Control Systems – I
(EE503)

Online Courseware (OCW)

B.TECH (3^{rd} YEAR – 5^{th} SEM)

(2022-23)

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(Affiliated to MAKUT, West Bengal , Approved by AICTE - Accredited by NAAC – 'A+' Grade)
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Course Name: Control Systems – I
Course Code: EE503
Contact: 3L:0T:0P
Total Contact Hours: 36
Credit: 3

Prerequisite: Concept of Basic Electrical Engineering, Circuit Theory and Engineering Mathematics.

Course Outcomes: After successful completion of the course, student will be able to

- **CO1.** Calculate mathematical model and transfer function of the physical systems.
- **CO2.** Analyze the linear systems in time domain.
- **CO3.** Illustrate the linear systems in frequency domain.
- **CO4.** Design simple compensators and controllers for the given specifications.

CO-PO-PSO Mapping:

COs	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO1 0	PO1 1	PO1 2	PSO 1	PSO 2	PSo 3
CO1	2	2	2	-	2	-	-	-	-	-	-	2	2	1	1
CO2	2	2	2	-	2	-	-	-	-	-	-	2	1	3	1
CO3	2	3	2	-	2	-	-	-	-	-	-	2	1	3	1
CO4	2	2	3	-	2	-	-	-	-	-	-	2	1	3	1
Avg.	2	2.25	2.25		2	-	-	-	-	-	-	2	1.25	2.5	1

Course Content

Module 1: Systems and their Representations **6L**
Basic elements in control systems - open loop & closed loop - Transfer functions of mechanical, electrical and analogous systems. Block diagram reduction - signal flow graphs.

Module 2: Time Domain Analysis **6L**
Standard test signals, Time response of first and second order system, Time domain specifications, Steady state error, error constants, generalized error coefficient.

Module 3: Stability Analysis and Root Locus
6L
Stability - concept and definition, Characteristic equation – Location of poles – Routh Hurwitz criterion - Root locus techniques: construction, properties and applications.

Module 4: Frequency Response Analysis **5L**
Bode plot - Polar plot - Correlation between frequency domain and time domain specifications.

Module 5: Stability in Frequency Domain **5L**
Relative stability, Gain margin, Phase margin, stability analysis using frequency response methods, Nyquist stability criterion.

Module 6: Control Systems Design **8L**
Introduction to design problem and philosophy. Introduction to time domain and frequency domain design specification and its physical relevance. Effect of gain on transient and steady state response. Effect of addition of pole on system performance. Effect of addition of zero on system response.

Introduction to compensator. Design of Lag, lead lag-lead compensator in time domain & frequency domain using Bode plot. Design of P, PI, PD and PID controllers in time domain and frequency domain for first, second order systems.

Text Books:

1. Modern Control Engineering, K. Ogata, 4th Edition, Pearson Education.
2. Norman S. Nise, “Control System Engineering”, John Wiley & Sons, 6th Edition, 2011.
3. Benjamin C Kuo “Automatic Control System” John Wiley & Sons, 8th Edition, 2007.

Reference Books:

1. M. Gopal, “Control Systems-Principles And Design”, Tata McGraw Hill –4th Edition, 2012.
2. R.C. Dorf & R.H. Bishop, “Modern Control Systems”, Pearson Education, 11th Edition, 2008.
3. I. J. Nagrath and M.Gopal,” Control System Engineering”, New Age International Publishers, 4th Edition, 2006.
4. Graham C. Goodwin, Stefan F. Graebe, Mario E. Sagado, “Control System Design”, Prentice Hall, 2003.

[illegible] input [illegible] perform a specific function [illegible] system [illegible] the output quantity is controlled by varying the [illegible] [illegible] the system is called control system

[illegible] of a room temperature is achieved by switching ON and switching OFF the power supply to a heating appliance. Thus power supply to the appliance is switched on, if the room temp. is felt low and switched off when the desired temperature is reached

The above system can be modified if the duration of application of power is predetermined to achieve the room temp within desired limit.

However a further refinement can be made by measuring the difference between the actual room temperature and the desired room temp. and this difference being the error is used to control the element which in turn controls the o/p i.e. room temperature.

The above description indicates that in the former case the o/p (room temp.) has no control on the input. This type of system is called open loop control system whereas, in the latter case the control action is affected by a feedback received from the output to the input. This type of system is called closed loop (feedback) control system (eg. air conditioning system)

The control system without involving human intervention for normal operation are called automatic control system. eg. control of traffic lamps of 3 different colours.

(VII) Linear and Non-linear System

In a linear system the form of output does not depend on the magnitude of the input, when the input increases the output will also increases but the form remains same.

In non-linear system form may change with change in magnitude of the input. eg. if the input is sinusoidal the output of the linear system will also be sinusoidal but in non-linear the output will be non-sinusoidal.

[illegible] of these properties.
System does not obey at least any one of these properties.

Additivity – if the i/p signals $x_1(t)$ and $x_2(t)$ correspond to the o/p signals $y_1(t)$ and $y_2(t)$ respectively, then the i/p signal $\{x_1(t) + x_2(t)\}$ should correspond to the o/p signal $\{y_1(t) + y_2(t)\}$.

* Homogeneity – if the i/p signal $x_1(t)$ correspond to the o/p signal $y_1(t)$, then the i/p signal $a_1 x_1(t)$ should correspond to the o/p signal $a_1 y_1(t)$ for any constant

Combining these two properties, the condition for a linear system can be written as, if the i/p signals $x_1(t)$ and $x_2(t)$ correspond to the o/p signals $y_1(t)$ and $y_2(t)$, then the i/p signal $\{a_1 x_1(t) + a_2 x_2(t)\}$ should correspond to the o/p signal $\{a_1 y_1(t) + a_2 y_2(t)\}$ for any constants a_1 and a_2.

Robustness

A controller designed for a perticular set of parameters is said to be robust if it would also work well under a different set of assumptions. High gain feedback is a simple example of a robust control method. With sufficiently high gain, the effect of any parameters variations will be negligible.

High gain feedback is the principle that allows simplified models of op-amp and emitter-degenerated bipolar transistors to be used in a variety of different settings.

Sensitivity

The parameters of a control system may have a tendency to vary due to changing environment conditions and this variation in parameters affects the desired performance of a control system. The use of feedback in control system reduces the effect of parameter variations.

The term sensitivity in relation to control system gives an assessment of the system performance as affected due to parameter variation.

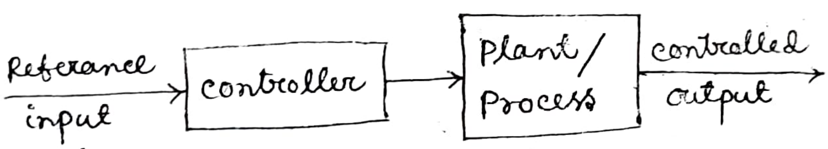
... element K ... is given ...
then the control system sensitivity is expressed as

$$\text{sensitivity} = \frac{\%\ \text{change in A}}{\%\ \text{change in K}}$$

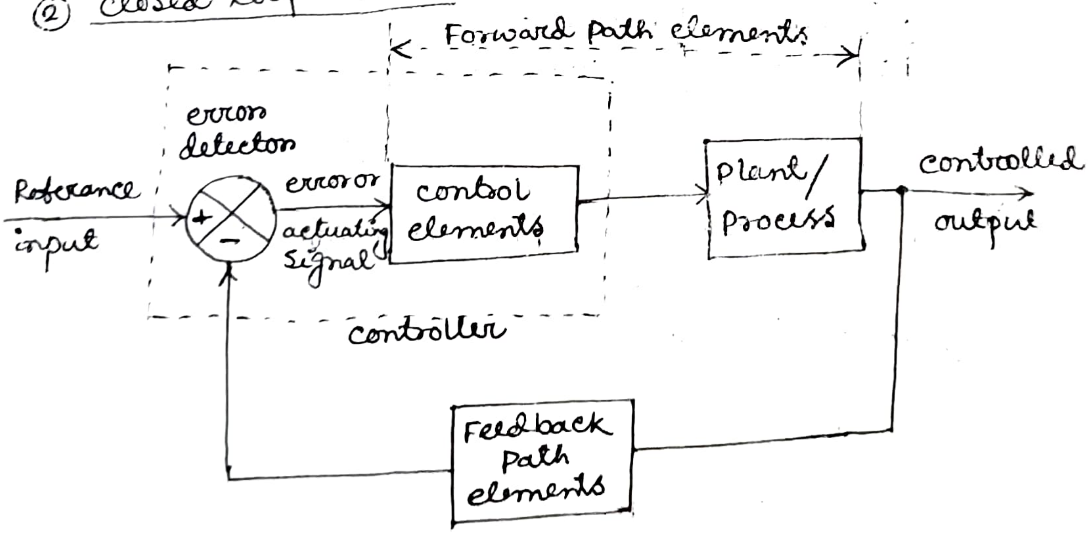
Different types of control system

* based on different angle
  1. Manual control.
  2. Automatic control.
* with respect to the data.
  1. Analog control
  2. Digital control.
* with respect to the feedback.
  1. open loop (non-feedback) control
  2. closed loop (feedback) control

① open loop control



② closed loop control



Transfer function

Transfer function is defined as the ratio of Laplace transform of output to the Laplace transform of input with all initial conditions is zero. The concept of transfer function is applicable to single input - single output, linear time invariant systems.

The dynamics of a linear time invariant system are represented by a linear differential equation

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1}\frac{d^{n-1} c(t)}{dt^{n-1}} + \dots + a_1 \frac{dc(t)}{dt} + a_0 c(t)$$
$$= b_m \frac{d^m r(t)}{dt^m} + \dots + b_1 \frac{dr(t)}{dt} + b_0 r(t)$$

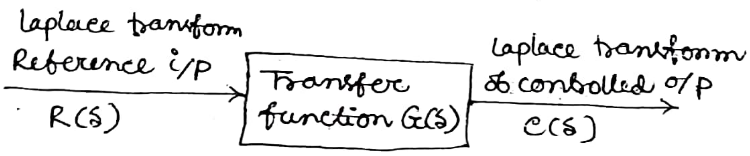
Laplace transform of the above eq. with all initial conditions is zero

$$(a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0) C(s) = (b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0) R(s)$$

$$\text{or, } \frac{C(s)}{R(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} \quad \dots \quad (1)$$

$$\therefore \text{Transfer function, } G(s) = \frac{C(s)}{R(s)}$$

$$= \left.\frac{\text{Laplace Transform of output}}{\text{Laplace Transform of input}}\right|_{\text{with zero initial condition}}$$



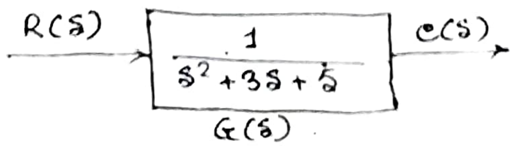
Prob (1) $\frac{d^2c}{dt^2} + 3\frac{dc}{dt} + 5c = r$. Find Transfer function $G(s)$

Soln Taking laplace transform.

$$s^2 C(s) + 3s\, C(s) + 5C(s) = R(s)$$

$$\text{or, } (s^2 + 3s + 5) C(s) = R(s)$$

$$\text{or, } \frac{C(s)}{R(s)} = \frac{1}{s^2 + 3s + 5} = G(s)$$



[illegible]

The numerator and denominator of equation (1) can be factored into n and m respectively. With such a factorisation the expression will be

$$G(s) = \frac{C(s)}{R(s)} = \frac{K(s-s_1)(s-s_2)\ldots(s-s_m)}{(s-s_a)(s-s_b)\ldots(s-s_n)} \qquad (2)$$

where K gain factor of the transfer function.

In eq (2) if s is put equal to $s_a, s_b, s_c \ldots s_n$, then the value of transfer function is infinite. Hence $s_a, s_b \ldots s_n$ are called Poles of the Transfer function. A pole is indicated by a small cross 'x'.

In eq (2) if s is put equal to $s_1, s_2 \ldots s_m$, then the value of the transfer function is zero. Hence $s_1, s_2 \ldots s_m$ are called zeros of the Transfer function. A zero is indicated by a small circle 'o'.

Prob (2) The T.F. of a system is given below

$$G(s) = \frac{8(s+3)(s+4)}{s(s+2)^2(s^2+2s+5)}$$

Determine the poles and zeros and show the Pole-zero configuration in s-plane.

Solu^n The poles are determined from the equation

$$s(s+2)^2(s^2+2s+5) = 0$$

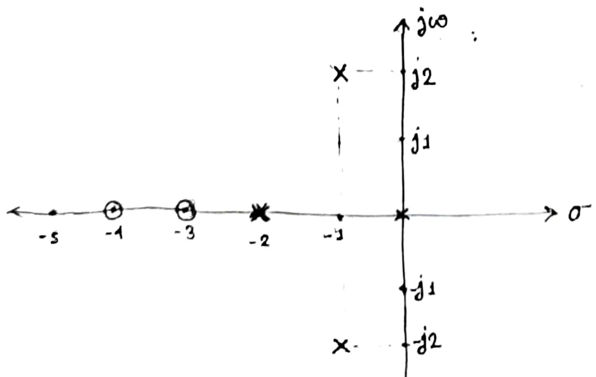
$\therefore$ poles are, $s_a = 0$, $s_b = s_c = -2$, $s_d, s_e = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$= \frac{-2 \pm \sqrt{(-2)^2 - 4.1.5}}{2}$$

$$= -1 \pm j2$$

The zeros are determined from the equation, $(s+3)(s+4) = 0$

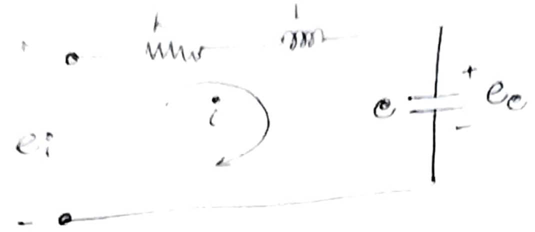
$\therefore$ zeros are, $s_1 = -3$, $s_2 = -4$.



$s = \sigma + j\omega$

(6)

Prob 3: Determine the transfer function of the



Soln: KVL equation of the circuit is given as,

$$e_i = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \ldots\ldots (1)$$

Taking Laplace transform of equation (1)

$$E_i(s) = R\,I(s) + Ls\,I(s) + \frac{1}{Cs}I(s)$$

$$= \left(R + Ls + \frac{1}{Cs}\right) I(s)$$

$$\therefore\ I(s) = \frac{E_i(s)}{\left(R + Ls + \frac{1}{Cs}\right)}$$

Now, voltage across the capacitor

$$e_c = \frac{1}{C}\int i\,dt.$$

taking Laplace Transform

$$E_c(s) = \frac{1}{Cs}I(s)$$

$$= \frac{1/Cs\ E_i(s)}{\left(R + Ls + \frac{1}{Cs}\right)}$$

$$\therefore \text{Transfer function} = \frac{E_c(s)}{E_i(s)} = \frac{1/Cs}{R + Ls + \frac{1}{Cs}}$$

$$= \frac{1}{s^2LC + sRC + 1}$$

$$= \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$E_i(s) \longrightarrow \boxed{\frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}} \longrightarrow E_c(s)$$

Find $\frac{V_{R2}(s)}{E_i(s)}$

Soln:

For loop 1

$$e_i = R_1 i_1 + \frac{1}{C}\int (i_1 - i_2)\,dt$$

L.T. $\Rightarrow$ $E_i(s) = R_1 I_1(s) + \frac{1}{Cs}\left[I_1(s) - I_2(s)\right]$

or, $E_i(s) = \left(R_1 + \frac{1}{Cs}\right) I_1(s) - \frac{1}{Cs} I_2(s)$

or, $I_1(s) = \dfrac{E_i(s) + \frac{1}{Cs} I_2(s)}{\left(R_1 + \frac{1}{Cs}\right)}$

For loop 2

$$0 = L\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C}\int (i_2 - i_1)\,dt$$

L.T. $\Rightarrow$ $0 = Ls I_2(s) + R_2 I_2(s) + \frac{1}{Cs}\left[I_2(s) - I_1(s)\right]$

or, $\frac{1}{Cs} I_1(s) = \left(Ls + \frac{1}{Cs} + R_2\right) I_2(s)$

or, $\dfrac{\frac{1}{Cs} E_i(s) + \left(\frac{1}{Cs}\right)^2 I_2(s)}{\left(R_1 + \frac{1}{Cs}\right)} = \left(R_2 + Ls + \frac{1}{Cs}\right) I_2(s)$

or, $\left[\left(R_1 + \frac{1}{Cs}\right)\left(R_2 + Ls + \frac{1}{Cs}\right) - \left(\frac{1}{Cs}\right)^2\right] I_2(s) = \frac{1}{Cs} E_i(s)$

or, $I_2(s) = \dfrac{\frac{1}{Cs} E_i(s)}{\left(R_1 + \frac{1}{Cs}\right)\left(R_2 + Ls + \frac{1}{Cs}\right) - \left(\frac{1}{Cs}\right)^2}$

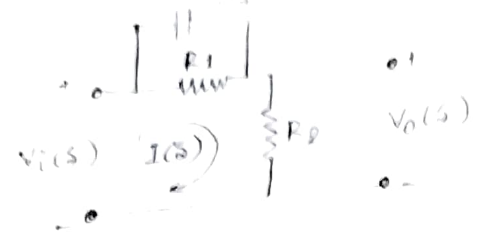
$\therefore$ voltage across R_2

$$v_{R2} = R_2 i_2$$

L.T. $\Rightarrow$ $V_{R2}(s) = R_2 I_2(s)$

$$\therefore \frac{V_{R2}(s)}{E_i(s)} = \frac{R_2/Cs}{\left(R_1 + \frac{1}{Cs}\right)\left(R_2 + Ls + \frac{1}{Cs}\right) - \left(\frac{1}{Cs}\right)^2}$$

(5)



Sol^n $V_i(s) = \left(\frac{1}{Cs} \parallel R_1\right) I(s) + R_2\, I(s)$

$$= \frac{R_1/Cs}{\frac{1}{Cs} + R_1}\, I(s) + R_2\, I(s)$$

$$\therefore I(s) = \frac{V_i(s)}{\frac{R_1/Cs}{R_1 + 1/Cs} + R_2} = \frac{V_i(s)}{\frac{R_1}{1 + R_1 Cs} + R_2}$$

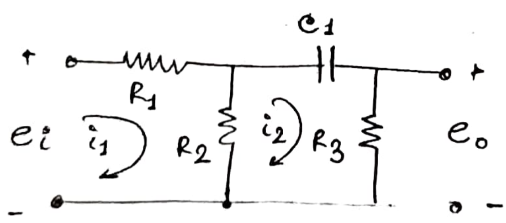
$$= \frac{V_i(s)(1 + R_1 Cs)}{R_1 + R_2 + R_1 R_2 Cs}$$

$\therefore$ voltage across R_2

$V_o(s) = I(s) R_2$

$$\therefore \text{Transfer func}^n = \frac{V_o(s)}{V_i(s)} = \frac{R_2(1 + R_1 Cs)}{R_1 + R_2 + R_1 R_2 Cs}.$$

Prob. 6 Determine the Transferfunction of the network.



Sol^n For loop 1

$e_i = R_1 i_1 + (i_1 - i_2) R_2$

L.T $\Rightarrow$ $E_i(s) = R_1 I_1(s) + R_2 [I_1(s) - I_2(s)]$

$$= (R_1 + R_2) I_1(s) - R_2 I_2(s)$$

$$\therefore I_1(s) = \frac{E_i(s) + R_2 I_2(s)}{R_1 + R_2}$$

For Loop 2

$0 = R_2(i_2 - i_1) + \frac{1}{C_1}\int i_2\, dt + R_3 i_2$

or, $R_2 I_1(S) = \left[R_2 + R_3 + \frac{1}{C_1 S} \right] I_2(S)$

or, $R_2 \frac{E_i(S) + R_2 I_2(S)}{(R_1 + R_2)} = \left[R_2 + R_3 + \frac{1}{C_1 S} \right] I_2(S)$

or, $R_2 E_i(S) + \cancel{R_2^2 I_2(S)} = \left[R_1R_2 + R_1R_3 + \frac{R_1}{C_1 S} + \cancel{R_2^2} + R_2R_3 + \frac{R_2}{C_1 S} \right] I_2(S)$

or, $$I_2(S) = \frac{R_2 E_i(S) \cdot C_1 S}{(R_1 + R_2) + (R_1R_2 + R_2R_3 + R_3R_1) C_1 S}$$

∴ voltage across R_3.

$$e_o = R_3 i_2$$

L.T ⇒ $E_o(S) = R_3 I_2(S)$

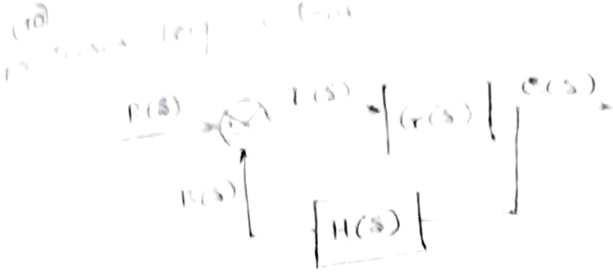
$$\frac{E_o(S)}{E_i(S)} = \frac{R_2 R_3 C_1 S}{(R_1 + R_2) + (R_1R_2 + R_2R_3 + R_3R_1) C_1 S}$$

Properties of Transfer function -

1. The Transfer function is defined only for a linear time-invarient system. It is not defined for non-linear systems.
2. The transfer function between a pair of input and output variables is the ratio of the Laplace transform of the output to the Laplace transform of the input.
3. All initial conditions of the systems are set to zero.
4. The Transfer function is independent of the input of the system.

Procedures to derive Transfer function of a system -

1. Develop the differential equation for the system by using the physical laws, eg. Newton's law and Kirchhoff's laws.
2. Take the laplace transform of the differential equation under the zero initial conditions.
3. Take the ratio of output $C(S)$ to the input $R(S)$. This ratio is Transfer function.



R(S) - Referance input

C(S) = controlled output

G(S) = Forward path Transfer function $= \frac{C(S)}{E(S)}$

$\therefore C(S) = G(S)E(S)$

E(S) = Error or actuating signal

$= R(S) - B(S)$

B(S) = Feedback signal $= C(S)H(S)$

H(S) = Transfer function of feedback element $= \frac{B(S)}{C(S)}$

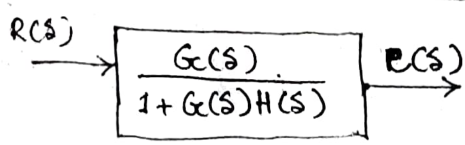
$$\therefore C(S) = G(S)E(S)$$

$$= G(S)[R(S) - B(S)]$$

$$= G(S)[R(S) - C(S)H(S)]$$

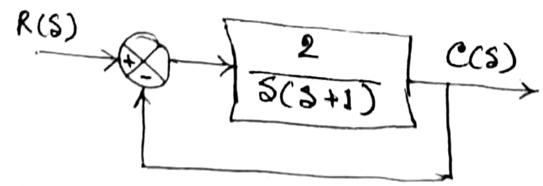
or, $[1 + G(S)H(S)]C(S) = G(S)R(S)$

$$\therefore \boxed{T(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}}$$ → closed loop Transfer function



Prob ⑦ A system having Loop transfer function $\frac{2}{S(S+1)}$. considering unity feedback (-ve) find out the closed loop Transfer function

Soln



$$\therefore \frac{C(S)}{R(S)} = \frac{2/S(S+1)}{1 + 2/S(S+1) \times 1}$$

$$= \frac{2}{S^2 + S + 2}$$

Mechanical System

Translatory System — Rotational System

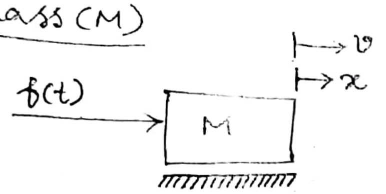
Translatory system.

The motion takes place along the straight line is known as Translational motion. There are three types of forces that resists motion.

Idealised Parameters

Mass (M) — Damper (B) — Spring (K)

① Mass (M)



x = displacement
v = velocity.

A force is applied to the mass produces an acceleration of the mass. The reaction force is equal to the applied force and is opposite in direction.

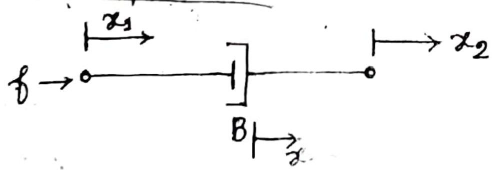
force = mass × acceleration

$$f = M \times \ddot{x}$$

$$= M\frac{d^2x}{dt^2} = M\frac{dv}{dt} = MDv$$

$$= MD^2x.$$

② Damper (B)



Reaction damping force = damping (B) × relative velocity of the two ends of the dashpot.

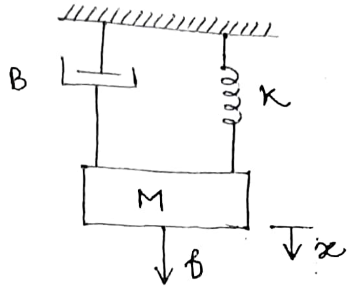
or, $f = B(\dot{x}_1 - \dot{x}_2)$

$= B(Dx_1 - Dx_2)$

Function force = ~~Ksat~~ stiffness of spring (K) × amount of deformation of the spring.

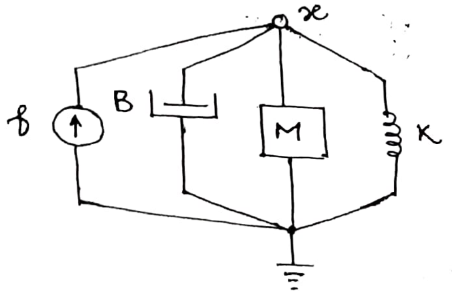
$f = K(x_1 - x_2)$

Prob(1) Find out the Transfer function for this physical model



Soln The fixed ends of B and K are considered as the reference points and the lower ends connected to the mass are considered as nodes.

