code
d. Parity bit

ANSWER: (a) Instantaneous code

73) The minimum distance for unextendedGolay code is

a. 8
b. 9
c. 7
d. 6

ANSWER: (c) 7

74) The Golay code (23,12) is a codeword of length 23 which may correct

a. 2 errors
b. 3 errors
c. 5 errors
d. 8 errors

ANSWER: (b) 3 errors

75) Orthogonality of two codes means

a. The integrated product of two different code words is zero
b. The integrated product of two different code words is one
c. The integrated product of two same code words is zero
d. None of the above
ANSWER: (a) The integrated product of two different code words is zero

76) The probability density function of a Markov process is

a. p(x1,x2,x3…….xn) = p(x1)p(x2/x1)p(x3/x2)…….p(xn/xn-1)
b. p(x1,x2,x3…….xn) = p(x1)p(x1/x2)p(x2/x3)…….p(xn-1/xn)
c. p(x1,x2,x3……xn) = p(x1)p(x2)p(x3)…….p(xn)
d. p(x1,x2,x3……xn) = p(x1)p(x2 * x1)p(x3 * x2)……..p(xn * xn-1)

ANSWER:(a) p(x1,x2,x3…….xn) = p(x1)p(x2/x1)p(x3/x2)…….p(xn/xn-1)

77) The capacity of Gaussian channel is

a. C = 2B(1+S/N) bits/s
b. C = B2(1+S/N) bits/s
c. C = B(1+S/N) bits/s
d. C = B(1+S/N)2 bits/s

ANSWER: (c) C = B(1+S/N) bits/s

78) For M equally likely messages, the average amount of information H is

a. H = log10M
b. H = log2M
c. H = log10M2
d. H = 2log10M

ANSWER:(b) H = log2M

79) The channel capacity is

a. The maximum information transmitted by one symbol over the channel
b. Information contained in a signal
c. The amplitude of the modulated signal
d. All of the above

ANSWER: (a) The maximum information transmitted by one symbol over the channel

80) The capacity of a binary symmetric channel, given H(P) is binary entropy function is

a. 1 – H(P)
b. H(P) – 1
c. 1 – H(P)2
d. H(P)2 – 1

ANSWER:(a) 1 – H(P)

81) According to Shannon Hartley theorem,

a. The channel capacity becomes infinite with infinite bandwidth
b. The channel capacity does not become infinite with infinite bandwidth
c. Has a tradeoff between bandwidth and Signal to noise ratio
d. Both b and c are correct

ANSWER: (d) Both b and c are correct

82) The negative statement for Shannon's theorem states that

a. If R > C, the error probability increases towards Unity
b. If R < C, the error probability is very small
c. Both a & b
d. None of the above

ANSWER: (a) If R > C, the error probability increases towards Unity

83) For M equally likely messages, M>>1, if the rate of information $R \leq C$, the probability of error is

a. Arbitrarily small
b. Close to unity
c. Not predictable
d. Unknown

ANSWER: (a) Arbitrarily small

84) For M equally likely messages, M>>1, if the rate of information $R > C$, the probability of error is

a. Arbitrarily small
b. Close to unity
c. Not predictable
d. Unknown

ANSWER: (b) Close to unity

85) In Alternate Mark Inversion (AMI) is

a. 0 is encoded as positive pulse and 1 is encoded as negative pulse
b. 0 is encoded as no pulse and 1 is encoded as negative pulse
c. 0 is encoded as negative pulse and 1 is encoded as positive pulse
d. 0 is encoded as no pulse and 1 is encoded as positive or negative pulse

ANSWER: (b) 0 is encoded as no pulse and 1 is encoded as positive or negative pulse

86) Advantages of using AMI

a. Needs least power as due to opposite polarity
b. Prevents build-up of DC
c. May be used for longer distance
d. All of the above