For developing a mechanical circuit diagram, the lower end of the mass is also considered as a fixed point although the same is not fixed.



f = Force
x = displacement
M = mass
B = friction coefficient
K = Spring constant

$$f = M\ddot{x} + B\dot{x} + Kx$$

$$= M\frac{d^2x}{dx^2} + B\frac{dx}{dt} + Kx.$$

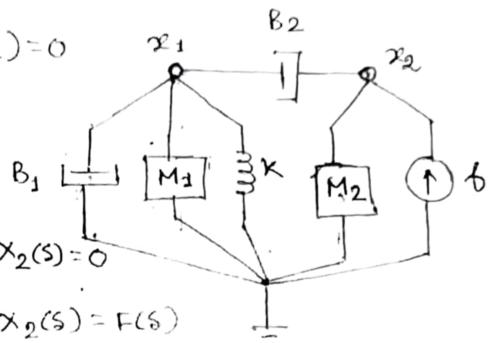
Taking Laplace Transform,

$$(MS^2 + BS + K)X(S) = F(S)$$

or, $$\boxed{\frac{X(S)}{F(S)} = \frac{1}{MS^2 + BS + K}}$$

Sol^n

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + Kx_1 + B_2(\dot{x}_1 - \dot{x}_2) = 0$$
$$M_2\ddot{x}_2 + B_2(\dot{x}_2 - \dot{x}_1) = f$$



Taking laplace Transform,

$$[M_1S^2 + (B_1+B_2)S + K]X_1(S) - B_2S\,X_2(S) = 0$$

and, $-B_2S\,X_1(S) + (M_2S^2 + B_2S)X_2(S) = F(S)$

$$\begin{bmatrix} M_1S^2 + (B_1+B_2)S + K & -B_2S \\ -B_2S & M_2S^2 + B_2S \end{bmatrix}\begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} = \begin{bmatrix} 0 \\ F(S) \end{bmatrix}$$

$$X_1(S) = \frac{\begin{vmatrix} 0 & -B_2S \\ F(S) & M_2S^2 + B_2S \end{vmatrix}}{\Delta} = \frac{B_2S\,F(S)}{\Delta}$$

where $\Delta = [M_1S^2 + (B_1+B_2)S + K](M_2S^2 + B_2S) - (B_2S)^2$.

$$\therefore \boxed{\frac{X_1(S)}{F(S)} = \frac{B_2S}{\Delta}}$$

and, $$X_2(S) = \frac{\begin{vmatrix} M_1S^2 + (B_1+B_2)S + K & 0 \\ -B_2S & F(S) \end{vmatrix}}{\Delta}$$

$$= \frac{[M_1S^2 + (B_1+B_2)S + K]\,F(S)}{\Delta}$$

$$\therefore \boxed{\frac{X_2(S)}{F(S)} = \frac{M_1S^2 + (B_1+B_2)S + K}{\Delta}}$$

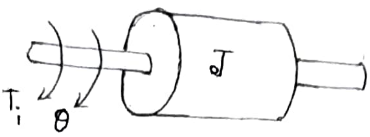
The [illegible] motion [illegible] body can be [illegible] motion of a body about a fixed axis. There are 3 types of torques resists the rotational motion

Rotational System

↓

Inertia Torque Damping Torque Spring Torque

① Inertia Torque (J)



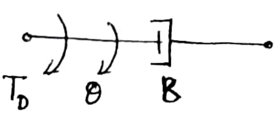
Inertia torque = moment of Inertia × angular acceleration

$$T_i = J \times \frac{d^2\theta}{dt^2} \quad (N\text{-}m)$$

$$= J \frac{d\omega}{dt}$$

[θ = angular displacement]
[ω = angular velocity]

② Damping Torque (T_D)

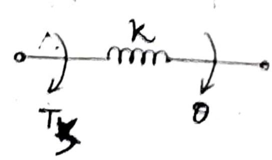


$$\therefore T_D = B \times \frac{d\theta}{dt}$$

$$= \underbrace{B}_{\text{damping coefficient}} \times \underbrace{\omega}_{\text{angular velocity}}$$

③ Spring Torque (T_s)

$T_s = K.\theta$



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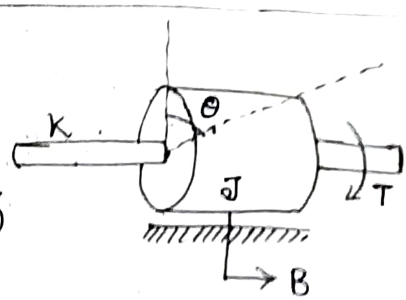
prob



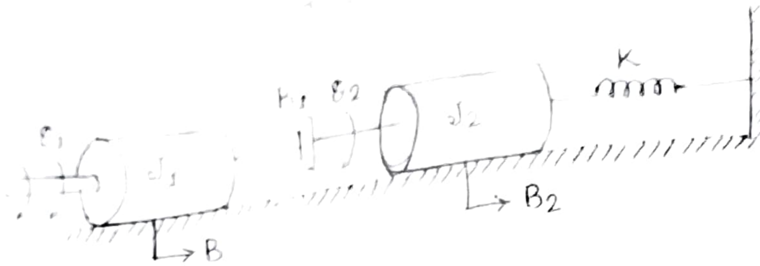
$$T = J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

L.T ⇒

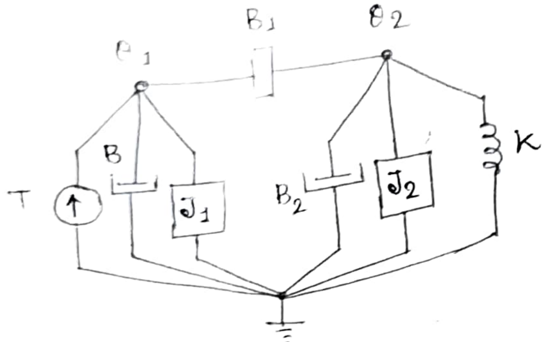
$$\therefore T(s) = Js^2\,\theta(s) + Bs\,\theta(s) + K\,\theta(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K}$$

$$T(s) \longrightarrow \boxed{\frac{1}{Js^2+Bs+K}} \longrightarrow \theta(s)$$



spring torque



angular acceler

angular displa

angular veloci

$$J_1 \frac{d^2\theta_1}{dt^2} + B\frac{d\theta_1}{dt} + B_1\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right) = T$$

$$J_2 \frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} + B_1\left(\frac{d\theta_2}{dt} - \frac{d\theta_1}{dt}\right) + K.\theta_2 = 0$$

$B \times \frac{d\theta}{dt}$

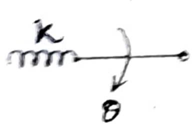
$B \times \omega$ → angular velocity.

king L.T ⇒

$$(J_1 s^2 + Bs + B_1 s)\,\theta_1(s) - B_1 s\,\theta_2(s) = T(s)$$

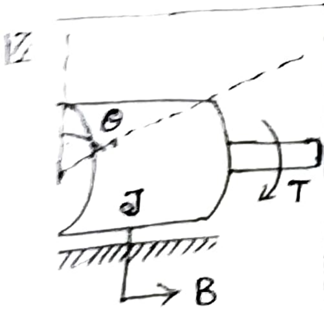
$$-B_1 s\,\theta_1(s) + (J_2 s^2 + B_2 s + B_1 s + K)\,\theta_2(s) = 0$$

$$\begin{bmatrix} J_1 s^2 + Bs + B_1 s & -B_1 s \\ -B_1 s & (J_2 s^2 + B_2 s + B_1 s + K) \end{bmatrix} \begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$$



$$\theta_1(s) = \frac{\begin{vmatrix} T(s) & -B_1 s \\ 0 & (J_2 s^2 + B_2 s + B_1 s + K) \end{vmatrix}}{\Delta}$$

$$\Delta = (J_1 s^2 + Bs + B_1 s)(J_2 s^2 + B_2 s + B_1 s + K) - (B_1 s)^2$$



$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{J_2 s^2 + (B_2 + B_1)s + K}{\Delta}}$$

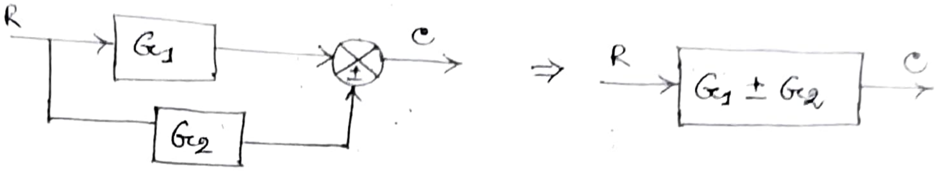
$$\theta_2(s) = \frac{\begin{vmatrix} (J_1 s^2 + Bs + B_1 s) & T(s) \\ -B_1 s & 0 \end{vmatrix}}{\Delta}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{B_1 s}{\Delta}}$$

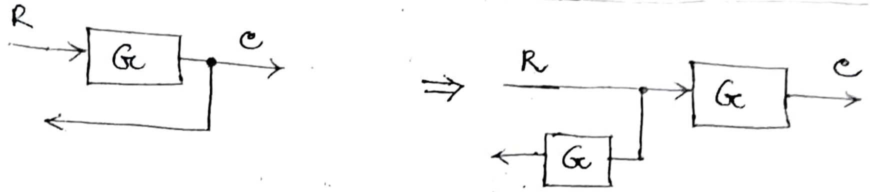
Block Diagram Reduction Technique

① Blocks in cascade / series

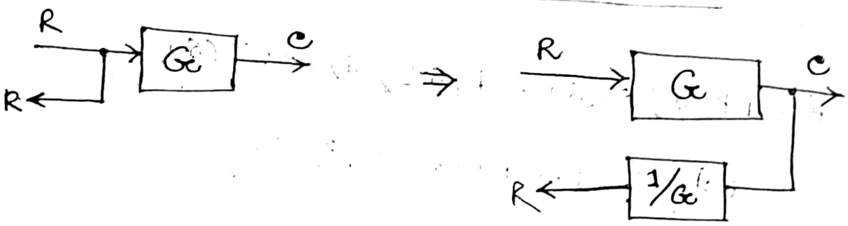
② Blocks in parallel



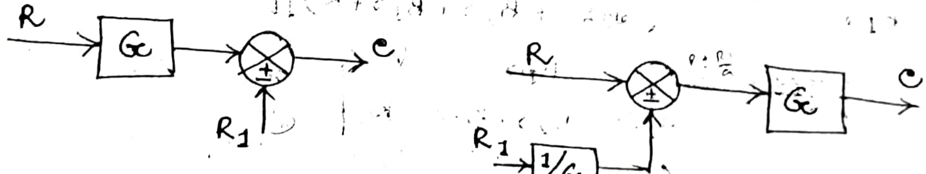
③ Moving the take off point comes ahead of a block



④ Moving the take off point after the block



⑤ Moving the summing points comes ahead of a block

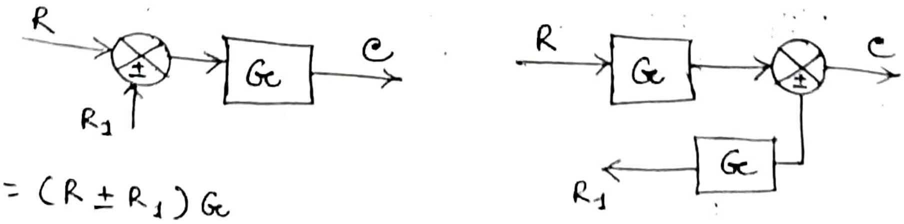


$C = (RG \pm R_1)$

$C = (R \pm \frac{R_1}{G})G$

$= RG \pm R_1$

⑥ Moving the summing point comes after of a block



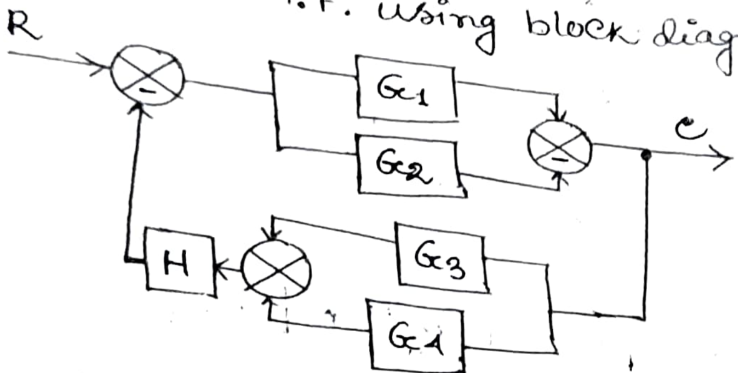
$C = (R \pm R_1)G$

$C = RG \pm R_1G$

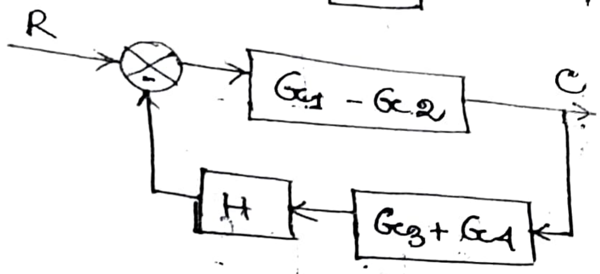
$= (R \pm R_1)G$

Ex.

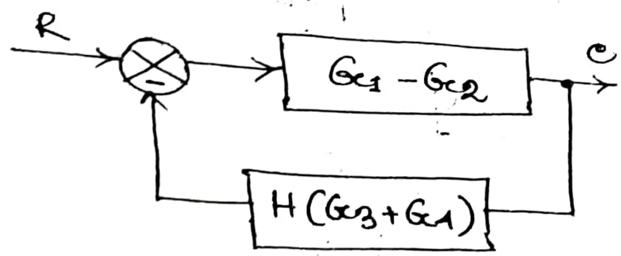
Prob ① Find the T.F. using block diagram Reduction Technique.



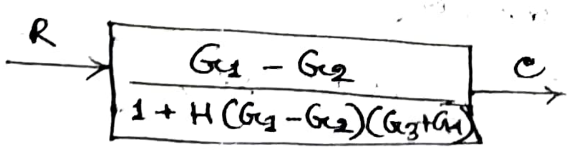
Soln

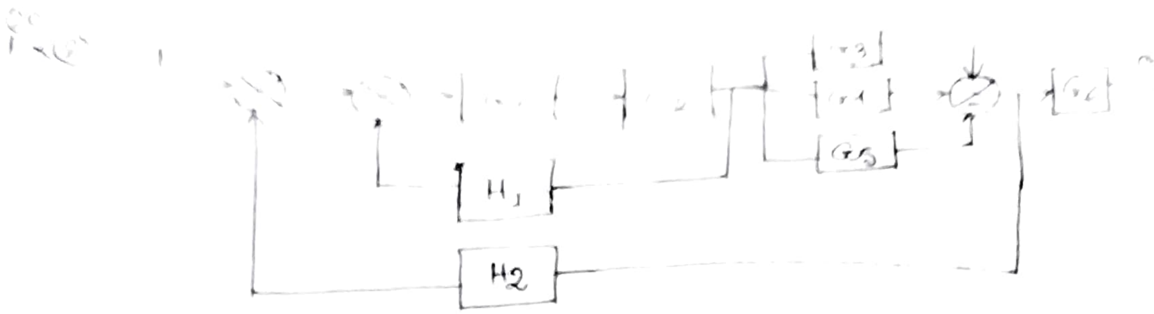


⇒

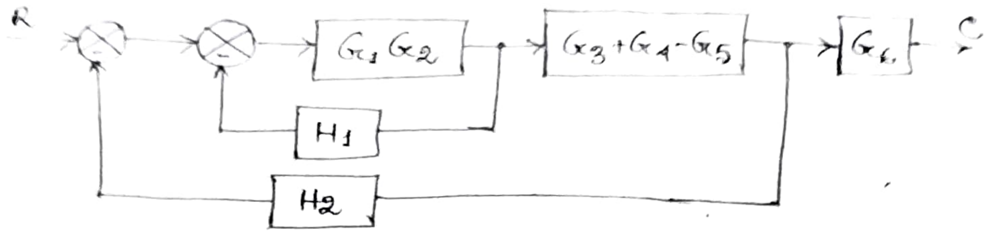


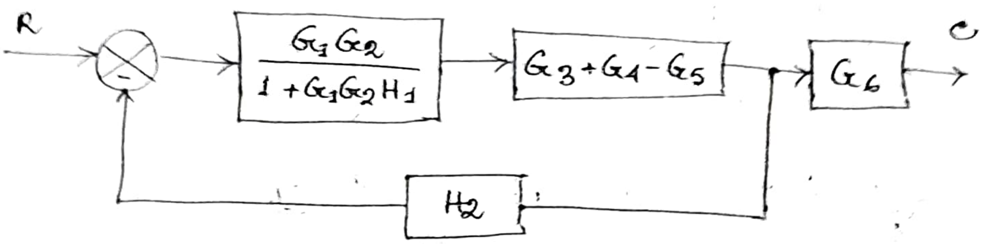
⇒

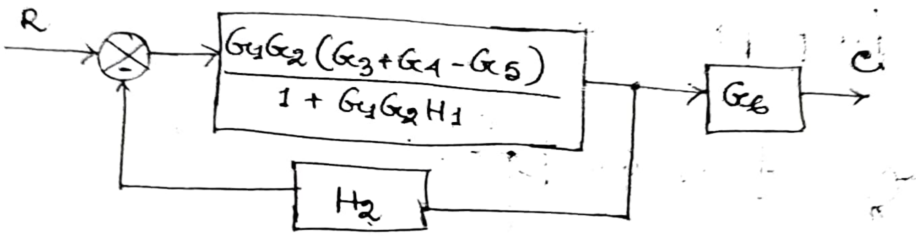


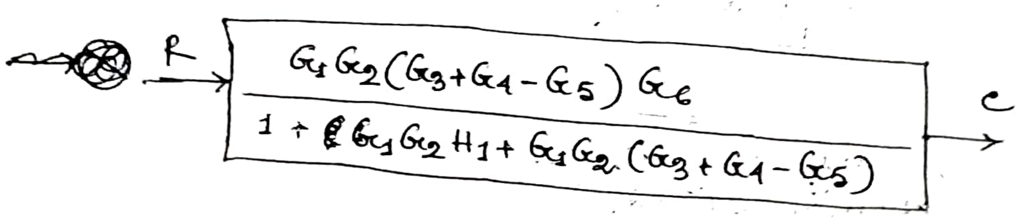


Soln

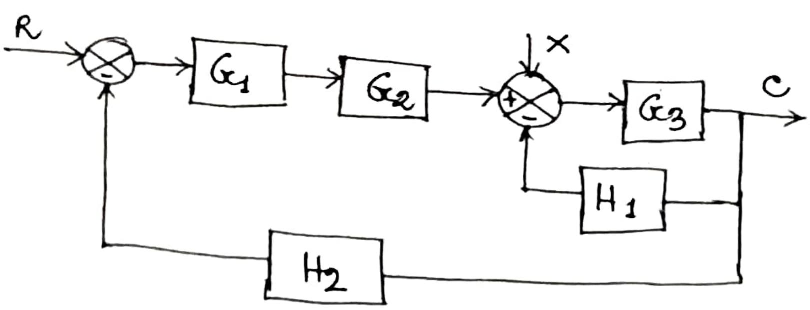








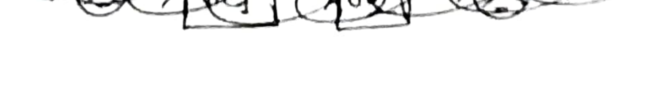
Ex. 3

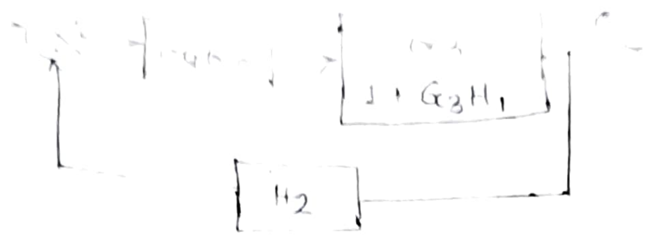


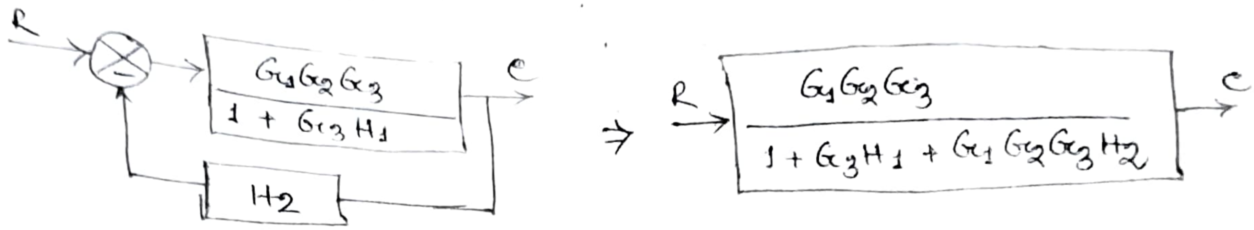
Find T.F. C/R and C/x.

Soln To find C/R,

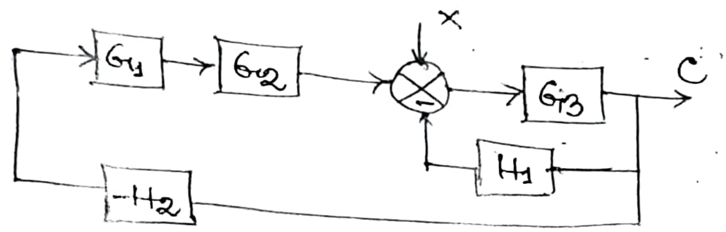
put $x = 0$,

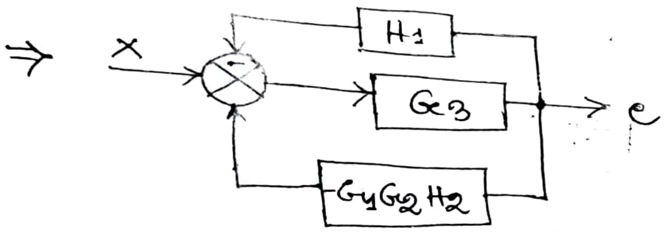


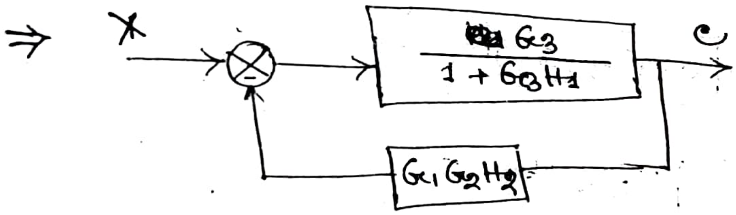


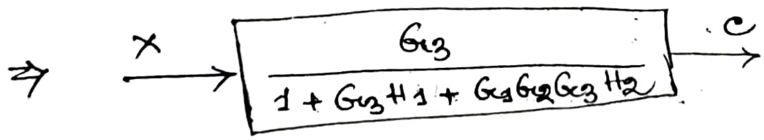


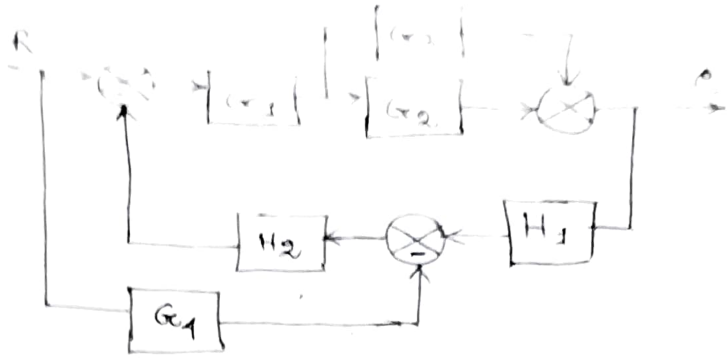
(*) To find C/X, put R=0,



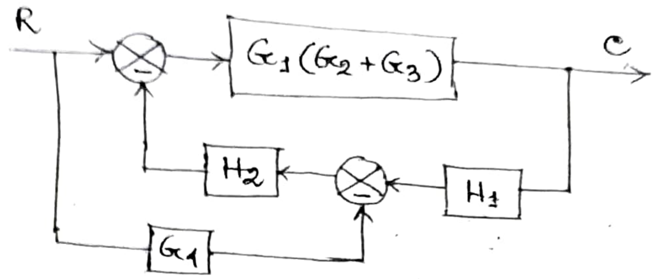




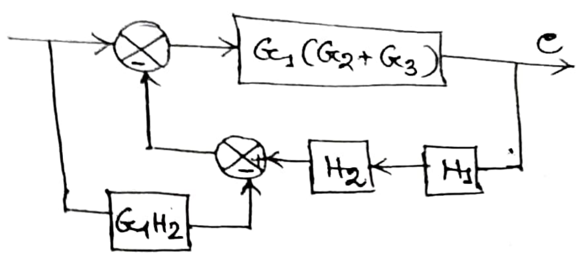



R
G_1
G_2
H_2
H_1
G_1
C

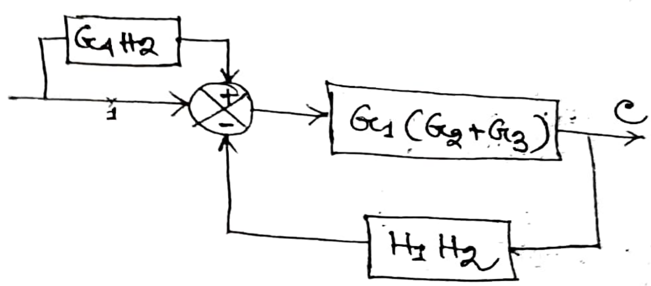
Soln


R
$G_1(G_2+G_3)$
C
H_2
H_1
G_1

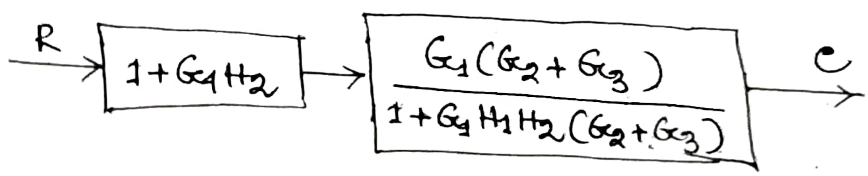
⇒


$G_1(G_2+G_3)$
C
H_2
H_1
G_1H_2

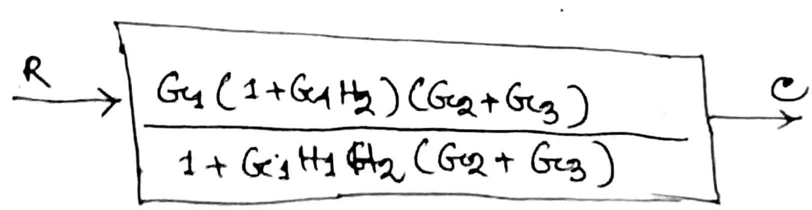
⇒

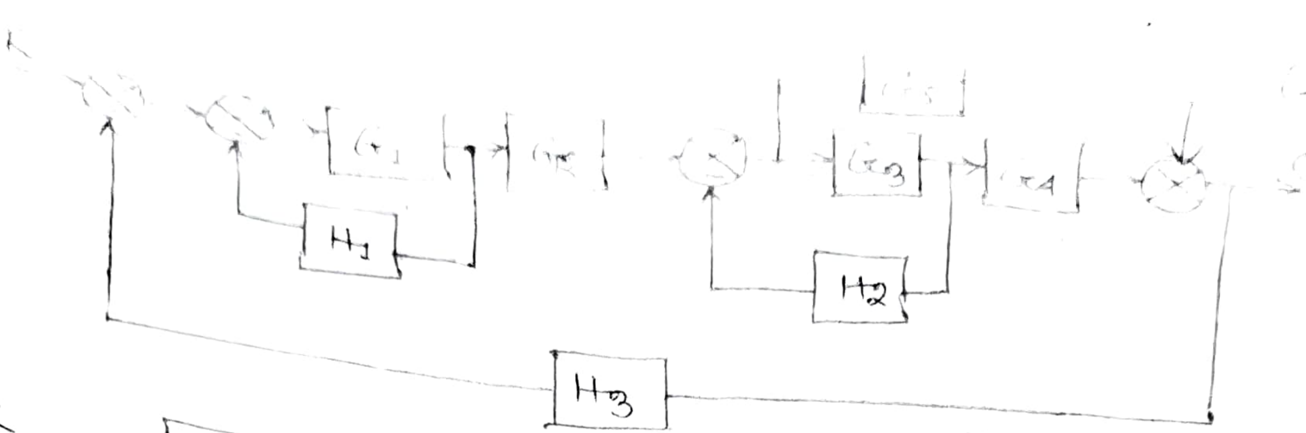

G_1H_2
1
$G_1(G_2+G_3)$
C
H_1H_2

⇒

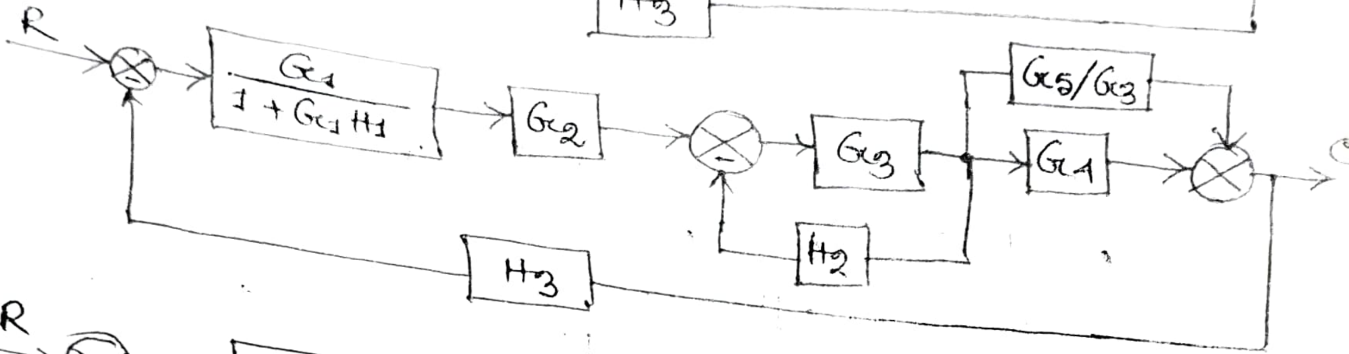

R
$1+G_1H_2$
$\frac{G_1(G_2+G_3)}{1+G_1H_1H_2(G_2+G_3)}$
C

⇒

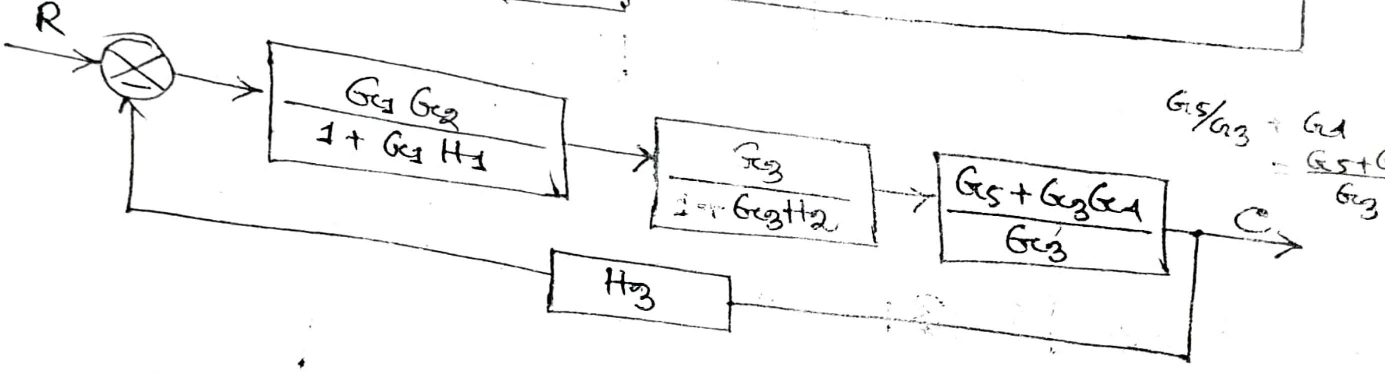

R
$\frac{G_1(1+G_1H_2)(G_2+G_3)}{1+G_1H_1H_2(G_2+G_3)}$
C



Sol:

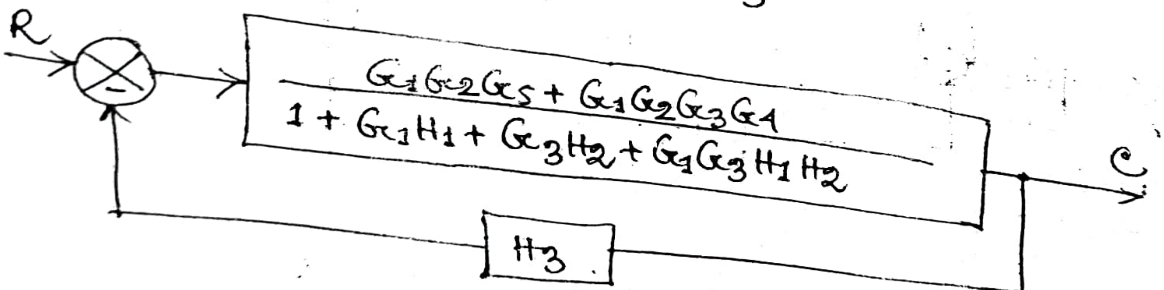


⇒

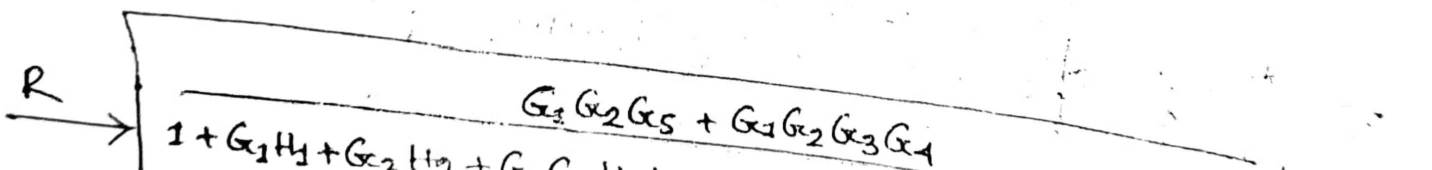


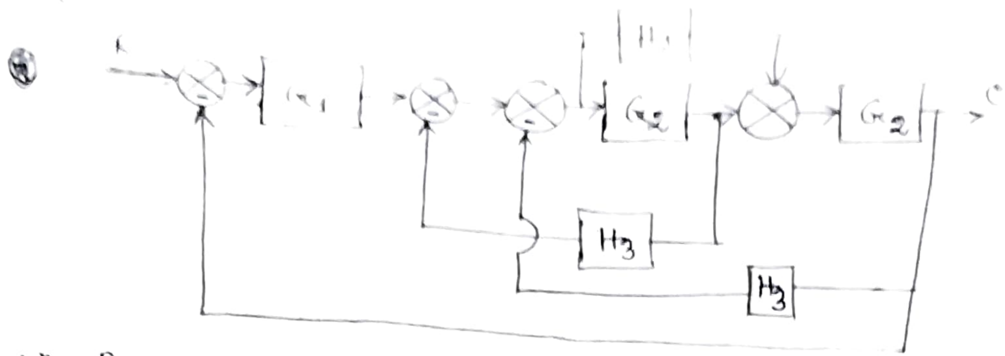
$$\frac{G_1 G_2}{(1+G_1H_1)} \cdot \frac{\cancel{G_3}}{(1+G_3H_2)} \cdot \frac{G_5+G_3G_4}{\cancel{G_3}}$$

⇒

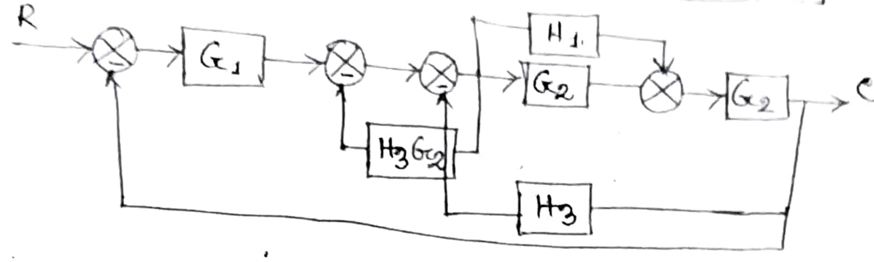


⇒

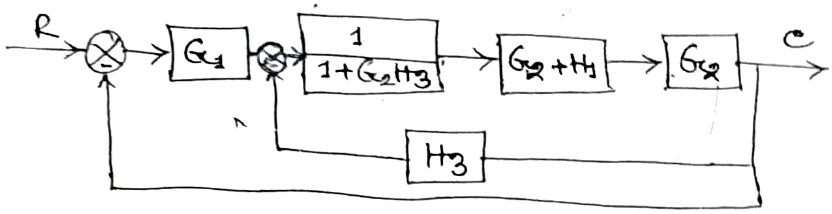



R
G_1
H_1
G_2
G_2
C
H_3
H_3

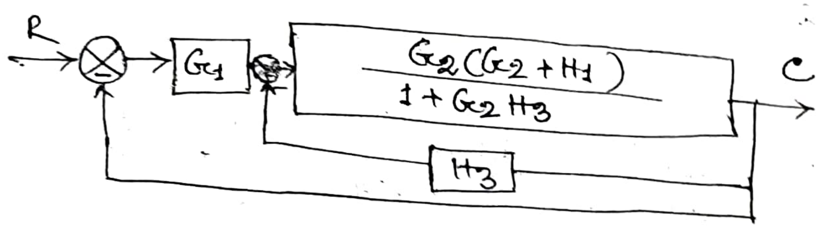
Soln


R
G_1
H_1
G_2
G_2
C
$H_3 G_2$
H_3

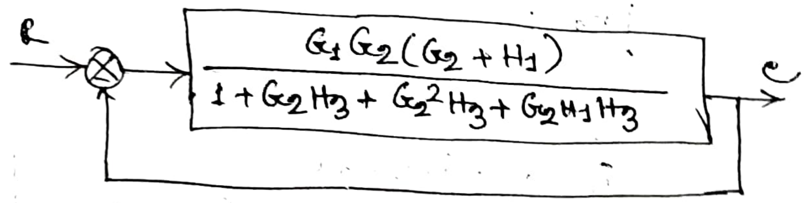
⇒


R
G_1
$\frac{1}{1+G_2H_3}$
G_2+H_1
G_2
C
H_3

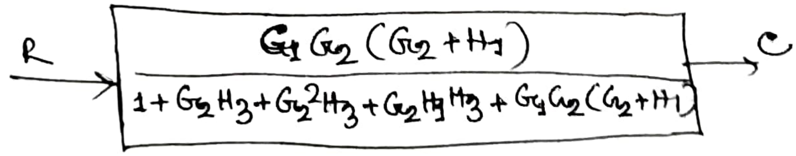
⇒


R
G_1
$\frac{G_2(G_2+H_1)}{1+G_2H_3}$
C
H_3

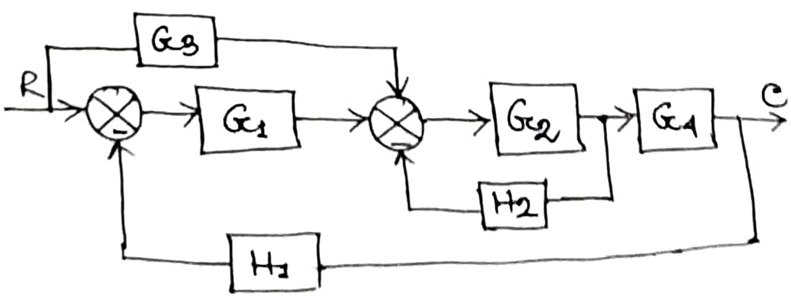
⇒

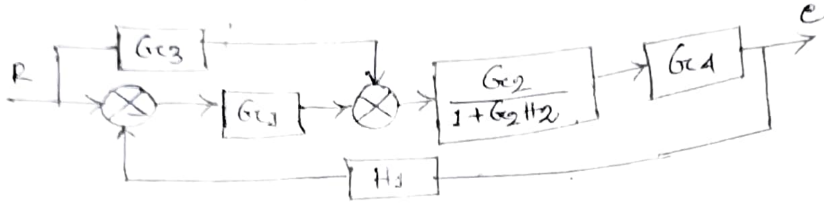

R
$\frac{G_1G_2(G_2+H_1)}{1+G_2H_3+G_2^2H_3+G_2H_1H_3}$
C

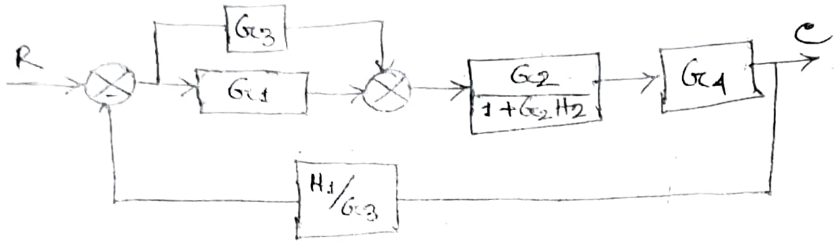
⇒

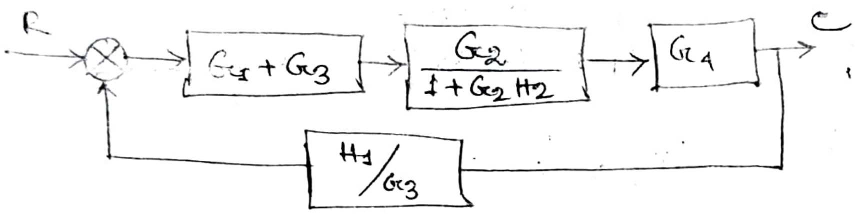

R
$\frac{G_1G_2(G_2+H_1)}{1+G_2H_3+G_2^2H_3+G_2H_1H_3+G_1G_2(G_2+H_1)}$
C

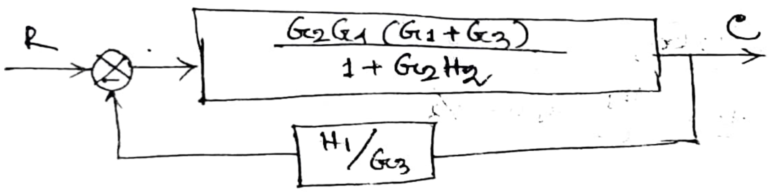
Ex. 7


G_3
R
G_1
G_2
G_4
C
H_2
H_1

⇒ 

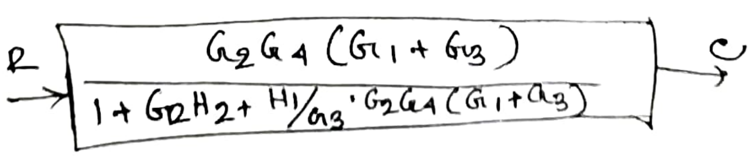
⇒ 

⇒ 

⇒ 

⇒

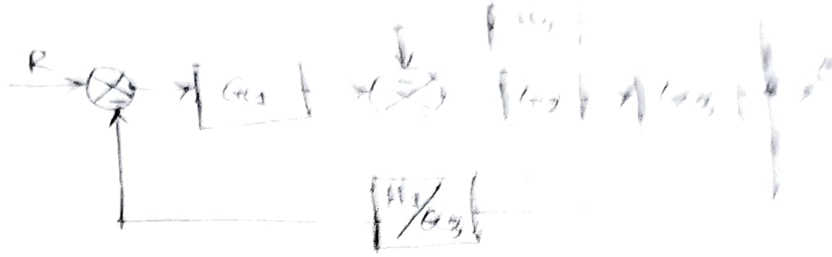
$$\frac{\dfrac{G_2G_4(G_1+G_3)}{1+G_2H_2}}{1+\dfrac{G_2G_4(G_1+G_3)}{1+G_2H_2}\cdot\dfrac{H_1}{G_3}} = \frac{G_2G_4(G_1+G_3)}{1+G_2H_2+\frac{H_1}{G_3}G_2G_4(G_1+G_3)}$$



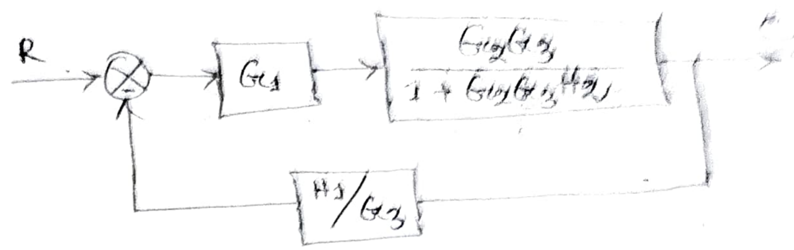
(26)

Ex (8)

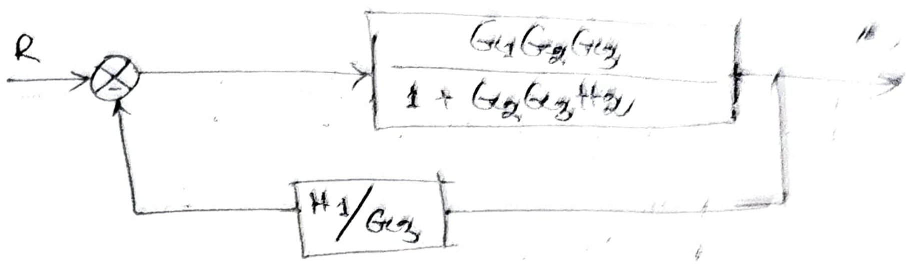
Soln



$\Rightarrow$



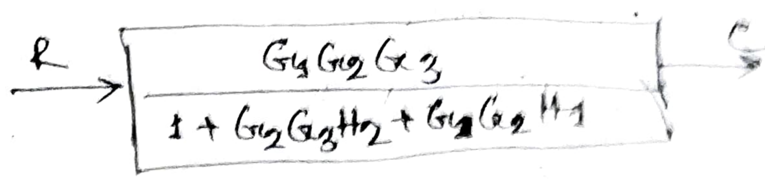
$\Rightarrow$

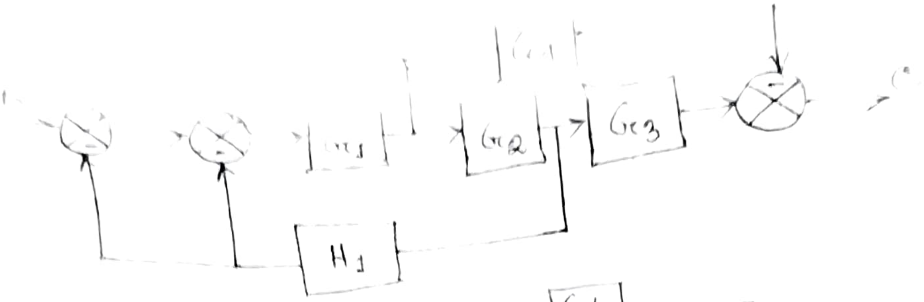


$$\frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 + \frac{H_1}{\cancel{G_3}} G_1 G_2 \cancel{G_3}}$$

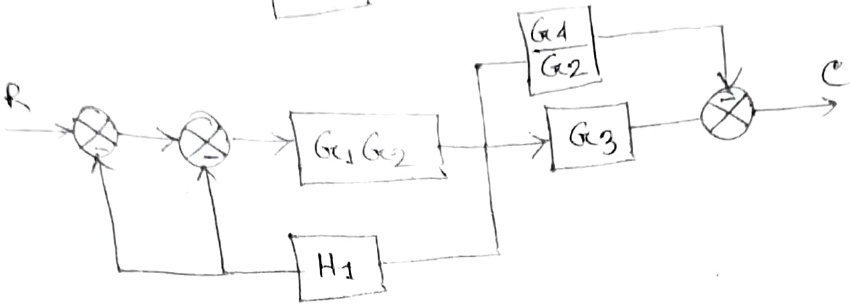
$$= \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 + G_1 G_2 H_1}$$

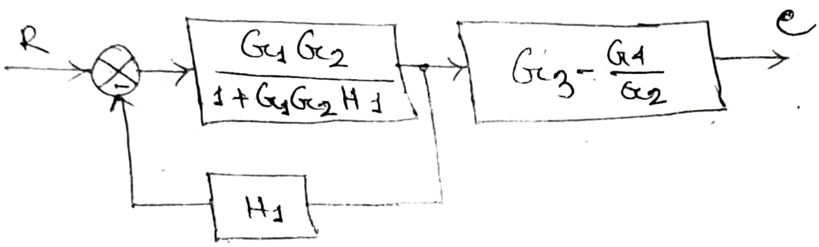
$\Rightarrow$





Soln

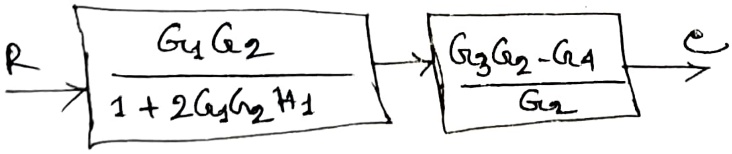


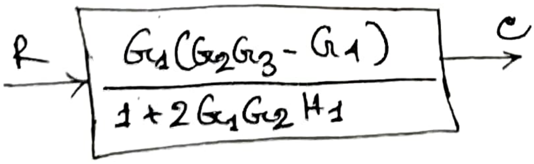
⇒ 

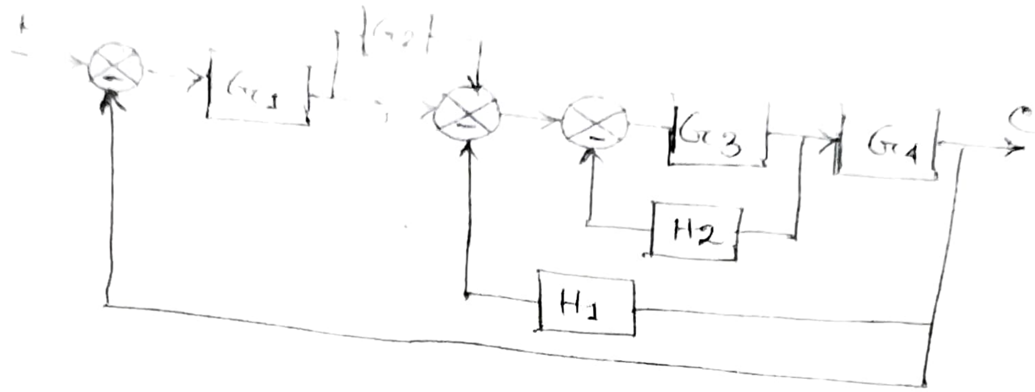
$$\frac{\dfrac{G_1G_2}{1+G_1G_2H_1}}{1+\dfrac{G_1G_2}{1+G_1G_2H_1}\cdot H_1} = \frac{G_1G_2}{1+G_1G_2H_1+G_1G_2H_1}$$

$$= \frac{G_1G_2}{1+2G_1G_2H_1}$$

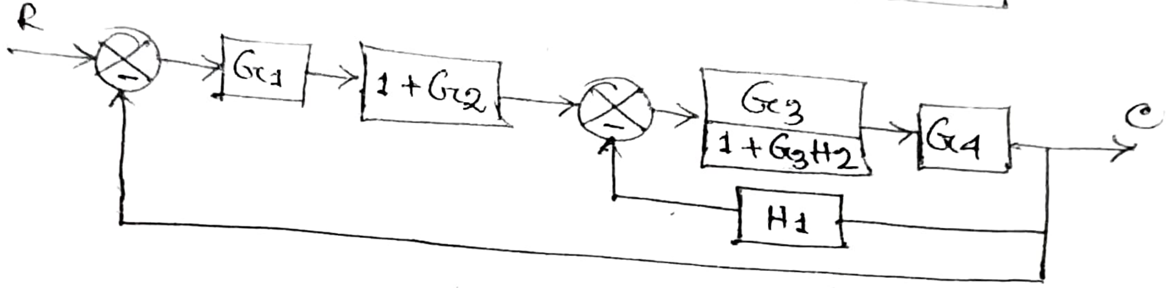
$$G_3-\frac{G_4}{G_2} = \frac{G_3G_2-G_4}{G_2}$$

⇒ 

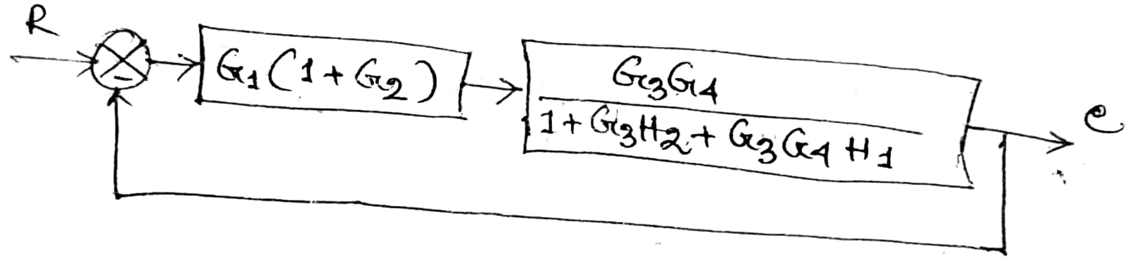
⇒ 


G1
G2
G3
G4
H2
H1
C

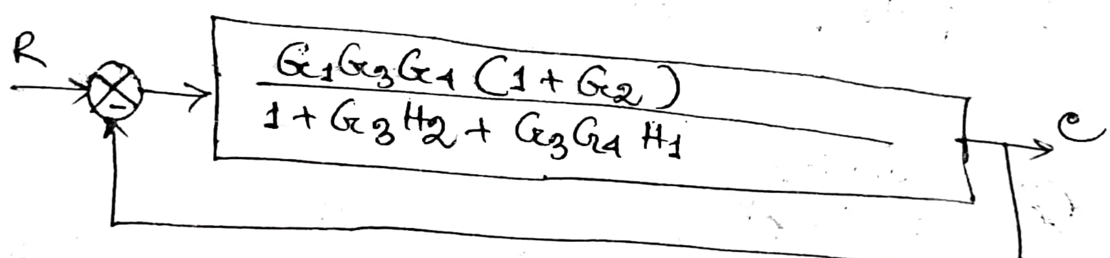
Soln


R
G1
1 + G2
G3 / 1 + G3H2
G4
H1
C

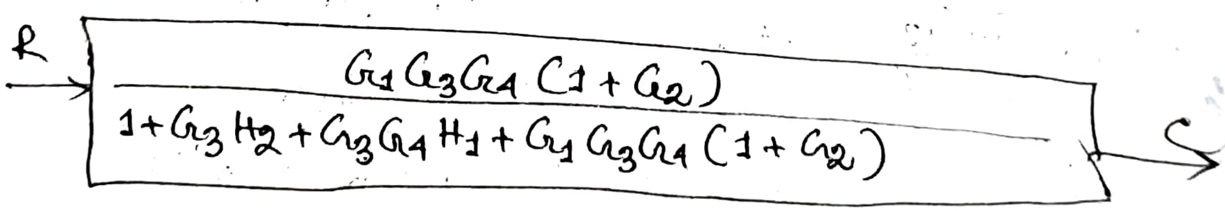
⇒


R
G1(1 + G2)
G3G4 / 1 + G3H2 + G3G4H1
C

⇒


R
G1G3G4(1 + G2) / 1 + G3H2 + G3G4H1
C

⇒


R
G1G3G4(1 + G2) / 1 + G3H2 + G3G4H1 + G1G3G4(1 + G2)
C

[illegible] terms of mathematical model

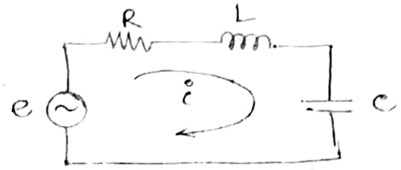
[illegible] translational mechanical system

$$\boxed{M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f}$$

② Force - voltage analogy

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = e$$

Now, $i = \frac{dq}{dt}$

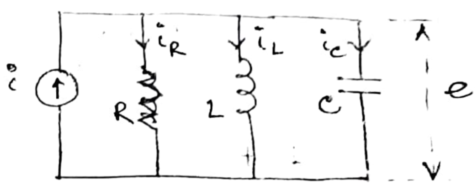


$$\boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = e}$$

③ Force current analogy

$$i = i_R + i_L + i_C$$

or, $C\frac{de}{dt} + \frac{e}{R} + \frac{1}{L}\int e\,dt = i$

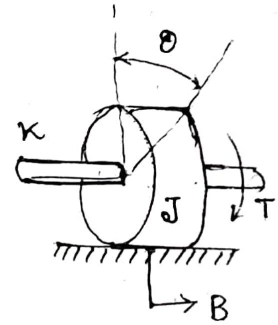


Now, $e = \frac{d\phi}{dt}$

$$\therefore \boxed{C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} = i}$$

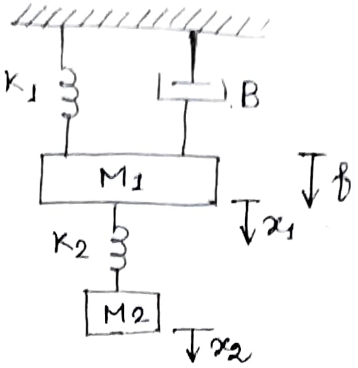
④ Rotational mechanical system

$$\boxed{J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta = T}$$

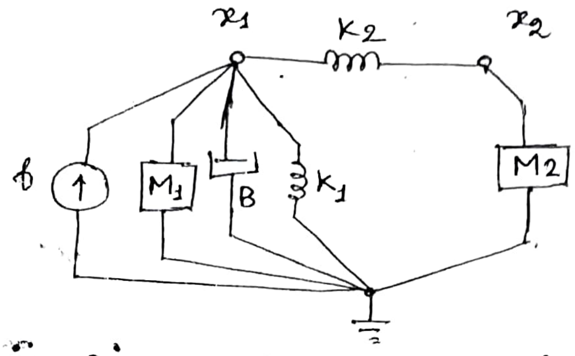


Translation mechanical system	Translation Rotational system	Force-voltage (f-v) analogy	Force-current (f-i) analogy
f	T	e	i
M	J	L	C
B	B	R	1/R
K	K	1/C	1/L
x	θ	q	φ
v	ω	i	e

Ex. Draw the electrical analogous ckt using force voltage analogy for the mechanical system as shown in fig.



Solu^n



$\begin{cases} s = \text{deri} \\ 1/s = \text{inte} \end{cases}$

$$M_1 \ddot{x}_1 + B\dot{x}_1 + K_1 x_1 + K_2(x_1 - x_2) = f$$

$$\text{L.T.} \rightarrow (M_1 s^2 + Bs + K_1 + K_2) X_1(s) - K_2 X_2(s) = F(s) \quad \cdots (1)$$

$$K_2(x_2 - x_1) + M_2 \ddot{x}_2 = 0$$

$$\text{LT} \rightarrow -K_2 X_1(s) + (M_2 s^2 + K_2) X_2(s) = 0 \quad \cdots (2)$$

In force voltage analogy $\rightarrow$

$\begin{cases} sQ = I \\ \frac{dQ}{dt} = I \end{cases}$

from (1) $\Rightarrow$

$$\left(L_1 s^2 + Rs + \frac{1}{C_1} + \frac{1}{C_2}\right) Q_1(s) - \frac{1}{C_2} Q_2(s) = E(s)$$

$$\text{or, } \left(L_1 s + R + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right) I_1(s) - \frac{1}{C_2 s} I_2(s) = E(s)$$

$$\xrightarrow{\text{ILT}} \text{or, } L_1 \frac{di_1}{dt} + Ri_1 + \frac{1}{C_1}\int i_1\,dt + \frac{1}{C_2}\int i_1\,dt - \frac{1}{C_2}\int i_2\,dt = e$$

$$\text{or, } L_1 \frac{di_1}{dt} + Ri_1 + \frac{1}{C_1}\int i_1\,dt + \frac{1}{C_2}\int (i_1 - i_2)\,dt = e \quad \cdots (3)$$

from (2) $\Rightarrow$

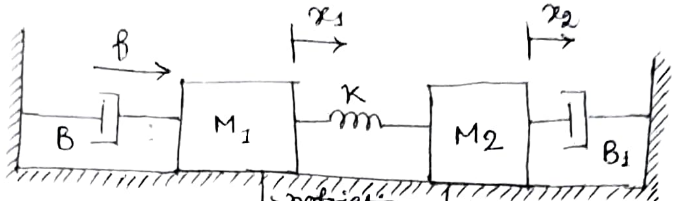
$$-\frac{1}{C_2} Q_1(s) + \left(L_2 s^2 + \frac{1}{C_2}\right) Q_2(s) = 0$$

$$\text{or, } -\frac{1}{C_2 s} I_1(s) + \left(L_2 s + \frac{1}{C_2 s}\right) I_2(s) = 0$$

$$\xrightarrow{\text{ILT}} \text{or, } -\frac{1}{C_2}\int i_1\,dt + L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt = 0$$

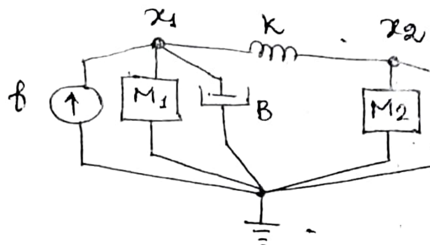
$$\text{or, } L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1)\,dt = 0 \quad \cdots (4)$$

Ex②



Find out the equivalent electrical ckt by using force-voltage and force-current analogy.