ANSWER: (d)All of the above

87) The interference caused by the adjacent pulses in digital transmission is called

a. Inter symbol interference
b. White noise
c. Image frequency interference
d. Transit time noise

ANSWER: (a) Inter symbol interference

88) Eye pattern is

a. Is used to study ISI
b. May be seen on CRO
c. Resembles the shape of human eye
d. All of the above

ANSWER: (d) All of the above

89) The time interval over which the received signal may be sampled without error may be explained by

a. Width of eye opening of eye pattern
b. Rate of closure of eye of eye pattern
c. Height of the eye opening of eye pattern
d. All of the above

ANSWER:(a) Width of eye opening of eye pattern

90) For a noise to be white Gaussian noise, the optimum filter is known as

a. Low pass filter
b. Base band filter
c. Matched filter
d. Bessel filter

ANSWER:(c) Matched filter

91) Matched filters are used

a. For maximizing signal to noise ratio
b. For signal detection
c. In radar
d. All of the above

ANSWER: (d) All of the above

92) The number of bits of data transmitted per second is called

a. Data signaling rate
b. Modulation rate
c. Coding
d. None of the above

ANSWER: (a) Data signaling rate

93) Pulse shaping is done

a. to control Inter Symbol Interference
b. by limiting the bandwidth of transmission
c. after line coding and modulation of signal
d. All of the above

ANSWER: (d) All of the above

94) The criterion used for pulse shaping to avoid ISI is

a. Nyquist criterion
b. Quantization
c. Sample and hold
d. PLL

ANSWER: (a) Nyquist criterion

95) The filter used for pulse shaping is

a. Raised – cosine filter
b. Sinc shaped filter
c. Gaussian filter
d. All of the above

ANSWER: (d) All of the above

96) Roll – off factor is defined as

a. The bandwidth occupied beyond the Nyquist Bandwidth of the filter
b. The performance of the filter or device
c. Aliasing effect
d. None of the above

ANSWER: (a) The bandwidth occupied beyond the Nyquist Bandwidth of the filter 97) Nyquist criterion helps in

a. Transmitting the signal without ISI
b. Reduction in transmission bandwidth
c. Increase in transmission bandwidth
d. Both a and b

ANSWER: (d) Both a and b

98) The Nyquist theorem is

a. Relates the conditions in time domain and frequency domain
b. Helps in quantization
c. Limits the bandwidth requirement
d. Both a and c

ANSWER: (d) Both a and c

99) The difficulty in achieving the Nyquist criterion for system design is

a. There are abrupt transitions obtained at edges of the bands
b. Bandwidth criterion is not easily achieved
c. Filters are not available
d. None of the above

ANSWER: (a) There are abrupt transitions obtained at edges of the bands

100) Equalization in digital communication

a. Reduces inter symbol interference
b. Removes distortion caused due to channel
c. Is done using linear filters
d. All of the above

ANSWER: (d) All of the above

101) Zero forced equalizers are used for

a. Reducing ISI to zero
b. Sampling
c. Quantization
d. None of the abov

ANSWER: (a)Reducing ISI to zero

102) The transmission bandwidth of the raised cosine spectrum is given by

a. $Bt = 2w(1 + \alpha)$
b. $Bt = w(1 + \alpha)$
c. $Bt = 2w(1 + 2\alpha)$
d. $Bt = 2w(2 + \alpha)$

ANSWER: (a) $Bt = 2w(1 + \alpha)$

103) The preferred orthogonalization process for its numerical stability is

a. Gram- Schmidt process
b. House holder transformation
c. Optimization
d. All of the above

ANSWER: (b) House holder transformation

104) For two vectors to be orthonormal, the vectors are also said to be orthogonal. The reverse of the same

a. Is true
b. Is not true
c. Is not predictable
d. None of the above

ANSWER: (b) Is not true

105) Orthonormal set is a set of all vectors that are

a. Mutually orthonormal and are of unit length
b. Mutually orthonormal and of null length
c. Both a & b
d. None of the above