Soln



$\frac{dQ}{dt} = I$

$sQ(s) = I(s)$

$$M_1\ddot{x}_1 + B\dot{x}_1 + K(x_1 - x_2) = f$$

LT → $(M_1 s^2 + Bs + K)X_1(s) - K X_2(s) = F(s)$ ..... ①

$$M_2\ddot{x}_2 + B_1\dot{x}_2 + K(x_2 - x_1) = 0$$

LT → $-KX_1(s) + (M_2 s^2 + B_1 s + K)X_2(s) = 0$ .... ②

In force voltage analogy

From ① ⇒ $(L_1 s^2 + Rs + \frac{1}{C})Q_1(s) - 1/C\, Q_2(s) = E(s)$

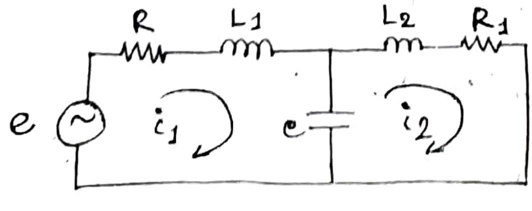
$(L_1 s + R + \frac{1}{Cs}) I_1(s) - \frac{1}{Cs} I_2(s) = E(s)$

taking ILT → $L_1\frac{di_1}{dt} + Ri_1 + \frac{1}{C}\int(i_1 - i_2)\,dt = e$ ... ③

From ② ⇒ $-1/C\, Q_1(s) + (L_2 s^2 + R_1 s + 1/C)Q_2(s) = 0$

$(L_2 s + R_1 + \frac{1}{Cs}) I_2(s) - \frac{1}{Cs} I_1(s) = 0$

taking ILT → $L_2\frac{di_2}{dt} + R_1 i_2 + \frac{1}{C}\int(i_2 - i_1)\,dt = 0$ ... ④



In force ...

From ① ⇒ $\left(C_1 s^2 + \frac{1}{R} s + \frac{1}{L}\right)\phi_1(s) - \frac{1}{L}\phi_2(s) = I(s)$

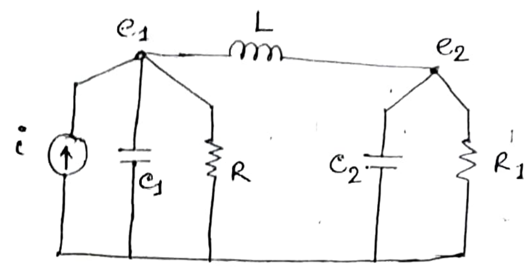
or, $\left(C_1 s + \frac{1}{R} + \frac{1}{Ls}\right)E_1(s) - \frac{1}{Ls}E_2(s) = I(s)$

taking ILT ⇒ $$C_1 \frac{de_1}{dt} + \frac{1}{R} e_1 + \frac{1}{L}\int (e_1 - e_2)\,dt = i \quad (5)$$

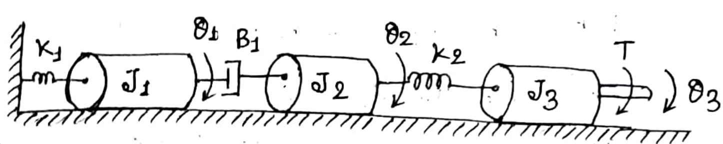
From ② ⇒ $-\frac{1}{L}\phi_1(s) + \left(C_2 s^2 + \frac{1}{R_1} s + \frac{1}{L}\right)\phi_2(s) = 0$

or, $\left(C_2 s + \frac{1}{R_1} + \frac{1}{Ls}\right)E_2(s) - \frac{1}{Ls}E_1(s) = 0$

taking ILT ⇒ $$C_2 \frac{de_2}{dt} + \frac{1}{R_1} e_2 + \frac{1}{L}\int (e_2 - e_1)\,dt = 0 \quad (6)$$

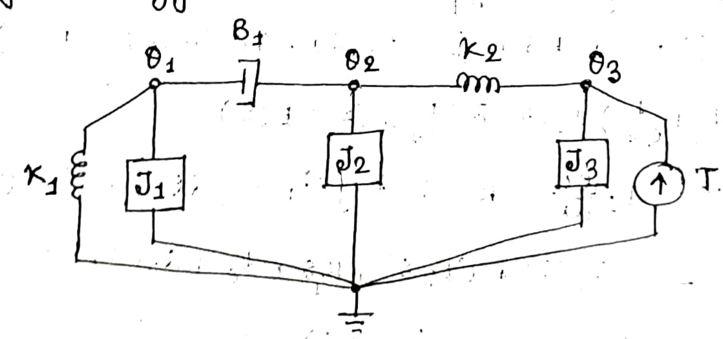


Ex③



What is the equivalent electrical system by using force voltage analogy.

Solu^n.



$$J_1\ddot{\theta}_1 + K_1\theta_1 + B_1(\dot{\theta}_1 - \dot{\theta}_2) = 0$$

LT → $(J_1 s^2 + K_1 + B_1 s)\,\theta_1(s) - B_1 s\,\theta_2(s) = 0$ ...... ①

$$J_2\ddot{\theta}_2 + B_1(\dot{\theta}_2 - \dot{\theta}_1) + K_2(\theta_2 - \theta_3) = 0$$

LT → $(J_2 s^2 + B_1 s + K_2)\,\theta_2(s) - B_1 s\,\theta_1(s) - K_2\,\theta_3(s) = 0$ .. ②

$(J_3 s^2 + K_2)\theta_3(s) \quad \ldots \quad T(s)$ (3)

In force voltage analogy

from ① ⇒

$$\left(L_1 s^2 + \frac{1}{C_1} + R_1 s\right) Q_1(s) - R_1 s\, Q_2(s) = 0$$

$$\text{or, } \left(L_1 s + \frac{1}{C_1 s} + R_1\right) I_1(s) - R_1 I_2(s) = 0.$$

taking ILT ⇒

$$L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1\, dt - R_1(i_1 - i_2) = 0 \quad \ldots \text{④}$$

from ② ⇒

$$\left(L_2 s^2 + R_1 s + \frac{1}{C_2}\right) Q_2(s) - R_1 s\, Q_1(s) - \frac{1}{C_2} Q_3(s) = 0$$

$$\text{or, } \left(L_2 s + R_1 + \frac{1}{C_2 s}\right) I_2(s) - R_1 I_1(s) - \frac{1}{C_2 s} I_3(s) = 0$$

taking ILT ⇒

$$L_2 \frac{di_2}{dt} + R_1(i_2 - i_1) + \frac{1}{C_2}\int (i_2 - i_3)\, dt = 0 \quad \ldots \text{⑤}$$

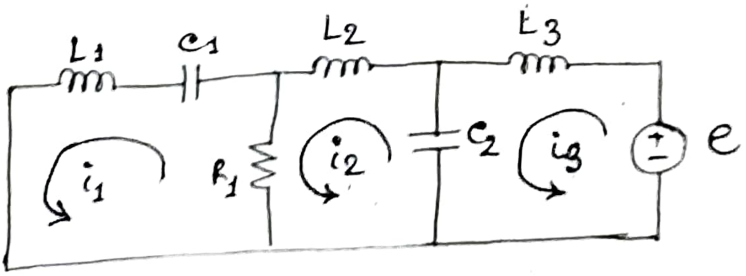
from ③ ⇒

$$\left(L_3 s^2 + \frac{1}{C_2}\right) Q_3(s) - \frac{1}{C_2} Q_2(s) = E(s)$$

$$\text{or, } \left(L_3 s + \frac{1}{C_2 s}\right) I_3(s) - \frac{1}{C_2 s} I_2(s) = E(s)$$

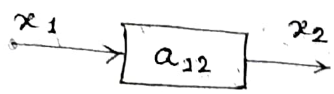
taking ILT ⇒

$$L_3 \frac{di_3}{dt} + \frac{1}{C_2}\int (i_3 - i_2)\, dt = e \quad \ldots \text{⑥}$$



Signal Flow Graph (SFG)

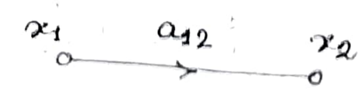
Block diagram gives a pictorial representation of a control system by way of short handing the transfer function. Signal Flow Graph (SFG) further shortens the representation of a control system by way of eleminating summing point. take of point and block. This elemination is achived by representing the variables by points called 'nodes' and the Transfer function is termed as 'transmittance' which is represented by a line called 'branch'. a signal on the branch travels in the direction of a arrow indicated on the branch.



Block diagram

$x_2 = a_{12} x_1$

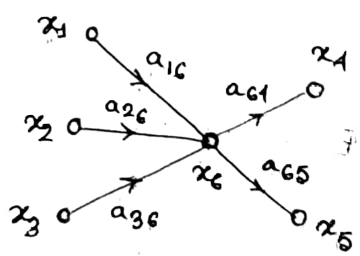
$\Rightarrow$



SFG

$x_2 = a_{12} x_2$

(*)



$x_6 = a_{16} x_1 + a_{26} x_2 + a_{36} x_3$

$x_4 = a_{64} x_6$

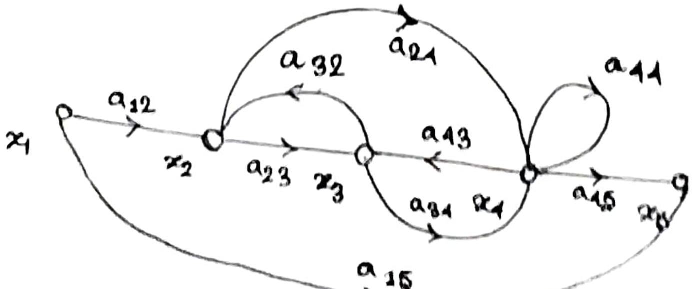
$x_5 = a_{65} x_6$

(*) $x_2 = a_{12} x_1 + a_{32} x_3$

$x_3 = a_{23} x_2 + a_{43} x_4$

$x_4 = a_{34} x_3 + a_{24} x_2 + a_{44} x_4$

$x_5 = a_{45} x_4 + a_{15} x_1$



signal

Branch — Two nodes connected by a directed line called branch

Input Node / Source — It is the node which has only outgoing branches.

Output node / Sink — It is the node which has only incoming branches.

~~Partia~~ Mixed node — A node having incoming & outgoing branches is known as mixed nodes.

Forward path — It is the path which originates from the input node and terminates at the output node and along which no node is traversed more than once.

Loop — It is a special type of path which originates and terminate in the same point and along which no ~~other~~ other node is traversed more than once.

~~Self loop~~

Self loop — It is a path which originates and terminates on the same node.

non-touching loop — Loops are said to be non-touching, if they have no common nodes.

Mason's Gain Formula

The overall transmittance (gain) in a SFG between the source node and sink node is given by Mason's gain formula

$$\boxed{T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K}$$

K = No. of forward path

P_K = gain of the K^{th} forward path.

T = overall Transfer function

Δ = Determinant of SFG.

= 1 − (Sum of all Individual loop gain)

+ (Sum of loop gain products of all possible combination of 2- non touching loops)

− (Sum of loop gain products of all possible combination of 3 - non touching loops)

+ .... − ....

Δ_K = The part of Δ not touching the K^{th} forward path.

Ex. 1

$x_2 = x_1 + 3x_3$

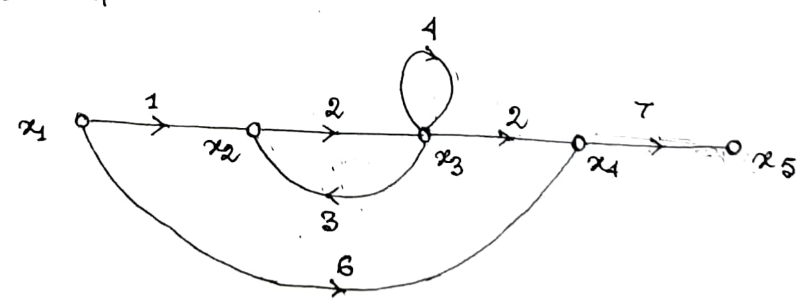
$x_3 = 2x_2 + 4x_3$

$x_4 = 6x_1 + 2x_3$

$x_5 = 7x_4$

what will be the overall Transfer function.

Solun.



$$\boxed{T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K}$$

* here no. of forward path, $K = 2$

$$\therefore \boxed{T = \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2)}$$

No. of individual loops = 2

$L_1 = \ldots 6$

$L_2 = 1$

(*) $\Delta = 1 - (L_1 + L_2)$

$= 1 - (6+1)$

$= -9$

(*) $\Delta_1 = 1$

$\Delta_2 = 1 - (L_1 + L_2)$

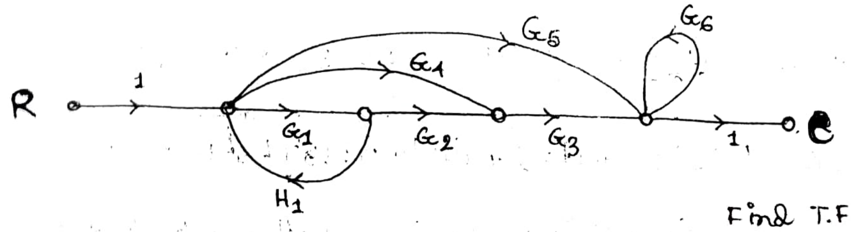
$= 1 - (6+1)$

$= -9$

(*) $\therefore T = \frac{x_5}{x_1} = \frac{(28 \times 1) + (42 \times -9)}{-9}$

$= \frac{350}{9}$

Ex.(2)



Find T.F, C/R.

Solⁿ (*) here no. of forward path, (K) = 3

$\therefore \boxed{T = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3)}$

(*) $P_1 = 1 \times G_1 \times G_2 \times G_3 \times 1 = G_1 G_2 G_3$

$P_2 = 1 \times G_4 \times G_3 \times 1 = G_3 G_4$

$P_3 = 1 \times G_5 \times 1 = G_5$

(*) No. of individual loops = 2 , $L_1 = G_1 H_1$, $L_2 = G_6$

(*) No. of Two non-touching loops = 1 , $L_1 L_2 = G_1 G_6 H_1$

(*) $\Delta = 1 - (L_1 + L_2) + L_1 L_2$

$= 1 - G_1 H_1 - G_6 + G_1 G_6 H_1$

(*) $\Delta_1 = 1$, $\Delta_2 = 1$, $\Delta_3 = 1$

(*) $\frac{C}{R} = \frac{G_1 G_2 G_3 + G_3 G_4 + G_5}{1 - G_1 H_1 - G_6 + G_1 G_6 H_1}$

Ex. ③

Find $\frac{C}{R}$

solⁿ ⊛ here $K = 1$

$$\boxed{T = \frac{P_1 \Delta_1}{\Delta}}$$

⊛ $P_1 = 1 \times G_1 \times 1 \times G_2 \times G_3 \times 1$

$= G_1 G_2 G_3$

⊛ No. of individual Loops = 3

$L_1 = G_1 H_1$

$L_2 = G_2 H_2$

$L_3 = G_4$

⊛ two non-touching loops $L_1 L_2 = G_1 G_2 H_1 H_2$

$L_2 L_3 = G_2 G_4 H_2$

$L_3 L_1 = G_1 G_4 H_1$

⊛ three non-touching loops $L_1 L_2 L_3 = G_1 G_2 G_4 H_1 H_2$

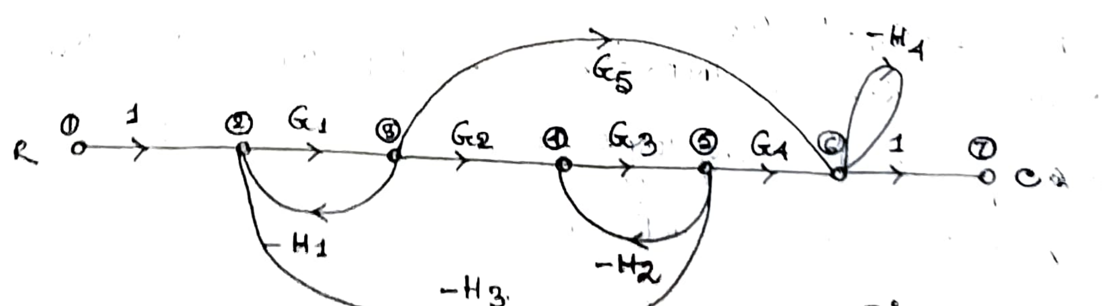
⊛ $\therefore \Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_2 L_3 + L_3 L_1) - L_1 L_2 L_3$

$= 1 - (G_1 H_1 + G_2 H_2 + G_4) + (G_1 G_2 H_1 H_2 + G_2 G_4 H_2 + G_1 G_4 H_1)$
$- G_1 G_2 G_4 H_1 H_2$

⊛ $\Delta_1 = 1$

⊛ $T = \frac{C}{R} = \frac{G_1 G_2 G_3}{\Delta}$

Ex. ④



Find C/R

$$\frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

* $P_1 \; G_1G_2G_3G_4$

$P_2 \; G_1G_5$

* No. of individual loops = 4

$L_1 = -G_1H_1$

$L_2 = -G_3H_2$

$L_3 = -G_1G_2G_3H_3$

$L_4 = -H_4$

* two non-touching loops, $L_1L_2 = +G_1G_3H_1H_2$

$L_1L_4 = G_1H_1H_4$

$L_2L_4 = G_3H_2H_4$

$L_3L_4 = G_1G_2G_3H_3H_4$

* Three non-touching loops, $L_1L_2L_4 = -G_1G_3H_1H_2H_4$

* $\Delta = 1 - (L_1+L_2+L_3+L_4) + (L_1L_2 + L_1L_4 + L_2L_4 + L_3L_4)$
$- L_1L_2L_4$

$= 1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_1G_3H_1H_2 + G_1H_1H_4$
$+ G_3H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_3H_1H_2H_4$

* $\Delta_1 = 1$

$\Delta_2 = 1 - (-G_3H_2)$

$= 1 + G_3H_2$

* $$T = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_1G_3H_1H_2 + G_1H_1H_4 + G_3H_2H_4 + G_1G_2G_3H_3H_4 + G_1G_3H_1H_2H_4}$$

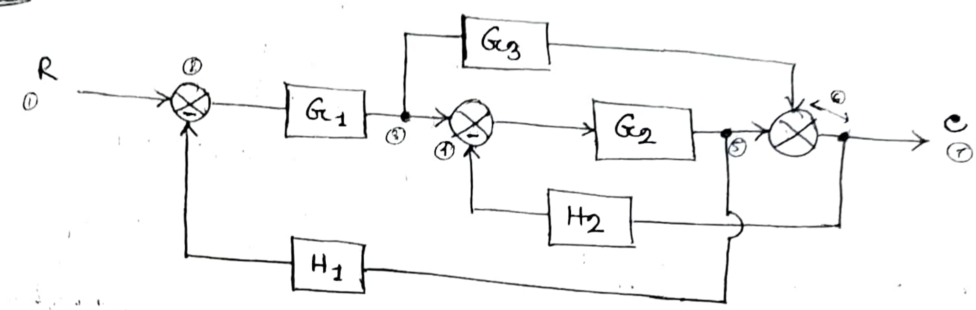
Construction of signal flow graph (SFG) from block diagram.

① All variables, summing points and take off points are represented by nodes.

② If a summing point is placed before a take off point in the direction of signal flow, then the summing point and take off point are represented by a single node.

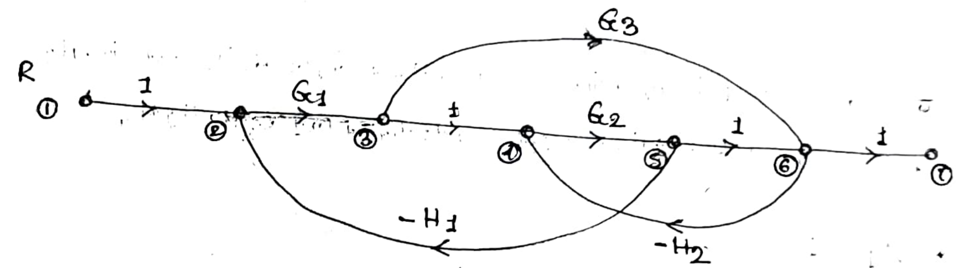
③ If a summing point is placed after a take off point in the direction of signal flow, then the summing point and take off point are represented by seperate node.

Ex. ⑤



Find Transfer function C/R using SFG.

Soln.



* here $K = 2$

$$\therefore \boxed{T = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)}$$

* $P_1 = G_1G_2$

  $P_2 = G_1G_3$

* no. of individual loops = 3

$L_1 = -G_1G_2H_1$

$L_2 = -G_2H_2$

$L_3 = G_1 \times G_3 \times (-H_2) \times G_2 \times (-H_1)$

$= G_1G_2G_3H_1H_2$

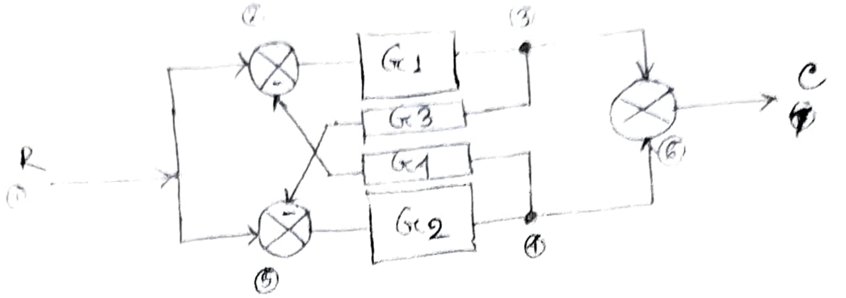
* two non touching loops and three non touching loops = 0

* $\Delta = 1 - (L_1 + L_2 + L_3)$

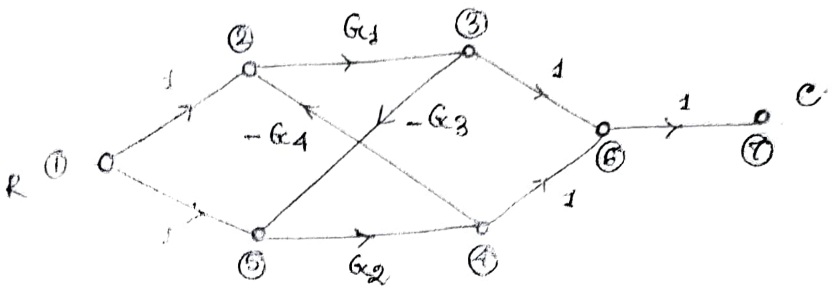
$= 1 + G_1G_2H_1 + G_2H_2 - G_1G_2G_3H_1H_2$

$1 + G_1G_2H_1 + G_2H_2 - G_1G_2G_3H_1H_2$

Ex. 6



Soln



* here $K = 4$

$$\boxed{T = \frac{1}{\Delta}\left(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4\right)}$$

* $P_1 = G_1$

  $P_2 = G_2$

  $P_3 = -G_1G_2G_3$

  $P_4 = -G_1G_2G_4$

* no. of individual loop $= 1$

  $L_1 = G_1G_2G_3G_4$

* $\Delta = 1 - L_1$

  $= 1 - G_1G_2G_3G_4$

* $\Delta_1 = 1, \quad \Delta_2 = 1, \quad \Delta_3 = 1, \quad \Delta_4 = 1.$

* $\therefore \dfrac{C}{R} = \dfrac{G_1 + G_2 - G_1G_2G_3 - G_1G_2G_4}{1 - G_1G_2G_3G_4}$

Type and order of a system

Highest power of s in the denominator of any Transfer function indicate the order of a system

No. of pole at origin of any system indicates the type of any system

(i) $G(s) = \frac{5}{s+1} \rightarrow$ Type $\rightarrow 0$, order $\rightarrow 1$

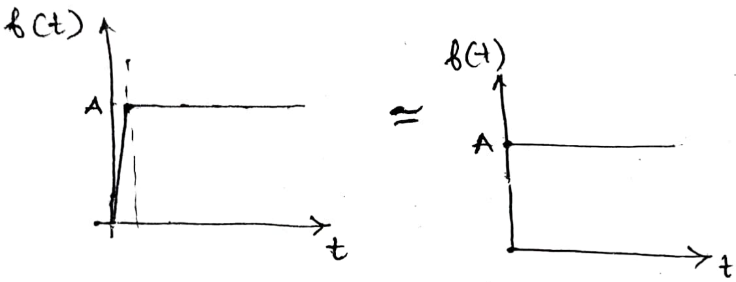
(ii) $G(s) = \frac{2(s+2)}{s(s+1)} \rightarrow$ Type $\rightarrow 1$, order $\rightarrow 2$

(iii) $G(s) = \frac{2(s+2)}{s^2(s+1)}$ $\rightarrow$ Type $\rightarrow 2$, $\rightarrow$ order $\rightarrow 3$

Standard Test Signals

① Step signal

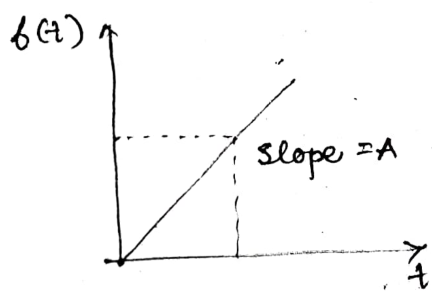
It is the signal which changes its level from one to another in a very short time.



$f(t) = A\,u(t)$

$F(s) = \frac{A}{s}$

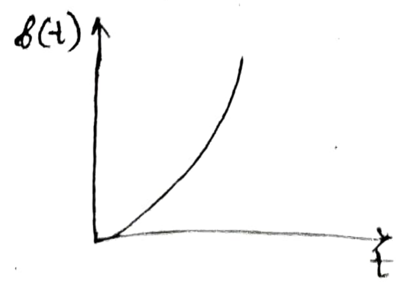
② Ramp signal



$f(t) = At\,u(t)$

$F(s) = \frac{A}{s^2}$

③ Parabolic signal

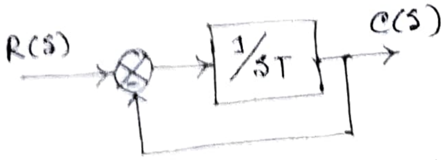


$f(t) = t^2/2$

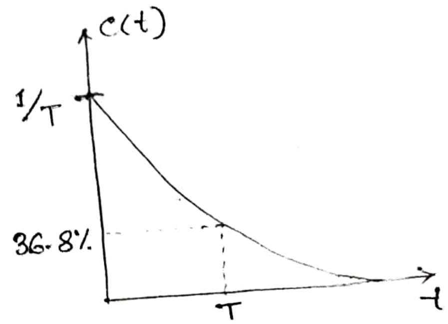
$F(s) = \frac{1}{s^3}$

$\left\{ t^n \rightarrow \frac{n!}{s^{n+1}} \right.$

Impulse Response of 1^{st} order system -



$$\frac{C(s)}{R(s)} = \frac{1/sT}{1 + 1/sT} = \frac{1/T}{s + 1/T}$$



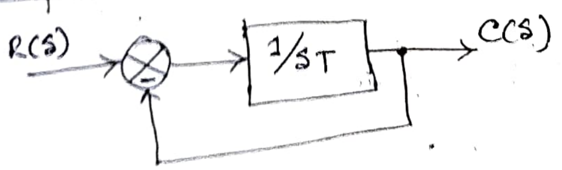
for impulse $r(t) = \delta(t)$

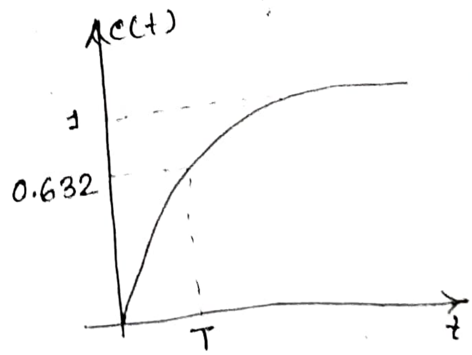
$\therefore R(s) = 1$

$$\therefore C(s) = \frac{1/T}{s + 1/T}$$

$$\therefore \boxed{c(t) = 1/T \; e^{-t/T}}$$

Step Response of 1^{st} order system





$$\frac{C(s)}{R(s)} = \frac{1/T}{s + 1/T}$$

for step, $r(t) = u(t)$

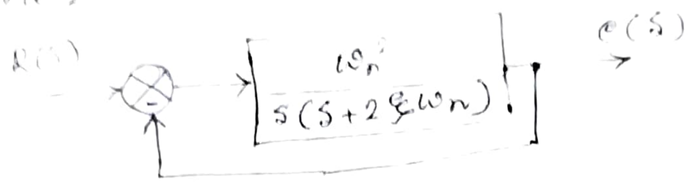
$R(s) = 1/s$

$$\therefore C(s) = \frac{1}{s} \cdot \frac{1/T}{(s + 1/T)} = \frac{1}{s} - \frac{1}{s + 1/T}$$

$$\therefore \boxed{c(t) = 1 - e^{-t/T}}$$

Step Response of a standard 2nd order system

Torm a 2nd order system



$$\therefore G(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where, ω_n = Natural frequency

$\omega_d = \omega_n\sqrt{1-\xi^2}$ = damped natural frequency

ξ = Damping Ratio.

$\alpha = \xi\omega_n$ = Damping factors

for step input. $r(t) = u(t)$

$\therefore R(s) = 1/s$

$$\therefore C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$\boxed{C(s) = \frac{1}{s} - \underbrace{\frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}}_{C'(s)}}$$

$$\therefore \boxed{C'(s) = \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

Depending upon the different values of ξ. Systems are classified as,

if $\xi = 0$, then the system is known as <u>undamped</u>

if $\xi = 1$ " " " " critically damped

if $\xi > 1$ " " " " overdamped

if $0 < \xi < 1$ " " " " underdamped

$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

For underdamped system $(0 < \xi < 1)$ the roots will be,

$$s_1, s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$= -\alpha \pm j\omega_d$$

$$C'(s) = \frac{s + 2\alpha}{(s+\alpha - j\omega_d)(s+\alpha+j\omega_d)}$$

or, $$C'(s) = \frac{A}{s+\alpha-j\omega_d} + \frac{B}{s+\alpha+j\omega_d}$$

$$A = (s+\alpha-j\omega_d)\,C'(s)\Big|_{s=-\alpha+j\omega_d}$$

$$= \frac{\alpha + j\omega_d}{2j\omega_d}$$

$$B = (s+\alpha+j\omega_d)\,C'(s)\Big|_{s=-\alpha-j\omega_d}$$

$$= \frac{\alpha - j\omega_d}{-2j\omega_d}$$

$$\therefore C'(s) = \frac{\alpha+j\omega_d}{2j\omega_d}\cdot\frac{1}{s+\alpha-j\omega_d} - \frac{\alpha-j\omega_d}{2j\omega_d}\cdot\frac{1}{s+\alpha+j\omega_d}$$

$$c'(t) = \frac{\alpha+j\omega_d}{2j\omega_d}e^{(-\alpha+j\omega_d)t} - \frac{\alpha-j\omega_d}{2j\omega_d}e^{-(\alpha+j\omega_d)t}$$

$$= \frac{e^{-\alpha t}}{2j\omega_d}\left[(\alpha+j\omega_d)e^{j\omega_d t} - (\alpha-j\omega_d)e^{-j\omega_d t}\right]$$

$$= \frac{e^{-\alpha t}}{2j\omega_d}\left[\alpha(e^{j\omega_d t} - e^{-j\omega_d t}) + j\omega_d(e^{j\omega_d t} + e^{-j\omega_d t})\right]$$

$$= 2j\omega_d \times \frac{e^{-\alpha t}}{2j\omega_d}\left[\frac{\alpha}{\omega_d}\left(\frac{e^{j\omega_d t} - e^{-j\omega_d t}}{2j}\right) + \frac{j\omega_d}{j\omega_d}\left(\frac{e^{j\omega_d t} + e^{-j\omega_d t}}{2}\right)\right]$$

$$= e^{-\alpha t}\left[\frac{\alpha}{\omega_d}\sin\omega_d t + \cos\omega_d t\right]$$

$$= e^{-\xi\omega_n t}\left[\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t + \cos\omega_d t\right]$$

$$= \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\xi\sin\omega_d t + \sqrt{1-\xi^2}\cos\omega_d t\right]$$

$\sin(\cos^{-1}\xi)$

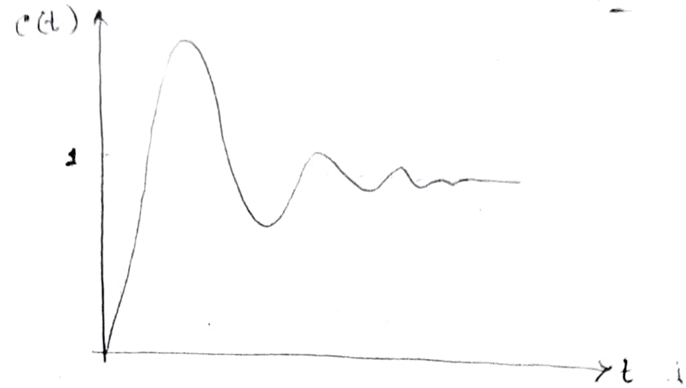
$$= \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \cos^{-1}\xi)\right]$$

$\sin^2 A + \cos^2 A = 1$

$\sin A = \sqrt{1-\cos^2 A}$

$\sqrt{1-\xi^2}$

This is the step response of a standard 2nd order system (underdamped)



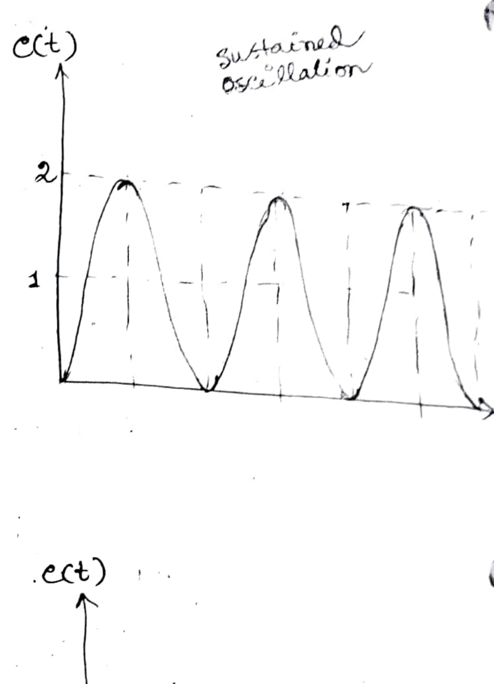
(*) For undamped system, $\xi = 0$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore C(s) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)}$$

$$= \frac{1}{s} - \frac{s}{\omega_n^2 + s^2}$$

$$\therefore \boxed{c(t) = 1 - \cos\omega_n t} \quad \cdots (2)$$



(*) For critically damped, $\xi = 1$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

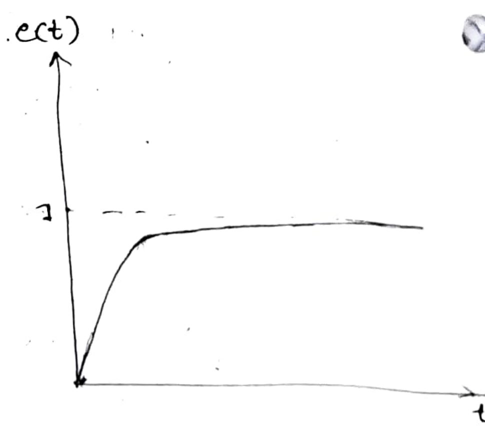
$$\therefore C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$= \frac{\omega_n^2}{s(s + \omega_n)^2}$$

$$= \frac{1}{s} - \frac{s + 2\omega_n}{(s + \omega_n)^2}$$

$$= \frac{1}{s} - \frac{1}{s + \omega_n} - \frac{\omega_n}{(s + \omega_n)^2}$$

$$\therefore \boxed{c(t) = 1 - e^{-\omega_n t} - \omega_n t\, e^{-\omega_n t}} \quad \cdots (3)$$



For an overdamped system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$= \frac{1}{s} \cdot \frac{\omega_n^2}{(s + \xi\omega_n)^2 - \omega_n^2(\xi^2 - 1)}$$

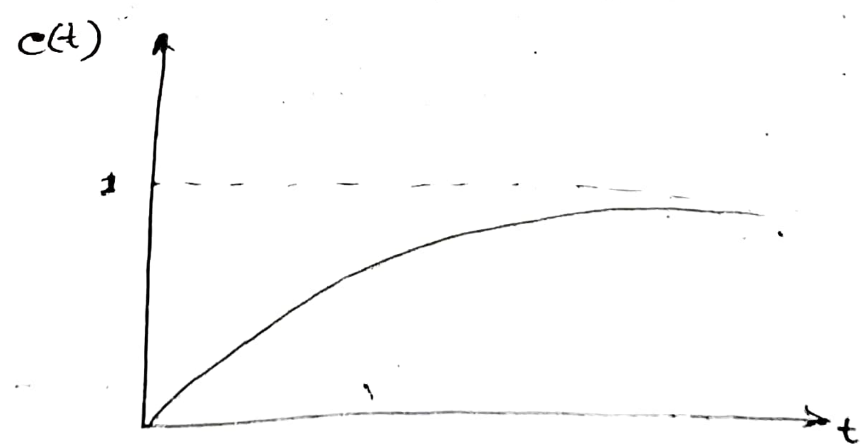
$$= \frac{1}{s} \cdot \frac{\omega_n^2}{\left[s + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1}\right] \cdot \left[s + \xi\omega_n - \omega_n\sqrt{\xi^2 - 1}\right]}$$

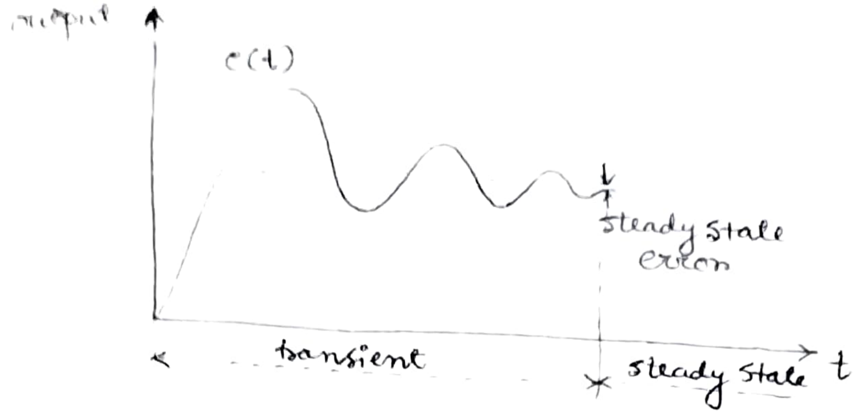
$$= \frac{1}{s} \cdot \frac{\omega_n^2}{\left[s + (\xi + \sqrt{\xi^2 - 1})\omega_n\right]\left[s + (\xi - \sqrt{\xi^2 - 1})\omega_n\right]}$$

$$= \frac{1}{s} - \frac{1}{2\sqrt{\xi^2 - 1}\,(\xi - \sqrt{\xi^2 - 1})\left[s + (\xi - \sqrt{\xi^2 - 1})\omega_n\right]}$$

$$+ \frac{1}{2\sqrt{\xi^2 - 1}\,(\xi + \sqrt{\xi^2 - 1})\left[s + (\xi + \sqrt{\xi^2 - 1})\omega_n\right]}$$

$$\boxed{c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2 - 1})\omega_n t}}{2\sqrt{\xi^2 - 1}\,(\xi - \sqrt{\xi^2 - 1})} + \frac{e^{-(\xi + \sqrt{\xi^2 - 1})\omega_n t}}{2\sqrt{\xi^2 - 1}\,(\xi + \sqrt{\xi^2 - 1})}} \quad \text{.....} (4)$$



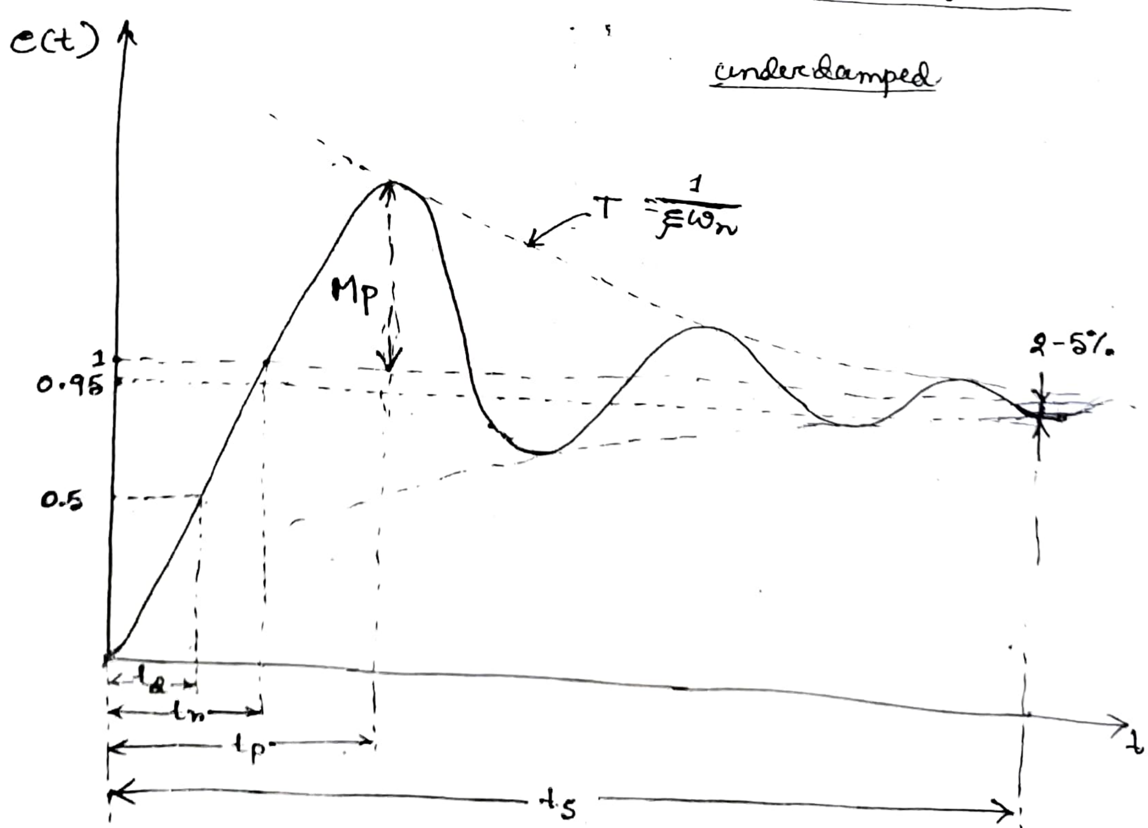


The initial part of time response of a control system transients appears and during the post transient steady state is achived. Steady state means a state of the o/p of a control system as the time approaches infinity after initiating of the i/p.

The transient part of time the response reveals the nature of response (i.e. oscillatory or overdamped) and also gives an indication about its speed.

The steady state part of time response reveals the accuracy of a control system. Steady state error is observed if the actual o/p does not exactly match with the i/p.

Transient Response specifications of 2nd order system



Delay time (t_d)

Delay time is the time required for the response to reach 50% of the final value in first time.

(2) Rise time (t_r) -

It is the time needed for the response to reach from 10% to 90% of its final value for overdamped system and 0 to 100% for underdamped system.

i.e. at $t = t_r$, $c(t) = 1$

∴ From ① ⇒

$$1 = 1 - \frac{e^{\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_n\sqrt{1-\xi^2}\,.t_r + \cos^{-1}\xi)$$

$$\therefore \frac{e^{\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_n\sqrt{1-\xi^2}\,.t_r + \cos^{-1}\xi) = 0.$$

Since $\frac{e^{\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \neq 0$ ~~as~~ or finite.

So, $\sin(\omega_n\sqrt{1-\xi^2}\,.t_r + \cos^{-1}\xi) = 0.$

$$\therefore \omega_n\sqrt{1-\xi^2}\,.t_r + \cos^{-1}\xi = n\pi = \pi \qquad [\text{at } n=1]$$

$$\therefore \boxed{t_r = \frac{\pi - \cos^{-1}\xi}{\omega_n\sqrt{1-\xi^2}}}$$

③ Peak time (t_p) -

The peak time is the time required for the response to reach the 1st peak of the time response or 1st peak overshoot.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \cos^{-1}\xi)$$

Now, $$\frac{dc(t)}{dt} = -\frac{1}{\sqrt{1-\xi^2}}\Big[-\xi\omega_n e^{-\xi\omega_n t} \sin(\omega_d t + \cos^{-1}\xi) + e^{-\xi\omega_n t}.\omega_d . \cos(\omega_d t + \cos^{-1}\xi)\Big]$$

at $t = t_p$, $\frac{dc(t)}{dt} = 0.$

$$\therefore -\xi\omega_n \sin(\omega_d t_p + \cos^{-1}\xi) + \omega_d \cos(\omega_d t_p + \cos^{-1}\xi) = 0.$$

or, $\xi \sin(\omega_d t_p + \cos^{-1}\xi) - \sqrt{1-\xi^2}\cos(\omega_d t_p + \cos^{-1}\xi) = 0.$

put, $\xi = \cos\theta$ $\quad \therefore \sqrt{1-\xi^2} = \sin\theta.$

$$\therefore \cos\theta \sin(\omega_d t_p + \theta) - \sin\theta\cos(\omega_d t_p + \theta) = 0$$

or, $\sin(\omega_d t_p) = 0$

$\omega_d t_p = n\pi$

at $n = 1$

$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

④ Maximum Overshoot (M_p) –

It is the difference between the peak value of the time response and the steady state value.

$$\therefore \quad \% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$c(t_p) - 1 = -\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \cos^{-1}\xi)$$

$c(\infty) = 1$

$$= -\frac{e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \cdot \frac{\pi}{\omega_d} + \cos^{-1}\xi\right)$$

$$= -\frac{e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin(\pi + \cos^{-1}\xi)$$

[now, $\sin(\pi + \cos^{-1}\xi) = -\sin\cos^{-1}\xi$

if $\cos^{-1}\xi = x$.

$\therefore \cos x = \xi \quad \therefore \sin x = \sqrt{1-\xi^2}$

or, $\sin\cos^{-1}\xi = \sqrt{1-\xi^2}$]

$$\therefore \quad c(t_p) - 1 = \frac{e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}{\cancel{\sqrt{1-\xi^2}}} \cancel{\sqrt{1-\xi^2}}$$

$$\therefore \quad \boxed{c(t_p) - c(\infty) = M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}}$$

$$\therefore \quad \boxed{\% M_p = 100\, e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}$$

(5) Settling time (t_s)

It is the time required for the response to reach and stay within a specified range (2% to 5%) of its final value.

For an underdamped system the magnitude of the oscillations present in the o/p time response decay exponentially with a time constant $\frac{1}{\xi\omega_n}$.

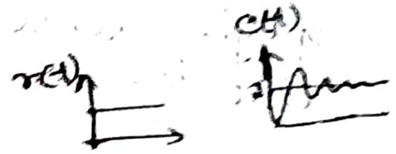
The settling time for a 2nd order control system is approximately 4-times of the time const. of the system.

$$t_s \approx \frac{4}{\xi\omega_n} \quad (2\%)$$

$$\approx \frac{3}{\xi\omega_n} \quad (5\%)$$

(6) Steady state Error (e_{ss})

It is the difference between actual output and desired o/p as time tends to zero.



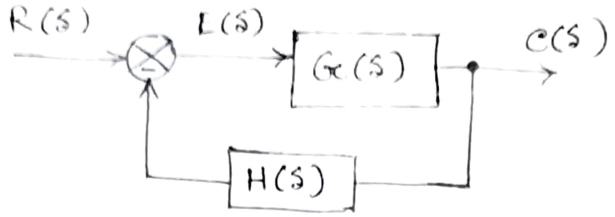
$$\therefore e_{ss} = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty} [r(t) - c(t)].$$

$y - 1 - e^{-\infty}$

$e^{-\infty} = 0$

Steady State Errors and Error constants



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

Now, $C(S) = E(S)G(S)$

$$\therefore \frac{E(S)}{R(S)} = \frac{1}{1+G(S)H(S)}$$

$$\therefore E(S) = \frac{R(S)}{1+G(S)H(S)}$$

$\therefore$ Steady state error $e_{ss} = \lim_{t \to \infty} e(t)$

applying final value theorem $\rightarrow$ $\boxed{e_{ss} = \lim_{S \to 0} S\,E(S)}$

$$\boxed{e_{ss} = \lim_{S \to 0} \frac{S\,R(S)}{1+G(S)H(S)}}$$

<u>If i/p is unit step</u> $\Rightarrow$

then $R(S) = \frac{1}{S}$

$$\therefore e_{ss} = \lim_{S \to 0} \frac{S \cdot 1/S}{1+G(S)H(S)}$$

$$= \frac{1}{1+\lim_{S \to 0} G(S)H(S)}$$

$$= \frac{1}{1+K_p}$$

where, $\boxed{K_p = \lim_{S \to 0} G(S)H(S)}$

= Displacement/proportional error constant.

then $R(S) = \frac{1}{S^2}$

$$e_{ss} = \lim_{S \to 0} \frac{S \cdot 1/S^2}{1 + G(S)H(S)}$$

$$= \lim_{S \to 0} \frac{1}{S + S\,G(S)H(S)}$$

$$= \frac{1}{\lim_{S \to 0} S\,G(S)H(S)} = \frac{1}{K_v}$$

$$\boxed{K_v = \lim_{S \to 0} S\,G(S)H(S)}$$

= Static velocity error constant.

<u>if i/p is unit parabolic</u>

then, $R(S) = 1/S^3$

$$\therefore e_{ss} = \lim_{S \to 0} \frac{S \cdot 1/S^3}{1 + G(S)H(S)}$$

$$= \frac{1}{\lim_{S \to 0} S^2 G(S)H(S)}$$

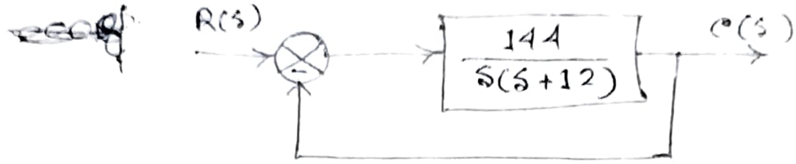
$$= \frac{1}{K_a}$$

where, $\boxed{K_a = \lim_{S \to 0} S^2 G(S)H(S)}$

= Static acceleration error constant.

(1) Determine the response specifications for a unit step input to a unity feedback system having $G(s) = \frac{144}{s(s+12)}$

Ans.



∴ closed loop T.F $= \frac{G(s)}{1+G(s)H(s)}$

$= \frac{144}{s^2 + 12s + 144}$

* $\omega_n^2 = 144$

  $\therefore \omega_n = 12$ rad/sec.

* $2\xi\omega_n = 12$

  $\therefore \xi = \frac{12}{2 \times 12} = 0.5$

* $\omega_d = \omega_n \sqrt{1-\xi^2} = 10.39$ rad/sec.

* $t_r = \frac{\pi - \cos^{-1}\xi}{\omega_n \sqrt{1-\xi^2}} = 0.2016$ sec.

* $t_p = \frac{\pi}{\omega_d} = 0.302$ sec.

* $\% MP = 100\, e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

  $= 100 \times e^{-\pi \times 0.5 / \sqrt{1-0.5^2}}$

  $= 16.30\%$

* settling time $(t_s) = \frac{4}{\xi\omega_n}$

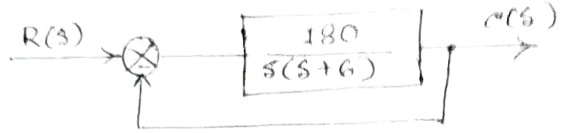
  $= 0.667$ sec.

2) A unity feedback system has

$$G(S) = \frac{180}{S(S+6)} \quad \text{and } r(t) = 4t$$

Determine (i) steady state error (ii) the value of K to reduce error by 6%

Ans.



(i) $$\boxed{e_{ss} = \lim_{S \to 0} \frac{S\,R(S)}{1 + G(S)H(S)}}$$

if $r(t) = 4t \quad \therefore R(S) = \frac{4}{S^2}$

$$\therefore e_{ss} = \lim_{S \to 0} \frac{S \cdot 4/S^2}{1 + \frac{180}{S(S+6)}}$$

$$= \lim_{S \to 0} \frac{4}{S + \frac{180}{S+6}} = \frac{4}{30} = 13.33\%$$

(ii) to reduce error by 6%,

new error $= (13.33 - 6)\% = 7.33\%$

let $G(S) = \frac{K}{S(S+6)}$

$$\therefore e_{ss} = \lim_{S \to 0} \frac{S \cdot 4/S^2}{1 + \frac{K}{S(S+6)}}$$

$$\therefore 0.0733 = \lim_{S \to 0} \frac{4}{S + \frac{K}{S+6}} = \frac{4 \times 6}{K} = \frac{24}{K}$$

$\therefore K = 327.42$

Characteristic equation

A general expression for the transfer function of a second order control system is given by

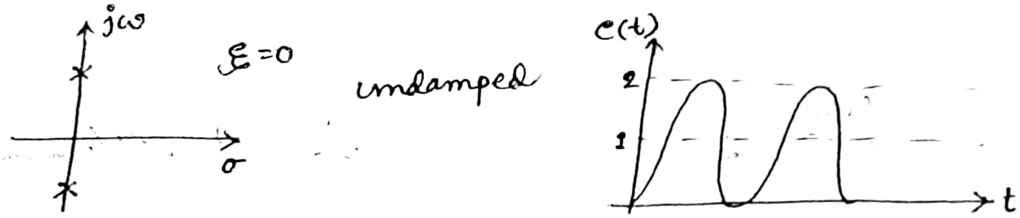
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

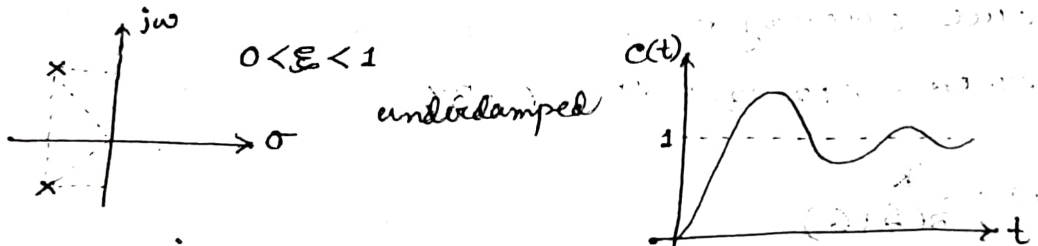
$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

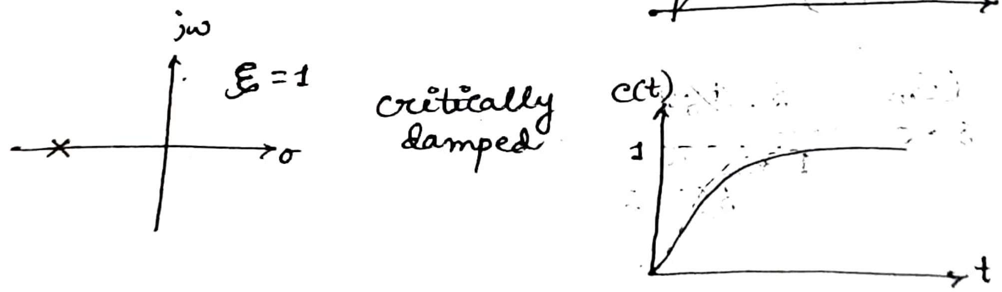
The roots are, $\boxed{s_1, s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}}$

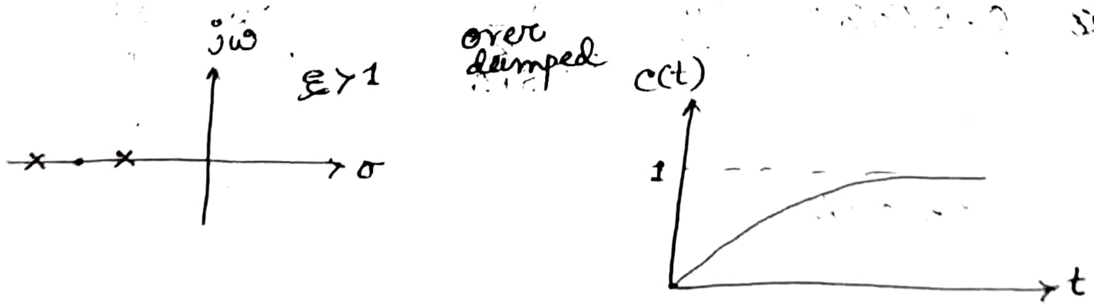
A study of roots s_1, s_2 gives a prediction about the nature of time response. The real part of the roots represents the damping and the imaginary part represents the damped frequency of oscillations.

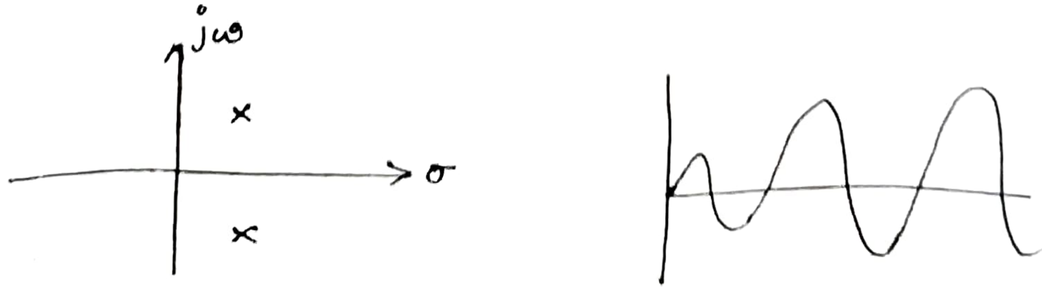
Therefore eq $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$, is called the characteristic equation of a 2nd order system.