ANSWER: (a) Mutually orthonormal and are of unit length

106) In On-Off keying, the carrier signal is transmitted with signal value '1' and '0' indicates

a. No carrier
b. Half the carrier amplitude
c. Amplitude of modulating signal
d. None of the above

ANSWER: (a) No carrier

107) ASK modulated signal has the bandwidth

a. Same as the bandwidth of baseband signal
b. Half the bandwidth of baseband signal
c. Double the bandwidth of baseband signal
d. None of the above

ANSWER: (a) Same as the bandwidth of baseband signal

108) Coherent detection of binary ASK signal requires

a. Phase synchronization
b. Timing synchronization
c. Amplitude synchronization
d. Both a and b

ANSWER: (d) Both a and b

109) The probability of error of DPSK is ______________ than that of BPSK.

a. Higher
b. Lower
c. Same
d. Not predictable

ANSWER: (a) Higher

110) In Binary Phase Shift Keying system, the binary symbols 1 and 0 are represented by carrier with phase shift of

a. Π/2
b. Π
c. 2Π
d. 0

ANSWER: (b) Π

111) BPSK system modulates at the rate of

a. 1 bit/ symbol
b. 2 bit/ symbol
c. 4 bit/ symbol
d. None of the above

ANSWER: (a) 1 bit/ symbol

112) The BPSK signal has +V volts and -V volts respectively to represent

a. 1 and 0 logic levels
b. 11 and 00 logic levels
c. 10 and 01 logic levels
d. 00 and 11 logic levels

ANSWER: (a) 1 and 0 logic levels

113) The binary waveform used to generate BPSK signal is encoded in

a. Bipolar NRZ format
b. Manchester coding
c. Differential coding
d. None of the above

ANSWER: (a) Bipolar NRZ format

114) The bandwidth of BFSK is ______________ than BPSK.

a. Lower
b. Same
c. Higher
d. Not predictable

ANSWER: (c) Higher

115) In Binary FSK, mark and space respectively represent

a. 1 and 0
b. 0 and 1
c. 11 and 00
d. 00 and 11

ANSWER: (a) 1 and 0

116) The frequency shifts in the BFSK usually lies in the range

a. 50 to 1000 Hz
b. 100 to 2000 Hz
c. 200 to 500 Hz
d. 500 to 10 Hz

ANSWER: (a) 50 to 1000 Hz

117) The spectrum of BFSK may be viewed as the sum of

a. Two ASK spectra
b. Two PSK spectra
c. Two FSK spectra
d. None of the above

ANSWER: (a) Two ASK spectra

118) The maximum bandwidth is occupied by

a. ASK
b. BPSK
c. FSK
d. None of the above

ANSWER: (c) FSK

119) QPSK is a modulation scheme where each symbol consists of

a. 4 bits
b. 2 bits
c. 1 bits
d. M number of bits, depending upon the requireme

ANSWER: (b) 2 bits

120) The data rate of QPSK is ___________ of BPSK.

a. Thrice
b. Four times
c. Twice
d. Same

ANSWER: (c) Twice

121) QPSK system uses a phase shift of

a. Π
b. Π/2
c. Π/4
d. 2Π

ANSWER: (b) Π/2

122) Minimum shift keying is similar to

a. Continuous phase frequency shift keying
b. Binary phase shift keying
c. Binary frequency shift keying
d. QPSK

ANSWER: (a) Continuous phase frequency shift keying

123) In MSK, the difference between the higher and lower frequency is

a. Same as the bit rate
b. Half of the bit rate
c. Twice of the bit rate
d. Four time the bit rate

ANSWER: (b) Half of the bit rate

124) The technique that may be used to reduce the side band power is

a. MSK
b. BPSK
c. Gaussian minimum shift keying
d. BFSK

ANSWER: (c) Gaussian minimum shift keying