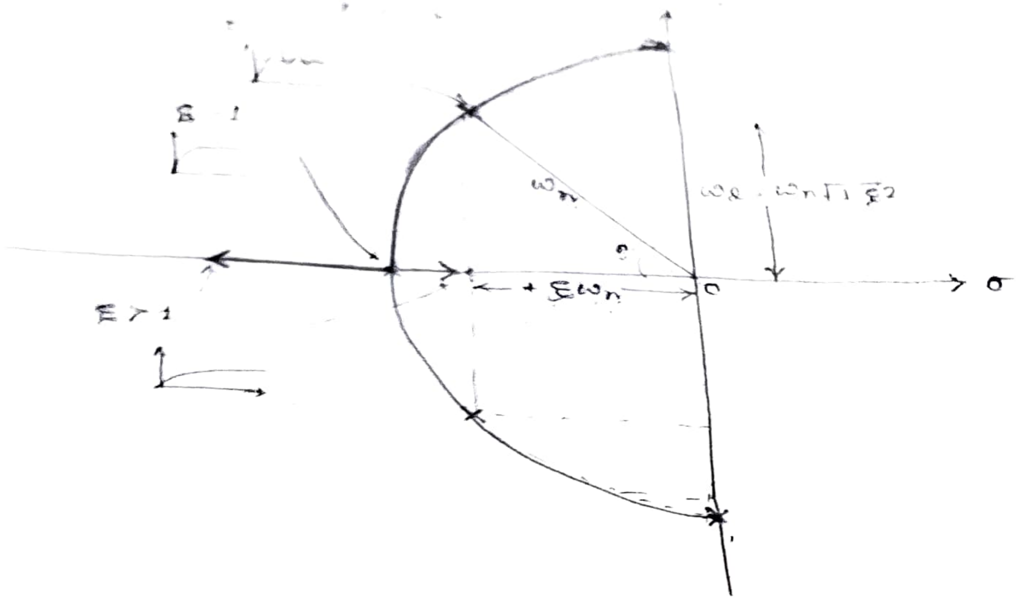












* Fig shows the general pole locus for a 2nd order system with fixed ω_n and variable damping ratio(ξ).
* It can be seen that as ξ increases the pole sketches a circular locus of radius ω_n and move away from imaginary axis. The locus meets the negative real axis at ω_n. At this point it seperates but travels along the real axis, one travels towards zero and other towards infinity.

The open loop transfer function of a unity feedback system is given by $G(s) = \frac{K}{s(Ts+1)}$, where K and T are +ve constants. Determine how much should be the amplifier gain be reduced so that the peak overshoot of unit step response of the system is reduced from 75% to 25%.

Ans $\boxed{\%\,MP = 100\, e^{-\pi\xi/\sqrt{1-\xi^2}}}$

with 75% peak overshoot,

$$e^{\frac{-\pi\xi_1}{\sqrt{1-\xi_1^2}}} = 0.75$$

$$\frac{+\pi\xi_1}{\sqrt{1-\xi_1^2}} = +0.28$$

$$\therefore \frac{\pi^2\xi_1^2}{(1-\xi_1^2)} = 0.082$$

$$\therefore \pi^2\xi_1^2 = 0.082 - 0.082\,\xi_1^2$$

or, $9.95\,\xi_1^2 = 0.082$

or, $\boxed{\xi_1 = 0.091}$

with 25% peak overshoot,

$$e^{\frac{-\pi\xi_2}{\sqrt{1-\xi_2^2}}} = 0.25$$

$$\frac{+\pi\xi_2}{\sqrt{1-\xi_2^2}} = +1.38$$

$$\therefore \frac{\pi^2\xi_2^2}{1-\xi_2^2} = 1.92$$

or, $\pi^2\xi_2^2 = 1.92 - 1.92\,\xi_2^2$

or, $11.79\,\xi_2^2 = 1.92$

or, $\boxed{\xi_2 = 0.403}$

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{Ts^2+s+K} = \frac{K/T}{s^2+s/T+K/T}$$

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\therefore \omega_n^2 = K/T$ $\qquad$ $2\xi\omega_n = 1/T$

or, $\omega_n = \sqrt{K/T}$ $\qquad$ $\xi = \frac{1}{2\omega_n T} = \frac{1}{2T}\sqrt{T/K} = \frac{1}{2\sqrt{KT}}$

$$\frac{K_1}{K_2} = 19.66$$

$$\boxed{K_1 = 19.66 K_2}$$

$\therefore$ % reduction in gain $= \frac{K_1 - K_2}{K_1} \times 100$

$$= \frac{(19.66 - 1) K_2}{19.66 K_2} \times 100$$

$$= 94.9\%$$

④ The open loop transfer function of unity feedback system is given by,

$$G(s) = \frac{50}{(1+0.1s)(s+10)}$$

Determine the static error coefficients K_p, K_v and K_a.

Ans. $K_p = \lim_{s \to 0} G(s) H(s)$

$$= \lim_{s \to 0} \frac{50}{(1+0.1s)(s+10)} = 5$$

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

$$= \lim_{s \to 0} s \frac{50}{(1+0.1s)(s+10)} = 0$$

$$K_a = \lim_{s \to 0} s^2 \frac{50}{(1+0.1s)(s+10)} = 0$$

Stability analysis: [illegible]

Stability [illegible] free system of the system takes towards the equilibrium point when time tends to infinity. the system is said to be Stable

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$$\frac{C(s)}{R(s)} = G(s)$$

$$\therefore C(s) = R(s)G(s) \quad \ldots (1)$$

The o/p time response $c(t)$ can be determined by taking ILT of eq (1). if $r(t) = \delta(t) =$ unit impulse at $t = 0$.

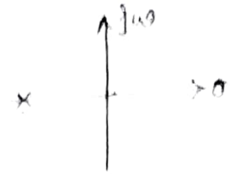
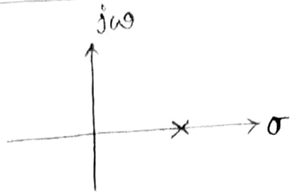
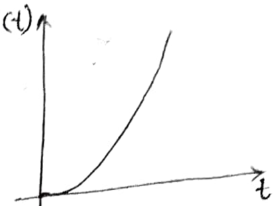
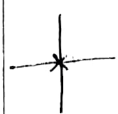
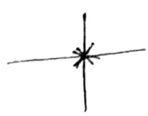
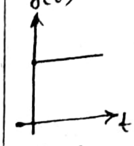
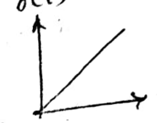
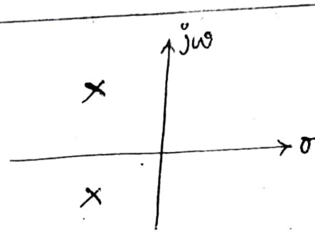
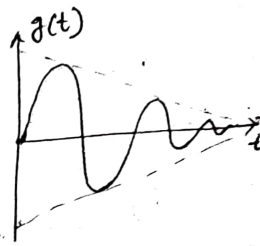
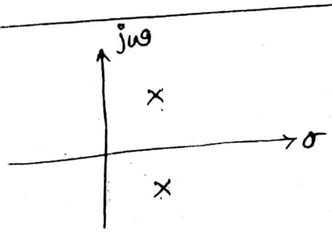
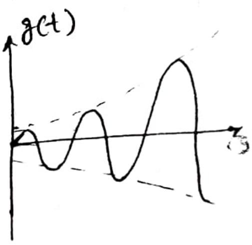
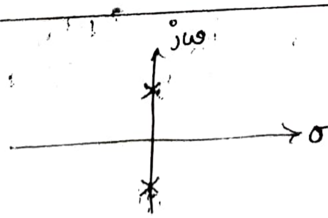
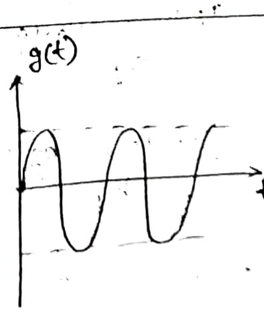
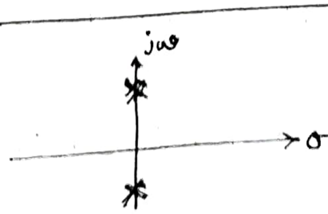
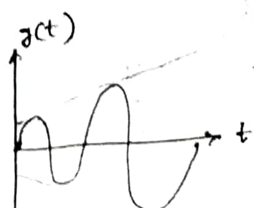
$$\therefore R(s) = 1.$$

$$\therefore C(s) = G(s)$$

ILT

$$\boxed{c(t) = g(t)}$$

- So, $g(t)$ is called impulse response of a system
- TF of a system is the Laplace transform of its impulse response.
- So a system is said to be Stable if its impulse response approaches zero for sufficient large time
- If the impulse response approaches infinity for sufficiently large time the system is said to be unstable.
- If the impulse response approaches a constant value for sufficiently large time the system is said to be marginally Stable.

① poles on -ve real axis			Stable
② poles on +ve real axis			unstable
③ poles at origin	 single pole  double pole	 single pole  double pole	unstable
④ complex pole on the left half of s-plane			stable
⑤ complex pole on the right half of s-plane			unstable
⑥ poles on the imaginary axis			marginally stable
⑦ repeated poles on imaginary axis			unstable

of s plane then the system is said to be stable.

(2) If any one pole lies to the right half of the s plane then the system is said to be unstable.

(3) If non repeated poles exists on imaginary axis the system is neither stable nor unstable i.e. marginally stable.

(4) if repeated poles exists on imaginary axis then the system is also unstable.

Routh-Hurwitz criterion

consider the following characteristic eq.

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0$$

the routh array can be formed by

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & \dots \\
s^{n-1} & a_1 & a_3 & a_5 & \dots \\
s^{n-2} & b_1 & b_2 & b_3 & \dots \\
s^{n-3} & c_1 & c_2 & c_3 & \dots \\
\vdots & & & & \\
s^0 & & & &
\end{array}$$

$$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1} \quad ; \quad b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$$

$$\text{Hlly } c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$$

* The Routh-Hurwitz criterion states that the system is stable if all the elements of the 1st column in the routh array is > 0.
* if any one comes -ve the system is said to be unstable

array indicates that the no. of poles in the right half of s plane.

ex. 1

$$s^3 + s^2 + 2s + 8 = 0$$

Sol:

s^3	1	2
s^2	1	8
s^1	-6	
s^0	8	

* so, one term comes -ve (-6), that i.e. the system is unstable.
* Here two sign changes occure (1 to -6 and -6 to 8), so two roots are present in the right half of s plane.

ex 2

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

s^4	1	18	5
s^3	8	16	
s^3	1	2	
s^2	16	5	
s^1	27/16		
s^0	5		

divide by 8

So all the terms in the 1st column in the routh array is +ve. So the system is stable.

$\dots + 2s^3 + 2s^2 + 3s + 5 = 0$

s^5	1	2	3
s^4	1	2	5
s^3	0	2	

replace the zero by a small +ve quantity 'ϵ' and continue the procedure.

s^5	1	2	3
s^4	1	2	5
s^3	ϵ	-2	
s^2	$\left(\frac{2\epsilon+2}{\epsilon}\right)$	5	
s^1	$\left(\frac{-4\epsilon-4-5\epsilon^2}{2\epsilon+2}\right) \rightarrow (-ve)$		
s^0	5		

$\epsilon \to 0$

* unstable
* 2 pole on the right half of s-plane.

~~when $\epsilon \to 0$~~

s^5	1	2	3
s^4	1	2	5
s^3	ϵ	2	
s^2	2	5	
s^1			
s^0			

ϵ = +ve

alternatively

put $s = 1/z$

$$5z^5 + 3z^4 + 2z^3 + 2z^2 + z + 1 = 0.$$

z^5	5	2	1
z^4	3	2	1
z^3	$-1/3$	$-2/3$	
z^2	$1/2$	-1	
z^1	2		
z^0	1		

* unstable
* 2 pole on the right half of s plane.

② if any row of the routh array is zero

Ex. ④ $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0.$

s^6	1	8	20	16
s^5	2	12	16	
s^5	1	6	8	
s^4	2	12	16	
s^4	1	6	8 → auxiliary equation	
s^3	0	0		

if any row in the routh array are coming zero, then we have to form the auxiliary polynomial by taking previous row.

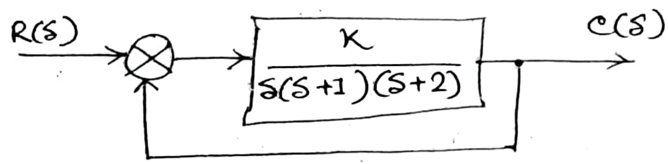
$$A(s) = s^4 + 6s^2 + 8.$$

$$\frac{dA(s)}{ds} = 4s^3 + 12s$$

s^6	1	8	20	16
s^5	2	12	16	
s^5	1	6	8	
s^4	2	12	16	
s^4	1	6	8	
s^3	4	12		
s^2	3	8		
s^1	1/3			
s^0	8			

{ (*) stable

EX (5)



For the system shown in fig find out the range of k for which the system is stable.

Solⁿ

$$T.F = \frac{G(S)}{1 + G(S)H(S)}$$

$\therefore$ characteristic equation $1 + G(S)H(S) = 0$.

$$\therefore \quad 1 + \frac{K}{S(S+1)(S+2)} = 0.$$

$$(S^2 + S)(S+2) + K = 0$$

or, $S^3 + 3S^2 + 2S + K = 0$.

S^3	1	2
S^2	3	K
S^1	$\frac{6-K}{3}$	
S^0	K	

for stable system,

(i) $K > 0$

(ii) $\frac{6-K}{3} > 0$

$6 - K > 0$

$K < 6$

$\therefore$ range of K $\quad 0 < K < 6$.

1×(5) The characteristic equation of feedback control system is

$$s^4 + 20s^3 + 15s^2 + 2s + K = 0$$

(a) Determine the range of K for the system to be stable.

(b) Can the system be marginally stable? If so find the value of K and the frequency of sustained oscillation.

Soln

s^4	1	15	K
s^3	20	2	
s^3	10	1	
s^2	14.9	K	
s^1	$\frac{14.9-10K}{14.9}$		
s^0	K		

(a) for stability $K > 0$.

$$14.9 - 10K > 0$$

$$10K < 14.9$$

$$\therefore K < 1.49$$

Hence range, $0 < K < 1.49$.

$14.9 - 10K = 0$
$K = 1.49$

(b) for marginally stable $K = 1.49$

$\therefore$ auxiliary equation $A(s) = 14.9s^2 + K = 0$

$$\therefore 14.9s^2 = -1.49$$

$$\therefore s^2 = -0.1$$

$$\therefore s = \pm j0.316$$

Now, $s = \sigma + j\omega$

$$\therefore j\omega = j0.316$$

$\therefore$ $\boxed{\omega = 0.316 \text{ rad/sec}}$ → frequency of sustained oscillation.

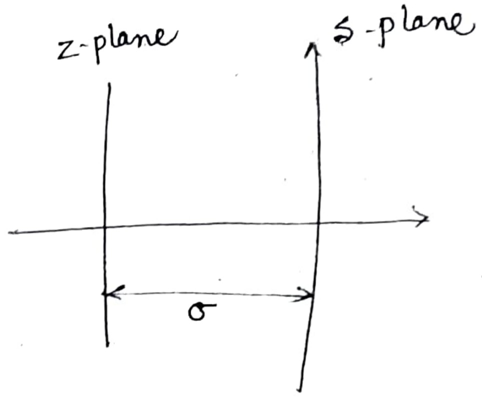
(70)

Relative stability

Routh stability gives the information about the absolute stability. ~~whereas~~ relative stability gives ~~the information about the~~

Relative stability can be examined by shifting the s plane and then apply routh stability criterion. The characteristic equation is modified by shifting the origin of s plane to $s_1 = -\sigma$ by the substitution

$s = z - \sigma$



Prob(6) A system having characteristic equation

$s^3 + 7s^2 + 14s + 8 = 0$

check whether any pole of this characteristic equation lie on the ~~left~~ right of -3.

$42 + 27 = 69$ $\quad$ $63 + 8 = 71$, $71 - 69 = 2$

Ans. $s = z - 3$

$(z-3)^3 + 7(z-3)^2 + 14(z-3) + 8 = 0$.

or, $z^3 + 9z^2 + 27z - 27 + 7z^2 - 42z + 63 + 14z - 42 + 8 = 0$

or, $z^3 - 2z^2 - z + 2 = 0$.

z^3	1	-1
z^2	-2	2
z^1	0	
z^0		

z^3	1	-1
z^2	-2	2
z^1	-4	
z^0	2.	

∴ unstable.

(*) 2 - poles on the right half of -3.

Prob. 7 check whether all the roots of the equation $s^3+7s^2+25s+39=0$, have real parts more negative than -1.

Soln

$$(z-1)^3+7(z-1)^2+25(z-1)+39=0.$$

$$\therefore z^3+4z^2+14z+20=0.$$

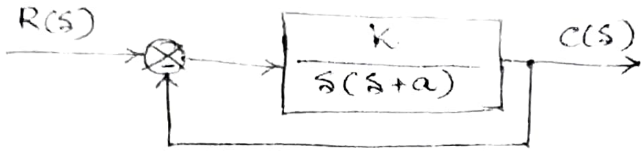
z^3	1	14
z^2	4	20
z^2	1	5
z^1	9	
z^0	5	

Since no sign change in the 1st column, the roots of the characteristic equation lie in the left of z-plane. It means all the roots of the original equation in s domain lie to the left of $s=-1$.

Root Locus

Root locus is a graphical method in which roots of the characteristics equation are plotted in s plane for the different values of parameters.

ex (1)



$$1 + G(S)H(S) = 0.$$

$$1 + \frac{K}{S^2 + aS} = 0.$$

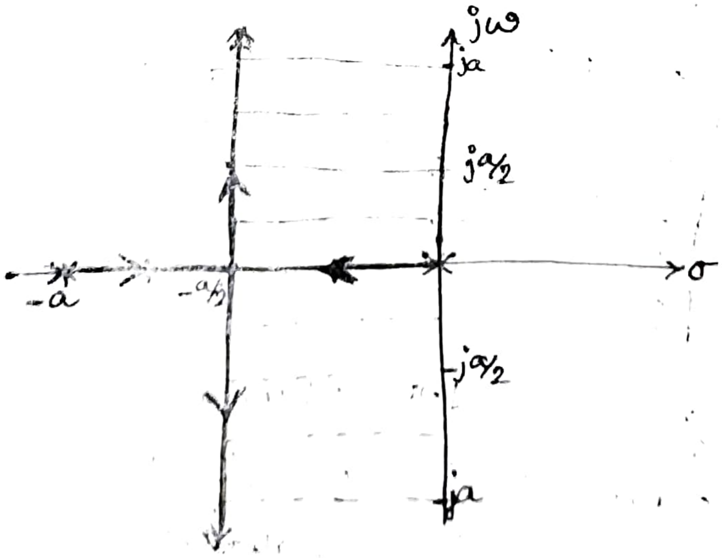
or, $\boxed{S^2 + aS + K = 0.}$

(*) $$S_1, S_2 = \frac{-a \pm \sqrt{a^2 - 4K}}{2}$$

$$= -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - K}$$

(*)

K	0	$a^2/4$	$a^2/2$	∞
S_1, S_2	$0, -a$	$-a/2, -a/2$	$-a/2 \pm j\frac{a}{2}$	$-\frac{a}{2} \pm j\infty$



Sol: Characteristic eq $1 + G(S)H(S) = 0$

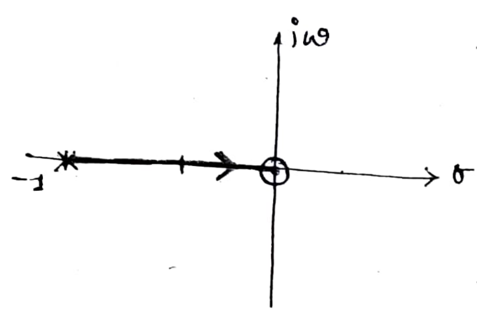
$$1 + \frac{KS}{S+1} = 0$$

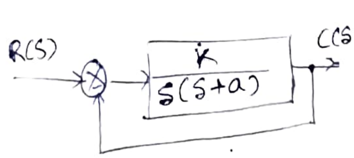
$$S + 1 + KS = 0$$

$$\text{or, } S(1+K) = -1$$

$$\text{or, } \boxed{S = -\frac{1}{1+K}} \rightarrow \text{root.}$$

K	0	1	5	9	∞
S	−1	−1/2	−0.166	−0.01	0





$1 + G(S)H(S) = 0.$

$S^2 + aS + K = 0.$

$$S_1, S_2 = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - K}.$$

K	0	$a^2/4$	$a^2/2$	∞
S_1, S_2	0, −a	−a/2, −a/2	$-a/2 \pm j\,a/2$	$-a/2 \pm j\infty$

[illegible] for plotting root locus

1. [illegible] the root locus starts (K = 0) from the open loop poles.

(2) Ending point – The root locus terminates (K = ∞) either on open loop zero or infinity.

(3) ~~Nn~~ Number of branches of root locus are (N)

$$N = P \quad \text{if } P > Z$$
$$= Z \quad \text{if } Z > P$$

where P = No. of poles. Z = No. of zeroes.

(4) Root Locus on Real axis – ~~The root locus~~ The existance of root locus on a section of real axis is confinemed if the sum of the open-loop poles and zeros to the right of the section is odd.

(5) Break in/away point – The intersection point between two root locus branches is known as break in or break away point.

$$G(s)H(s) = \frac{K \prod_{i=1}^{m} (s + Z_i)}{\prod_{j=1}^{n} (s + P_j)}$$

Ch. eq. $1 + G(s)H(s) = 0$.

$$\therefore \quad K = -\left(\frac{\prod_{j=1}^{n} (s + P_j)}{\prod_{i=1}^{m} (s + Z_i)} \right)$$

Put,

$\frac{dK}{ds} = 0$, and the valid root of this $\frac{dK}{ds} = 0$ givesus the break in and break away point.

(6) Let us consider no. of open loop poles = P and no. of open loop zeroes = Z, then Z no. of root locus will terminates at zeroes and (P-Z) no. of roots are terminates at infinity which is called asymptotes

no. of asymptotes = (P-Z)

कठिनाइयाँ

(8) Intersection of asymptotes on real axis: The asymptotes intersects at a point σ_A on the real axis given by

$$\sigma_A = \frac{\sum \text{Poles} - \sum \text{zeros}}{P - Z} \rightarrow \text{centroid}$$

(9) Intersection points on imaginary axis can be obtained from Routh array

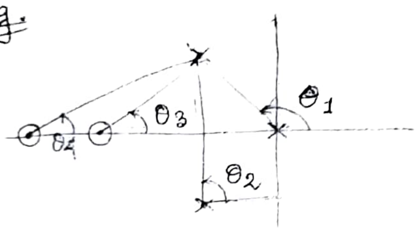
(10) Angle of departure from complex pole - is given by

$$\boxed{\phi_d = 180^\circ - (\phi_p - \phi_z)}$$

where, ϕ_p = sum of all the angles subtended by remaining poles.

ϕ_z = sum of all the angles subtended by zeroes.

eg.



$$\phi_d = 180^\circ - [\underbrace{(\theta_1 + \theta_2)}_{\phi_p} - \underbrace{(\theta_3 + \theta_4)}_{\phi_z}]$$

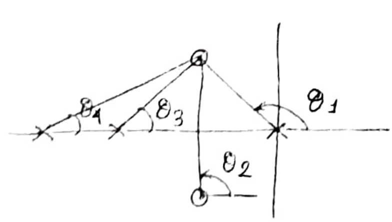
(11) Angle of arrival at complex zero - is given by.

$$\boxed{\phi_a = 180 - (\phi_z - \phi_p)}$$

where ϕ_z = sum of all the angles subtended by remaing zeroes.

ϕ_p = sum of all the angles subtended by zeroes.

eg.



$$\phi_a = 180^\circ - [\theta_2 - (\theta_1 + \theta_3 + \theta_4)]$$

(12) To find the gain for any point on the root locus, the formula is

$$K = \frac{\left| \prod_{j=1}^{n} (s_0 + P_j) \right|}{\left| \prod_{i=1}^{m} (s_0 + Z_i) \right|}$$

(76)

Ex: $G(s)H(s) = \frac{K}{s(s+a)}$ Find root locus and the value of gain (K) at break away point.

Soln:

No. of poles or open loop poles (P) = 2

poles are at $s = 0, -a$.

(*) No. of open loop zeroes (Z) = 0.

(*) here $P > Z$ $\therefore$ No. of branches in root locus (N) = P = 2

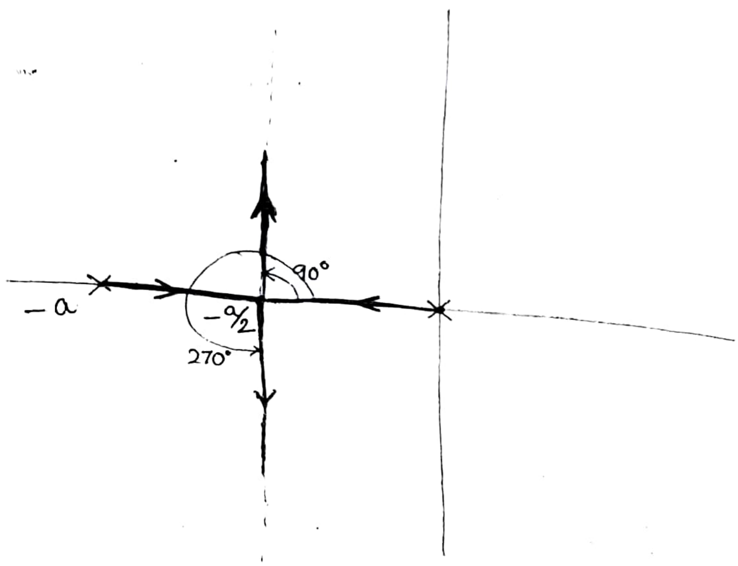
(*) No. of asymptotes = $(P - Z) = (2 - 0) = 2$.

(*) centroid (σ_A) = $\frac{-a-0}{2} = -\frac{a}{2}$

(*) Angle of asymptotes (ϕ_A) = $\frac{180(2q+1)}{2}$

here $q = (P - Z - 1) = 1$
$= 0, 1$

$= 90°, 270°$



(*) ch: $1 + G(s)H(s) = 0$.

$\therefore G(s)H(s) = -1$

$\therefore \frac{K}{s(s+a)} = -1$

or, $K = -s(s+a) = -s^2 - as$

$\therefore \frac{dK}{ds} = -2s - a$

Put $\frac{dK}{ds} = 0$

$\therefore -2s - a = 0$.

$\boxed{s = -a/2}$ → break away point

Ex② Sketch the root Locus for $G(S)H(S) = \frac{K}{S(S+1)(S+2)}$

(i) evaluate the value of K at the point where the root Locus crosses the imaginary axis. also determine the frequency.

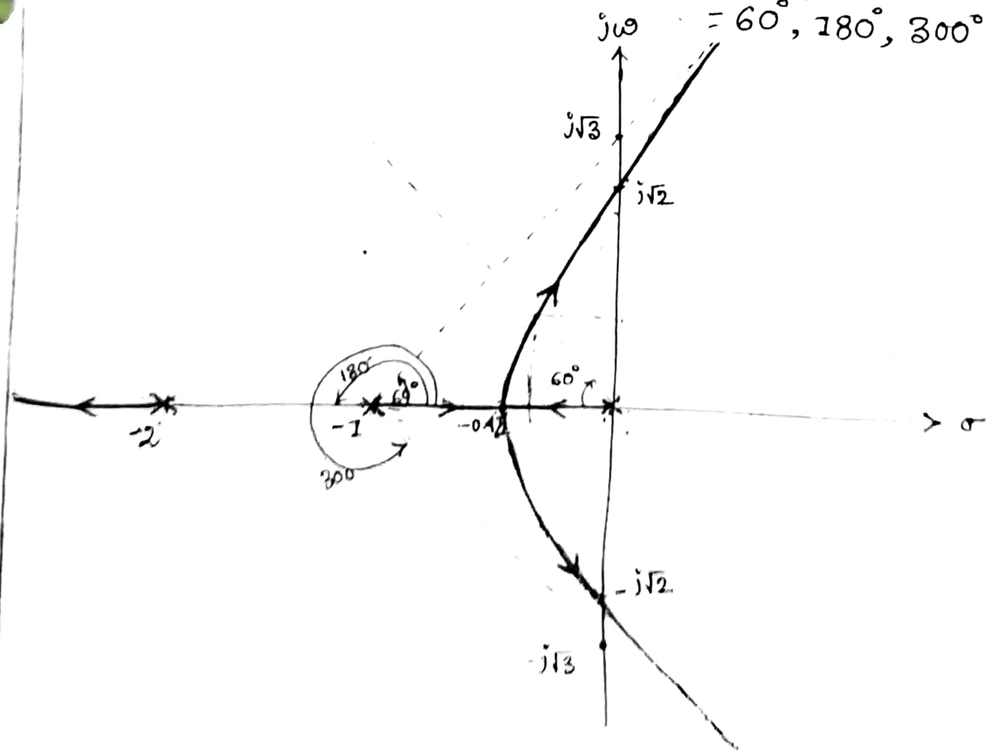
(ii) Determine the value of 'K' so that the dominant pair of complex poles of the system has a damping ratio of 0.5.

Sol^n

$$G(S)H(S) = \frac{K}{S(S+1)(S+2)}$$

* No. of open loop poles $(P) = 3$
  poles are at $S = 0, -1, -2$
* No. of open loop zeroes $(Z) = 0$
* here $P > Z$ $\therefore$ No. of branches in root locus $(N) = P = 3$
* No. of asymptotes $= (P-Z) = (3-0) = 3$
* centroid $(\sigma_A) = \frac{0-1-2-0}{3} = -1$
* Angle of asymptotes $(\phi_A) = \frac{180(2q+1)}{P-Z}$ here $q = 0, 1, 2$

$= 60°, 180°, 300°$



$\tan 60° = \frac{P}{1}$

$P = j\sqrt{3}$

$= 1.73j$

$K = -s(s^2+3s+2)$

$= -(s^3+3s^2+2s)$

Put $\frac{dK}{ds} = 0$

$3s^2+6s+2 = 0$

$s_1, s_2 = \frac{-6 \pm \sqrt{36-4.3.2}}{2.3}$

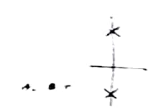
$= \boxed{-0.42}, -1.57 \rightarrow$ break away points.

Since, -1.57 is not the part of root locus, therefore breakaway point is -0.42

(K) (i) $1 + G(S)H(S) = 0$

$s^3 + 3s^2 + 2s + k = 0$

$$\begin{array}{l|ll} s^3 & 1 & 2 \\ s^2 & 3 & k \\ s^1 & \frac{6-k}{3} & \\ s^0 & k & \end{array}$$



for sustained oscillation $\frac{6-k}{3} = 0.$

$\therefore k = 6.$

$\therefore$ Auxiliary equation $3s^2 + 6 = 0$

$s^2 = -2$

$\therefore \boxed{s = \pm j\sqrt{2}.}$

$\therefore$ frequency of oscillation $= \sqrt{2}$ rad/sec.

for stability, $k > 0.$

$\frac{6-k}{3} > 0$ $\quad \boxed{0 < k < 6}$

$6-k > 0$

$6 > k$

Ex. ③ sketch the root Locus for $G(S)H(S) = \dfrac{K}{S(S+2)(S+4)}$

Ans (*) No. of open loop poles = 3 = P

poles are at 0, -2, -4.

(*) No of open loop zeroes = Z = 0.

(*) here P > Z. No. of root locus branches N = P = 3

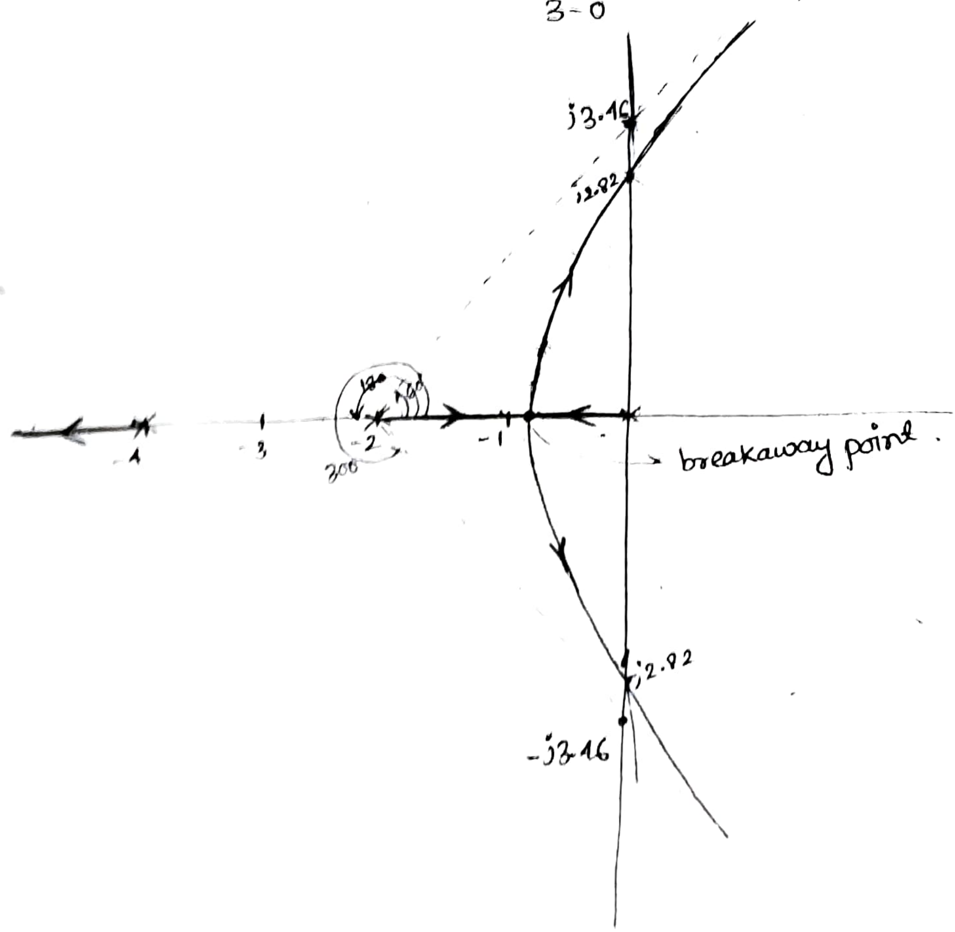
(*) No. of asymptotes = P - Z = 3-0 = 0.

(*) angle of asymptotes $(\Phi_A) = \dfrac{180(2q+1)}{P-Z}$ $\quad q = 0, 1, 2$

$= 60^\circ, 180^\circ, 300^\circ$

(*) centroid $(\sigma_A) = \dfrac{0-2-4-0}{3-0} = -2.$

$\tan 60^\circ = \dfrac{P}{2}$



(2)

[illegible]

$K = s(s+2)(s+4)$

$\therefore K = -(s^3 + 6s^2 + 8s)$

$$\frac{dK}{ds} = -(3s^2 + 12s + 8) = 0$$

$$s_1, s_2 = \frac{-12 \pm \sqrt{144 - 4.3.8}}{2.3}$$

$= -0.85, -3.15 \rightarrow$ break away point

(*) $1 + G(s)H(s) = 0$

$s^3 + 6s^2 + 8s + k = 6$

s^3	1	8
s^2	6	k
s^1	$\frac{48-k}{6}$	
s^0	k	

for sustained oscillation $\frac{48-k}{6} = 0$

$\therefore k = 48$.

$\therefore$ auxiliary eq $A(s) = 6s^2 + k = 0$

or, $6s^2 + 48 = 0$.

or, $s^2 = -8$

or, $s = \pm j2.82$

1\. (c) ~~Sketch the root locus for~~ A unity feedback system has an open loop transfer function (31)

$$G(s) = \frac{K}{s(s^2 + 4s + 13)}$$

Sketch the root locus. ~~branches~~

Sol.

(*) No. of open loop poles (P) = 3

poles are at $s_1 = 0$

$$s_2, s_3 = \frac{-4 \pm \sqrt{16 - 4.1.13}}{2.1} = -2 \pm j3$$

(*) No. of open loop zeroes (Z) = 0.

(*) here $P > Z$ ∴ No. of root locus branches (N) = P = 3

(*) No. of asymptotes = (P − Z) = 3 − 0 = 3

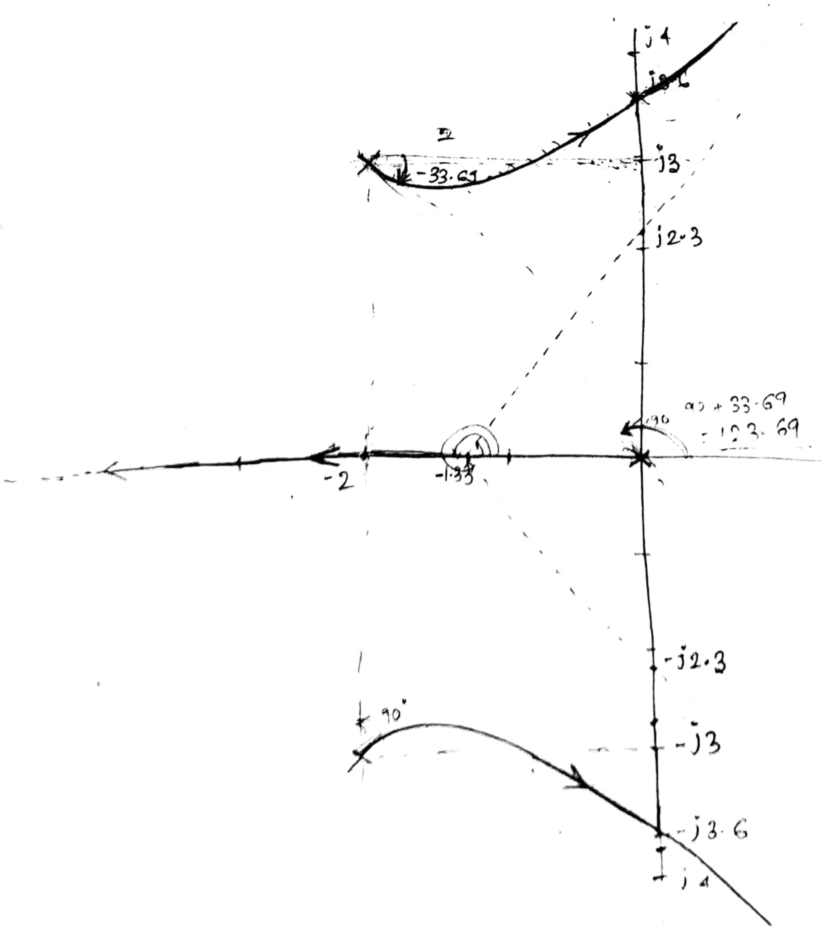
(*) angle of asymptotes (ϕ_A) $= \frac{180 \times (2q + 1)}{P - Z}$

$= 60^\circ, 180^\circ, 300^\circ$

(*) centroid (σ_A) $= \frac{0 - 2 + j3 - 2 - j3 - 0}{P - Z} = -\frac{4}{3} = -1.33$

$$\tan 60^\circ = \frac{P}{1.33}$$

$\{P = 2.3$



$\tan\theta = \frac{2}{3}$

$\theta = 33.69$

$\{\phi_A = 33.69 + 90$

(82)

$$K = -s(s^2+4s+13)$$

$$= -(s^3+4s^2+13s)$$

$$\frac{dK}{ds} = -(3s^2+8s+13) = 0$$

$$s = \frac{-8 \pm \sqrt{64 - 4.3.13}}{2.3} = \frac{-8 \pm j9.59}{6} = -1.33 \pm j1.59$$

Since it is a complex roots there will be no break-away point on real axis.

* $s^3 + 4s^2 + 13s + K = 0$

s^3	1	13
s^2	4	K
s^1	$\frac{52-K}{4}$	
s^0	K	

for sustained oscillation $\frac{52-K}{4} = 0$.

$\therefore K = 52$

$\therefore$ auxiliary equation $A(s) = 4s^2 + K = 0$

$\therefore 4s^2 + 52 = 0$

$\therefore s^2 = -13$

$s = \pm j3.6$

* angle of departure of upper complex pole –

$$\phi_d = 180^\circ - (123.69 + 90^\circ)$$

$$= -33.69^\circ$$

Ex.(5) A unity feedback system has an open loop transfer function $G(S) = \frac{K(S+1)}{S(S-1)}$

Sketch the root locus plot with 'K' as variable parameter and show that the Locii of complex roots are part of a circle with $(-1,0)$ as centre and radius $=\sqrt{2}$

Sol: (*) No. of open loop poles $(P) = 2$

poles are at $0, 1$

(*) No. of open loop zeroes $(Z) = 1$.

zero is at -1

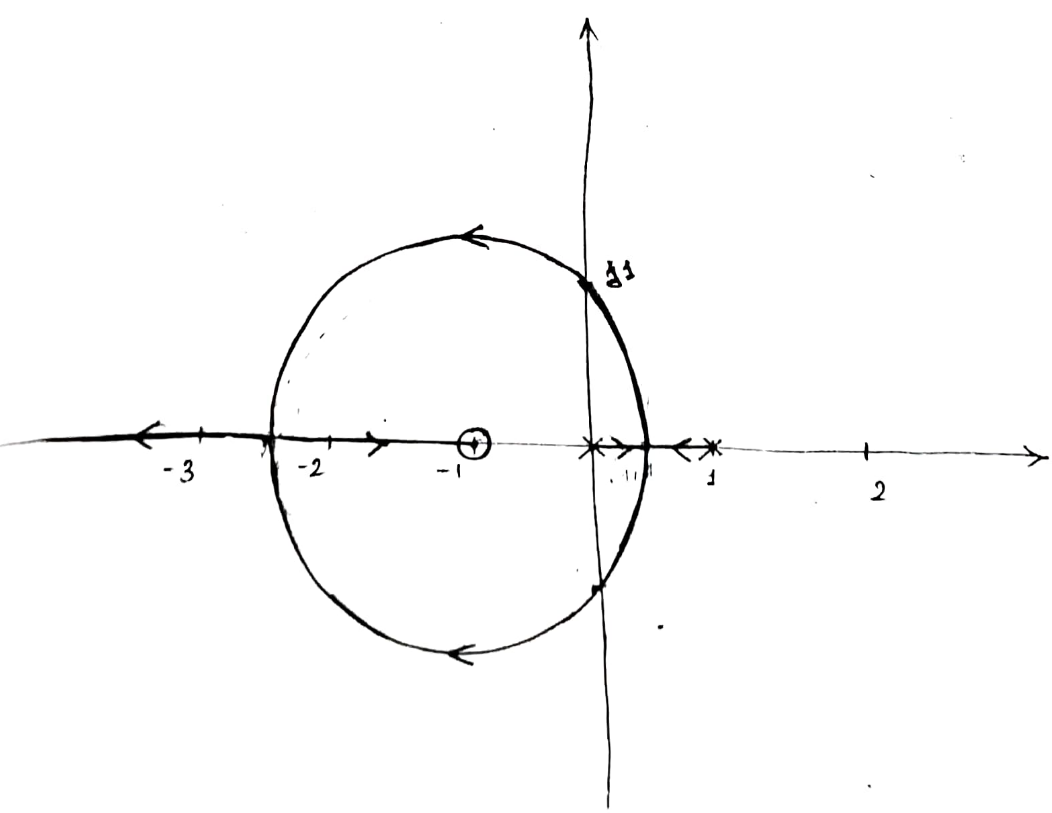
(*) here $P > Z$ $\therefore$ No of root locus branches $(N) = P = 2$.

(*) No. of asymptotes $= P - Z = 2 - 1 = 1$.

(*) angle of asymptotes $(\phi_A) = \frac{180(2q+1)}{P-Z}$

$= 180°$

(*) centroid $(\sigma_A) = \frac{0+1-(-1)}{P-Z} = 2$



(Q1)

$1 + G(s)H(s) = 0$

$K = \frac{s(s-1)}{s+1} \qquad = \frac{s^2 - s}{s+1}$

$$\frac{dK}{ds} = -\frac{(s+1)(2s-1) - (s^2 - s)(1)}{(s+1)^2} = 0$$

or, $(s+1)(2s-1) - (s^2 + s) = 0$

or, $2s^2 - \cancel{s} + 2s - 1 - s^2 + \cancel{s} = 0$

or, $s^2 + 2s - 1 = 0$

$$s = \frac{-2 \pm \sqrt{4 - 4.1.(-1)}}{2}$$

$= 0.414$ and -2.414

Since both values are the part of the root Locus, Hence there will be a breakaway point and and another will be break in point.

(*) $1 + G(s)H(s) = 0.$

$$1 + \frac{K(s+1)}{s(s-1)} = 0.$$

or, $s^2 - s + Ks + 1 = 0.$

or, $s^2 + s(K-1) + 1 = 0.$

s^2	1	1
s^1	$(K-1)$	
s^0	1	

for sustained oscillation $K - 1 = 0$ $\therefore K = 1.$

$\therefore A(s) = s^2 + 1 = 0.$

$s^2 = -1$

$s = \pm 1j.$

Ex. 6

$$G(s) = \frac{K(s+9)}{s(s^2+4s+1)}$$

Soln: (*) No. of open loop poles (P) = 3

poles are at, $s_1 = 0$

$$s_2, s_3 = \frac{-4 \pm \sqrt{16-4}}{2} = -0.26 \text{ and } -3.73$$

(*) No. of open loop zeroes (Z) = 1

zero is at -9.

(*) here $P > Z$ ∴ No. of root Locus branches (N) = P = 3

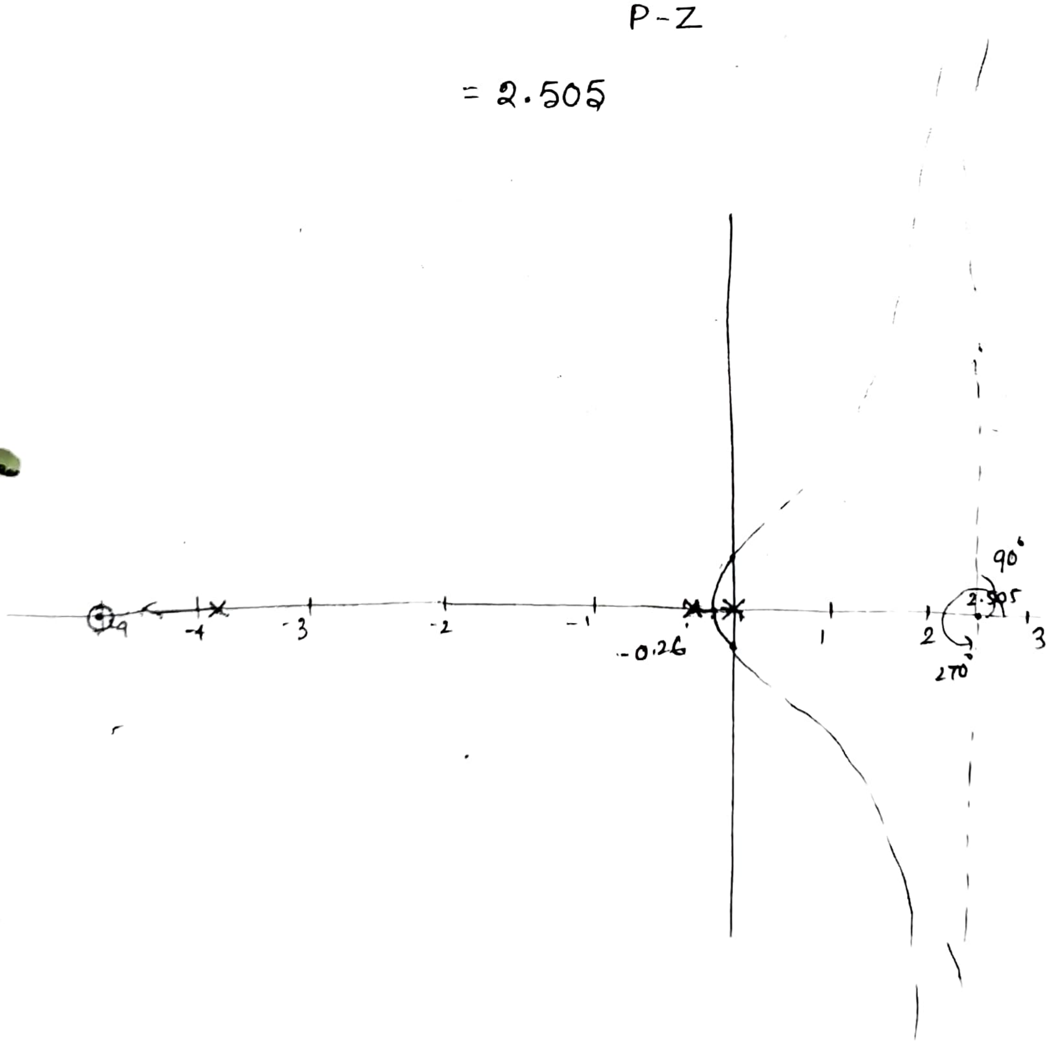
(*) No. of asymptotes $(P-Z) = 3-1 = 2$

(*) angle of asymptotes. $\frac{180(2q+1)}{P-Z}$ $q = 0, 1$.

$$= 90°, 270°$$

(*) centroid $(\sigma_A) = \frac{0 - 0.26 - 3.73 - (-9)}{P-Z}$

$$= 2.505$$



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(*) ch eq $\quad 1 + G(s)H(s) = 0$

$$K = -\frac{s^3 + 4s^2 + s}{s - 9}$$

$$\frac{dk}{ds} = -\left[\frac{(s-9).(3s^2 + 8s + 1) - (s^3 + 4s^2 + s)(1)}{(s-9)^2}\right]$$

Ex (7) $G(s)H(s) = \dfrac{K}{s(s^2+4s+8)}$

Sol^n: (*) No. of open loop poles (P) = 3

Locations are, $s_1 = 0$

$s_2, s_3 = \dfrac{-4 \pm \sqrt{16 - 4.1.8}}{2}$

$= -2 \pm j2$

(*) No. of open loop zeroes (Z) = 0.

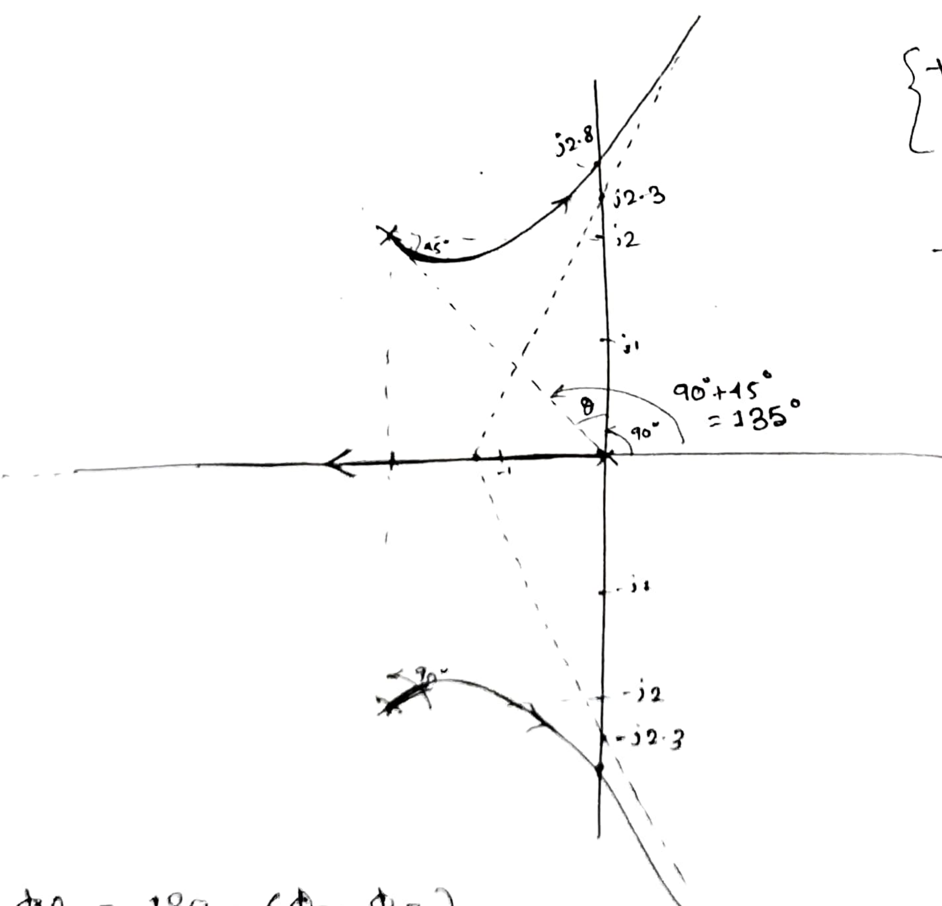
(*) here P > Z ∴ No. of root locus branches (N) = P = 3

(*) No. of asymptotes = (P - Z) = 3 - 0 = 3.

(*) angle of asymptotes $(\Phi_A) = \dfrac{180\,(2q+1)}{P-Z}$ $q = 0, 1, 2$

$= 60°, 180°, 300°$

(*) centroid $= \dfrac{0 - 2 + \cancel{j2} - 2 - \cancel{j2} - 0}{P - Z} = -4/3 = -1.33$



$\left\{ \tan 60° = \dfrac{P}{1.33} \right.$

$P = 2.3$

$\tan\theta = \dfrac{2}{2} = 1$

$\theta = 45°$

$\Phi_d = 180 - (\Phi_p - \Phi_z)$

$= 180 - 135 - 90°$

$= -45°$

(10)

when a pole is added in the forward path for [illegible] system

1. bandwidth decreases.
2. Rise time (t_r) increases.
3. Resonant peak (M_r) increases.
4. system becomes less stable.

when a zero is added in the forward path

1. Bandwidth increases.
2. Rise time (t_r) decreases.
3. Settling time (t_s)
4. System becomes non-stable.

~~Polar Plot~~

$$G(s) = \frac{1}{s(s+2)(s^2+4s+13)}$$

Q. For a unity feedback system the open loop Transfer function is given by.

$$G(S) = \frac{K}{S(S+2)(S^2+6S+25)}$$

(a) Sketch the root Locus.

(b) at what value of k the system becomes unstable

(c) at this point of instability, determine the frequency of oscillation

Ans. (*) No of Poles (P)=4.

Poles are at $S_1 = 0$, $S_2 = -2$.

$$S_3, S_4 = \frac{-6 \pm \sqrt{36-100}}{2} = -3 \pm j4.$$

(*) No of zeroes (Z)=0.

(*) here P>Z

So No of root Locus brunches (N) = P = 4.

(*) No. of Asymptotes = P-Z = 4.

(*) angle of Asymptotes $(\phi_A) = \frac{180(2q+1)}{P-Z}$ here q = 0, 1, 2, 3

$$= 45°, 135°, 225°, 315°$$

(*) centroid $(\sigma_A) = \frac{\sum \text{Poles} - \sum \text{zeroes}}{P-Z}$

$$= \frac{(0-2-3+j4-3-j4)-0}{4} = -2$$

(*) $1+G(S)H(S)=0$.

$$1 + \frac{K}{S(S+2)(S^2+6S+25)} = 0$$

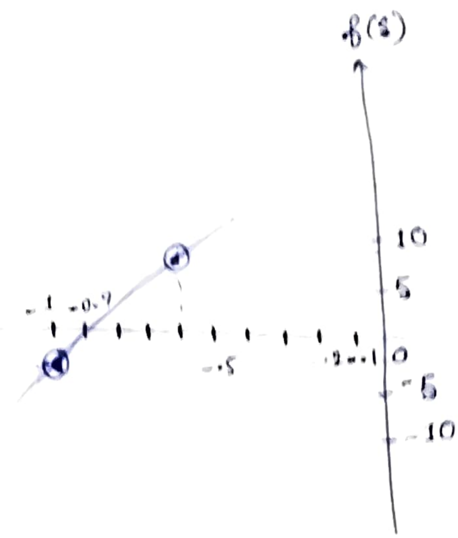
or, $K = -(S^4+8S^3+37S^2+50S)$

$\therefore \frac{dK}{dS} = -(4S^3+24S^2+74S+50) = 0$

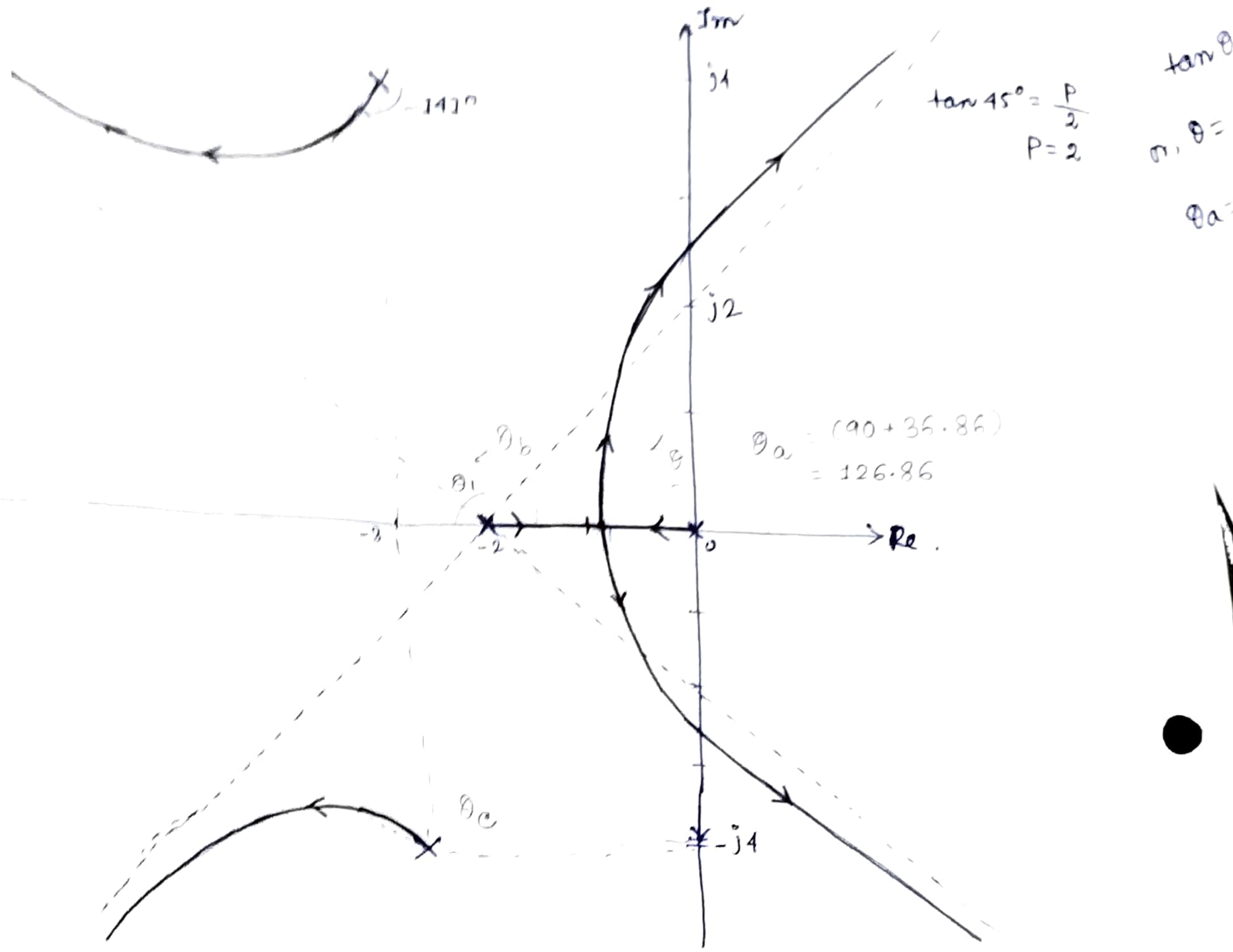
or $2S^3+12S^2+37S+25 = 0 = f(S)$

or $f(-0.5) = 9.25$

$f(-1) = -2$



hence the breakaway point is at $S = -0.9$



(+) $1 + G(S)H(S) = 0$

$$1 + \frac{K}{S(S+2)(S^2+6S+25)} = 0$$

or, $(S^2+2S)(S^2+6S+25) + K = 0$

or, $S^4 + 6S^3 + 2S^3 + 25S^2 + 12S^2 + 50S + K = 0$

or, $S^4 + 8S^3 + 37S^2 + 50S + K = 0$

S^4	1	37	K
S^3	8	50	
S^3	4	25	
S^2	30.75	K	
S^1	$\frac{768.75 - 4K}{30.75}$		
S^0	K		

For marginally stable condition,

$$\frac{768.75 - 4K}{30.75} = 0$$

or, $K = 192.1875$

Auxiliary equation,

$30.75S^2 + K = 0$

or, $30.75S^2 + 192.1875 = 0$

or, $S^2 = -6.25$

or, $S = \pm j2.5$

$\tan\theta = \frac{3}{4}$

or, $\theta = 36.86°$

$\theta_a = (90 + 36.86)$
$= 126.86°$

or, $\theta_1 = 76°$

$\theta_b = (180° - 76°)$
$= 101°$

$\therefore \theta_p = (126.86° + 101° + 90°)$
$= 320.86°$

$\therefore \theta_d = 180 - (\theta_p + \theta_z)$
$= 180 - 320.86° \simeq -141°$

(b) at $K > 192.1875$ the system become unstable.

(c) for oscillation $K = 192.1875$.

$\therefore 30.75 s^2 + K = 0.$

or $s = \pm j2.5$

$\therefore$ frequency of oscillation $\omega_n = 2.5$ rad/sec.

(Q) For a unity feedback system, the open loop transfer function is given by

$$G(s) = \frac{K(s+2)(s+3)}{s(s+1)}$$

Sketch the root locus for the system.

Ans. (*) $P = 2$ at $0, -1$,

(*) $Z = 2$ at, $-2, -3$

(*) $N = 2$

(*) No. of asymptotes $= (P - Z) = 0$.

(*) $1 + G(s)H(s) = 0$.

$$1 + \frac{K(s+2)(s+3)}{s(s+1)} = 0.$$

$$\text{or, } K = -\frac{s(s+1)}{(s+2)(s+3)} = -\frac{s^2+s}{s^2+5s+6}$$

$$\therefore \frac{dk}{ds} = -\frac{(s+2)(s+3)(2s+1) - s(s+1)(2s+5)}{[(s+2)(s+3)]^2} = 0.$$

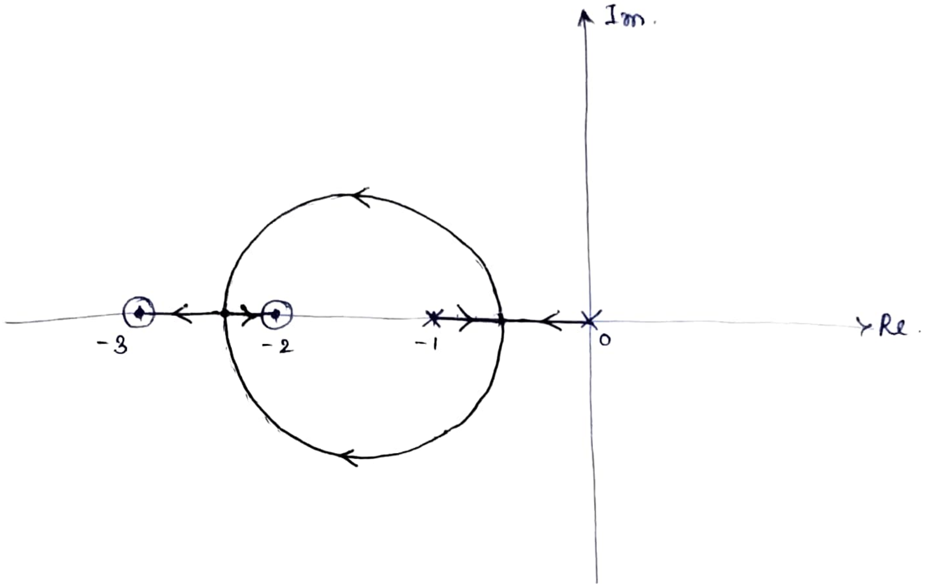
or, $(s+2)(s+3)(2s+1) - s(s+1)(2s+5) = 0$

or, $(s^2+5s+6)(2s+1) - (s^2+s)(2s+5) = 0$

or, $2s^3 + s^2 + 10s^2 + \cancel{5s} + 12s + 6 - \cancel{2s^3} - 5s^2 - 2s^2 - \cancel{5s} = 0$

or, $4s^2 + 12s + 6 = 0$

or, $s = \dfrac{-12 \pm \sqrt{144 - 4.4.6}}{2 \times 4} = -0.634, -2.366$



here, $s = -0.634$ is break away point and $s = -2.366$ is break in point.

(Q) The open loop transfer function of a system is given by,

$$G(s)H(s) = \frac{K(s+12)}{s^2(s+20)}$$

Sketch the root locus for the system.

Ans. (*) $P = 3$ at $0, 0, -20$

(*) $Z = 1$ at -12

(*) $N = P = 3$

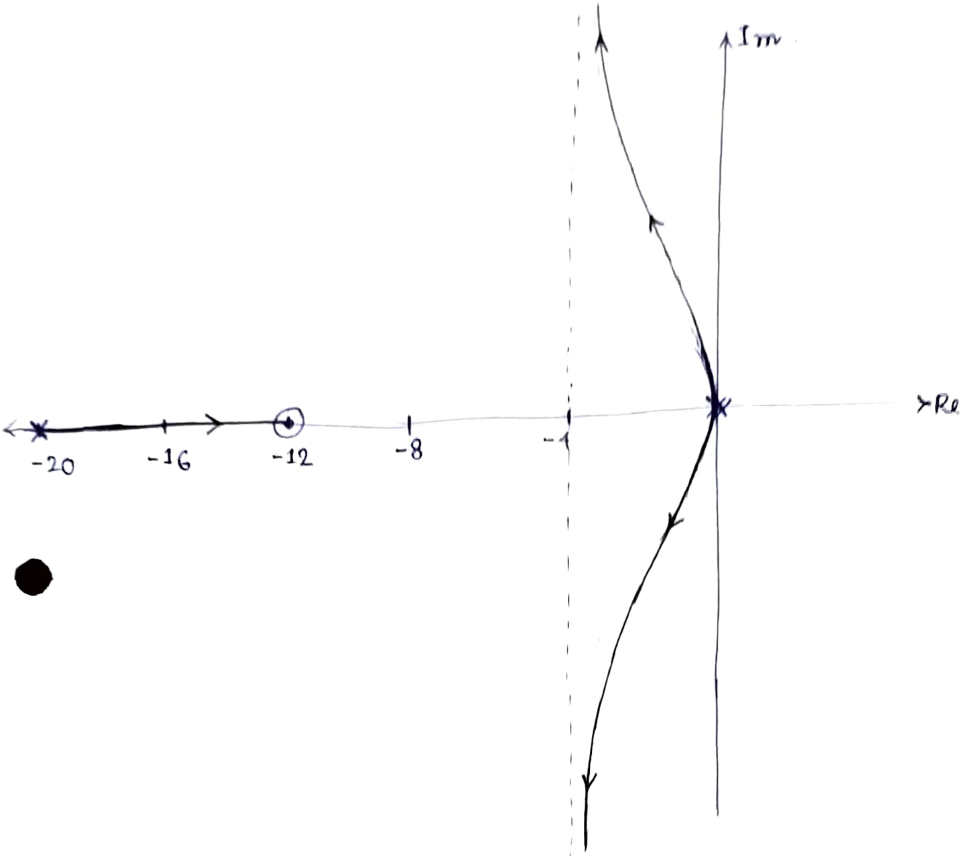
(*) No of asymptotes $= P - Z = 3 - 1 = 2$

(*) angle of asymptotes $= \dfrac{180(2q+1)}{P-Z}$ $q = 0, 1$

$= 90°, 270°$

(*) centroid (σ_A) = $\dfrac{\Sigma \text{Poles} - \Sigma \text{zeroes}}{P - Z}$

$$= \frac{(0+0-20)-(-12)}{2} = -4$$



(*) break away point –

$$1 + G(S)H(S) = 0.$$

or, $1 + \dfrac{K(S+12)}{S^2(S+20)} = 0.$

or, $K = -\dfrac{S^2(S+20)}{(S+12)}$

$\therefore \dfrac{dk}{dS} = -\dfrac{(S+12)(3S^2+40S) - S^2(S+20).1}{(S+12)2} = 0.$

or, $3S^3 + 40S^2 + 36S^2 + 480S - S^3 - 20S^2 = 0$

or, $2S^3 + 56S^2 + 480S = 0.$

or, $S(S^2 + 28S + 240) = 0.$

$\therefore S = 0$

$S = \dfrac{-28 \pm \sqrt{28^2 - 4\times 240}}{2} = -14 \pm j6.63.$

The complex points are neither break in or break away point, because the corrosponding gain value of K become complex quantities. Hence the breakaway point is at $S=0$.

~~Stability analysis in S domain~~

Frequency Response Analysis

The magnitude and phase relationship between sinusoidal input and steady state output of a system is known as frequency response.

Let a T.F. $G(S) = \frac{C(S)}{R(S)} = \frac{1+S}{2+S}$

$\therefore$ The sinusoidal form of the above T.F is

$$G(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{1+j\omega}{2+j\omega}$$

$$\therefore \text{Magnitude } (M) = \frac{\sqrt{1+(+\omega)^2}}{\sqrt{2^2+\omega^2}} = \frac{\sqrt{1+\omega^2}}{\sqrt{4+\omega^2}}$$

$$\phi = \tan^{-1}(+\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$a + jb$
$= M\angle\phi$
$M = \sqrt{a^2+b^2}$
$\phi = \tan^{-1}(b/a)$

$M\angle\phi$
$= M\cos\phi + jM\sin\phi$

* In steady state, i.e. in the limit $t \to \infty$, only the steady state part of the response exists. Hence there is no need to put $s = \sigma + j\omega$, instead we substitute $s = j\omega$ for steady state analysis.

* advantage of Frequency domain analysis

1. Frequency response tests are simple to perform.
2. T.F. can be obtained from frequency response of the system
3. Frequency response methods can be used to find the absolute as well as relative stability of a system.
4. The systems which do not have rational transfer function, freq. response can be applied to them.

* disadvantage

1. These method is applied to linear systems only.
2. For high time constant, frequency response method is not convenient.

(92)

Bode Plot

The variation of magnitude of sinusoidal T.F. expressed in decible and the corresponding phase angle in degree being plotted w.r.t. frequency on a logarithmic scale (i.e. $\log_{10}\omega$) in rectangular axes. This plot thus obtained is known as Bode plot.

Bode plot consists of two seperate plots

① a plot of the logarithm of magnitude of a sinusoidal transfer function (in dB) against the frequency ($\log_{10}\omega$)

② a plot of the phase angle against the freq. ($\log_{10}\omega$)

④ The magnitude of decible for the term of K is,

$$K\,(db) = 20\log_{10}(K)$$

$a + jb$

$M = \sqrt{a^2+b^2}$

① Graph for $\left(\frac{1}{s^n}\right)$

$$\boxed{G(s) = \frac{1}{s^n}}$$

$$\therefore G(j\omega) = \frac{1}{(j\omega)^n} \qquad \therefore |G(j\omega)| = \frac{1}{\omega^n}$$

$$\therefore \text{In db} \Rightarrow 20\log|G(j\omega)| = 20\log\left(\frac{1}{\omega^n}\right)$$

$$= 20\log(\omega^n)^{-1}$$

$$= -20\log\omega^n$$

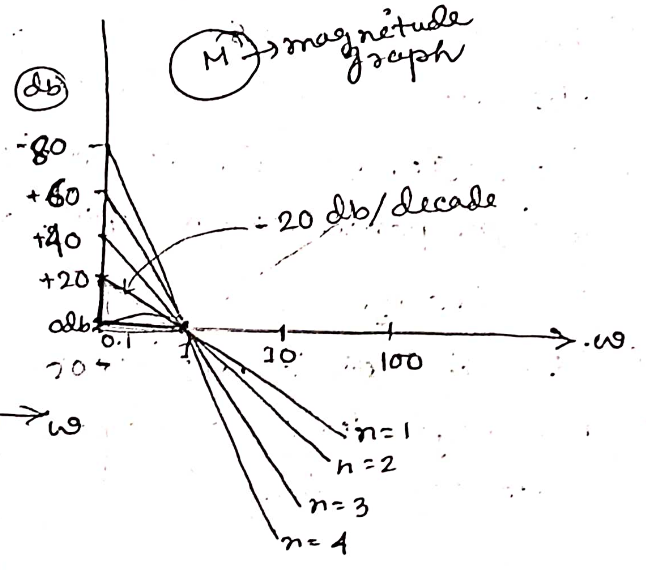
$$= -20n\log\omega$$

$$\phi = -\tan^{-1}\left(\frac{1/\omega^n}{0}\right)$$

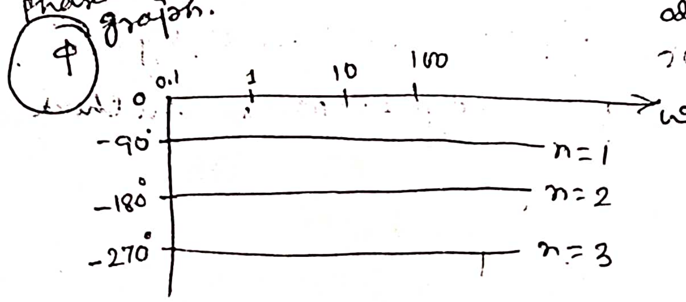
$$= -\tan^{-1}(\infty)$$

$$= -90n^\circ$$

M → magnitude graph



Phase angle graph.



(i) graph for the term $(1+ST)$ →

$$\boxed{G(S) = (1+ST)}$$

$G(j\omega) = 1 + j\omega T$ $|G(j\omega)| = \sqrt{1+\omega^2T^2}$

In db ⇒ $20\log|G(j\omega)| = 20\log\sqrt{1+\omega^2T^2} = 20\log(1+\omega^2T^2)^{1/2}$
$= 10\log(1+\omega^2T^2)$

(i) if $\omega T << 1$

then $20\log 1 = 0$ db ... (1)

(ii) if $\omega T >> 1$.

then $20\log\sqrt{\omega^2T^2} = 20\log(\omega T)$

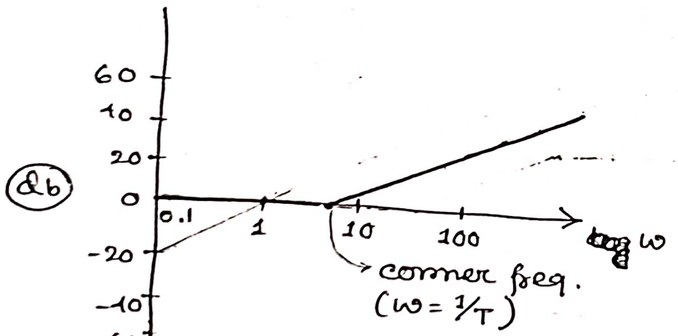
$= 20\log\omega - 20\log(1/T)$ ....(2)

Equate the above eq. to zero. [as, the graph of (1) lies on 0db axis and graph of (2) has a slope of +20db/decade. So these two graph intersect at a point on 0 db axis]

$\therefore 20\log\omega - 20\log(1/T) = 0.$

$\therefore \boxed{\omega = 1/T}$ → corner frequency
break frequency.

Thus the two parts of the graph intersects the 0db axis at $\omega = \frac{1}{T}$.



$\phi = \angle G(j\omega) = \tan^{-1}\left(\frac{\omega T}{1}\right) = \tan^{-1}\omega T$

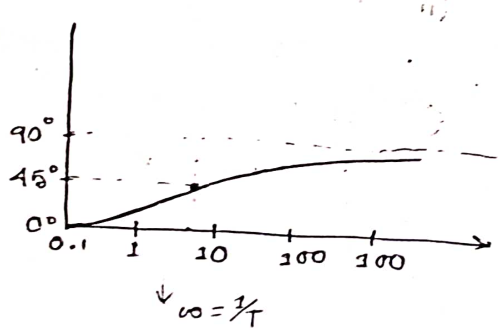
(i) at very low freq ωT is very very small

$\therefore \phi = \tan^{-1}(0) = 0°$

(ii) at very high freq

$\phi = \tan^{-1}(\infty) = 90°$

(iii) at $\omega = \frac{1}{T}$, $\phi = \tan^{-1}(1) = 45°$



(91)

③ graph for $\frac{1}{1+sT}$

$$\boxed{G(s) = \frac{1}{1+sT}}$$

$$G(j\omega) = \frac{1}{1+j\omega T} \qquad |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2T^2}}$$

In db. $20\log|G(j\omega)| = 20\log\frac{1}{\sqrt{1+\omega^2T^2}}$

$$= 20\cancel{\log 1}^{0} - 20\log\sqrt{1+\omega^2T^2}$$

$$= -20\log\sqrt{1+\omega^2T^2}$$

(i) if $\omega T \ll 1$ (very low freq)

$-20\log\sqrt{1} = 0$ db.

(ii) if $\omega T \gg 1$ (very high freq).

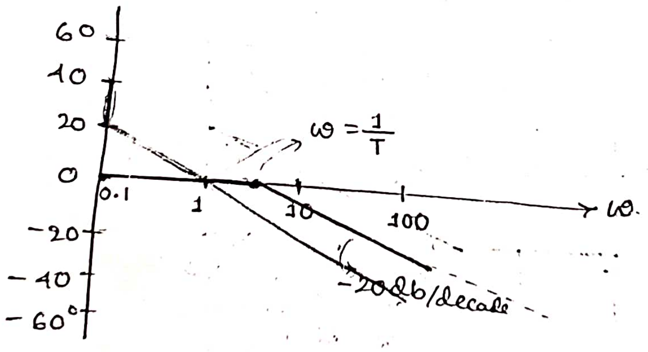
$$-20\log\sqrt{\omega^2T^2} = -20\log\omega T$$

$$= -20\log\omega + 20\log\left(\frac{1}{T}\right)$$

put, $-20\log\omega + 20\log\left(\frac{1}{T}\right) = 0$.

$\therefore \boxed{\omega = \frac{1}{T}} \longrightarrow$ corner freq
break freq

-20 db/decade



$\phi = \angle G(j\omega) = -\tan^{-1}\omega T$

(i) at very low freq.

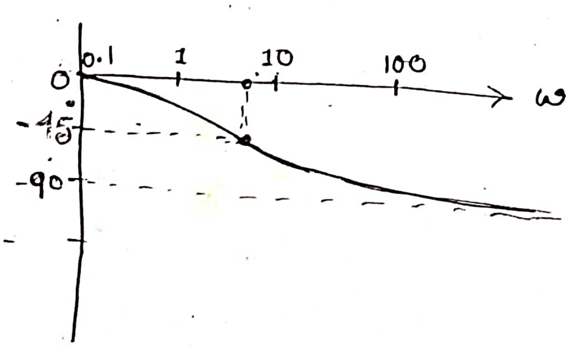
$\phi = -\tan^{-1}(0) = 0°$

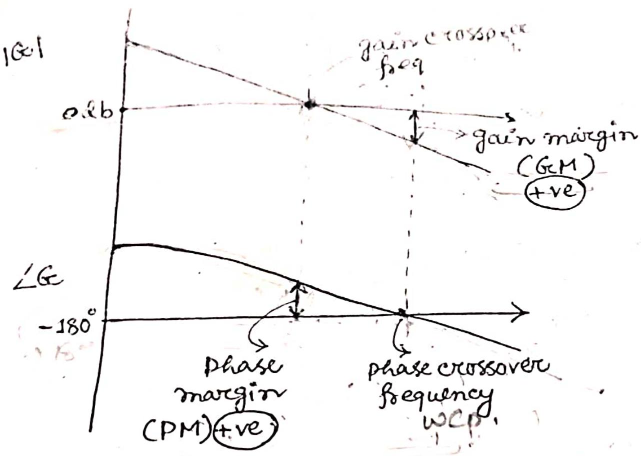
(ii) at very high freq.

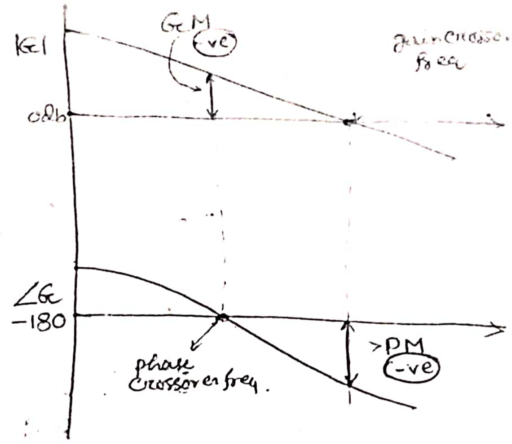
$\phi = -\tan^{-1}(\infty) = -90°$

(iii) at $\omega = \frac{1}{T}$

$\phi = -\tan^{-1}(1) = -45°$







* The point at which the magnitude curve crosses the 0db line is the gain crossover frequency. (ω_{cg})

* The point where the phase curve crosses the $-180°$ or $+180°$ line is called phase crossover frequency. (ω_{cp})

* Gain Margin – Gain margin is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability. Mathematically gain margin is defined as the reciprocal of the magnitude of $G(j\omega)H(j\omega)$ at phase crossover frequency.

$$G.M = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{cp}}}$$

In db ⇒ $G.M = 20\log\left(\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{cp}}}\right)$

$$= -20\log|G(j\omega)H(j\omega)|_{\omega=\omega_{cp}}$$

* Phase Margin – It is the additional phase angle which can be introduced in the system till system reaches on the verge of instability is called phase margin (PM). Mathematically.

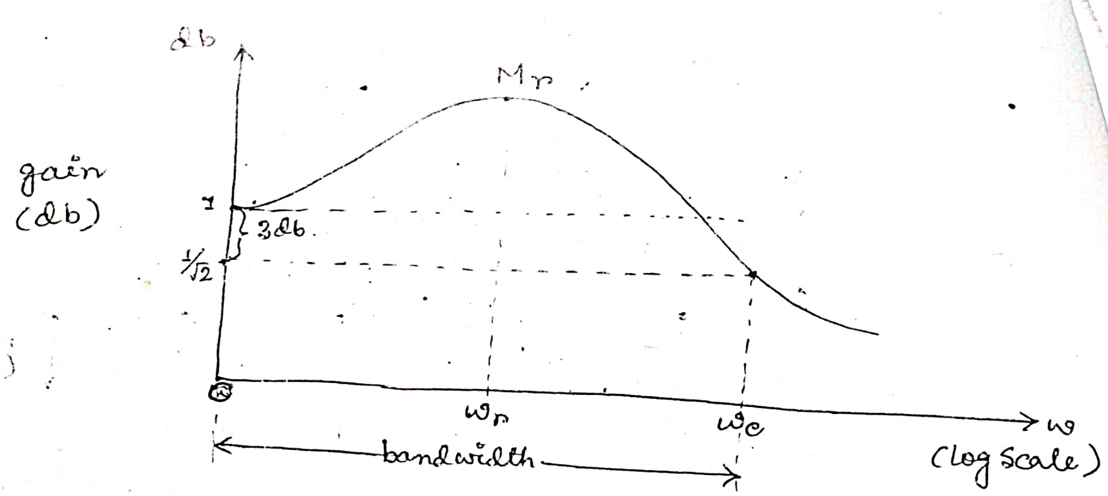
$$PM = \angle G(j\omega)H(j\omega)|_{\omega=\omega_{cg}} + 180°$$

* +ve gain margin (GM) means that the system is stable
 -ve gain margin (GM) means that the system is unstable

For min. phase system both PM and GM must be +ve for system to be stable.

Stable → $\omega_{cg} < \omega_{cp}$
unstable → $\omega_{cg} > \omega_{cp}$
marginally stable → $\omega_{cg} = \omega_{cp}$

Frequency Domain Specifications



① **Resonant Peak (M_r)** – The maximum value of magnitude is known as resonant peak. The magnitude of resonant peak gives the information about the relative stability of the system.

② **Resonant Frequency (ω_r)** – The frequency at which magnitude has max. value is known as resonant frequency. if ω_r is large, the time response is fast.

③ **Bandwidth** – The BW is defined as the range of freq. at which the magnitude gain of the frequency response plot reduces to $1/\sqrt{2} = 0.707$ i.e. 3db of its low freq value.

④ **Cut off frequency** – The frequency at which the magnitude is 3db below the zero frequency value is called cut off frequency (ω_c).

⑤ **Cut off rate** – The cut off rate is the slope of the log magnitude curve near the cut off frequency.

log x : 0.5

Ex. ① Draw the Bode plot for the system whose open loop transfer function is given by

$$G(s)H(s) = \frac{1000}{(1+0.1s)(1+0.001s)}$$

Determine ⓐ GM. ⓑ PM ⓒ closed loop stability.

Solⁿ
$$G(j\omega)H(j\omega) = \frac{1000}{(1+0.1j\omega)(1+0.001j\omega)}$$

Sl. No	Factors	Corner freq. ($\omega = 1/T$) rad/sec.	Slope contribution (db/decade)
1	1000	none = 0	$20\log 1000 = 60$ db.
2	$\frac{1}{1+0.1j\omega}$	$\omega = \frac{1}{0.1} = 10$	-20 db/decade
3	$\frac{1}{1+0.001j\omega}$	$\omega = \frac{1}{0.001} = 1000$	-20 db/decade.

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}(0.1\omega) - \tan^{-1}(0.001\omega)$$

ω	$\angle G(j\omega)H(j\omega)$
1	$-5.76°$
10	$-45.57°$
50	-81.55
100	$-90°$
150	-94.71
300	-104.79
500	-115.41
1000	-134.42
∞	$-180°$

From the bode plot,

$\omega_{cg} \simeq 3200$ rad/sec.

$\therefore PM = \angle G(j\omega)H(j\omega)|_{\omega=\omega_{cg}} + 180°$

$= -162.46 + 180°$

$= 17.53°$

$\omega_{cp} \simeq \infty$ rad/sec.

$\therefore GM = \infty$.

* The GM and PM both are +ve. so the system is stable.
* Further the GM is infinite. hence the system is inherently stable.

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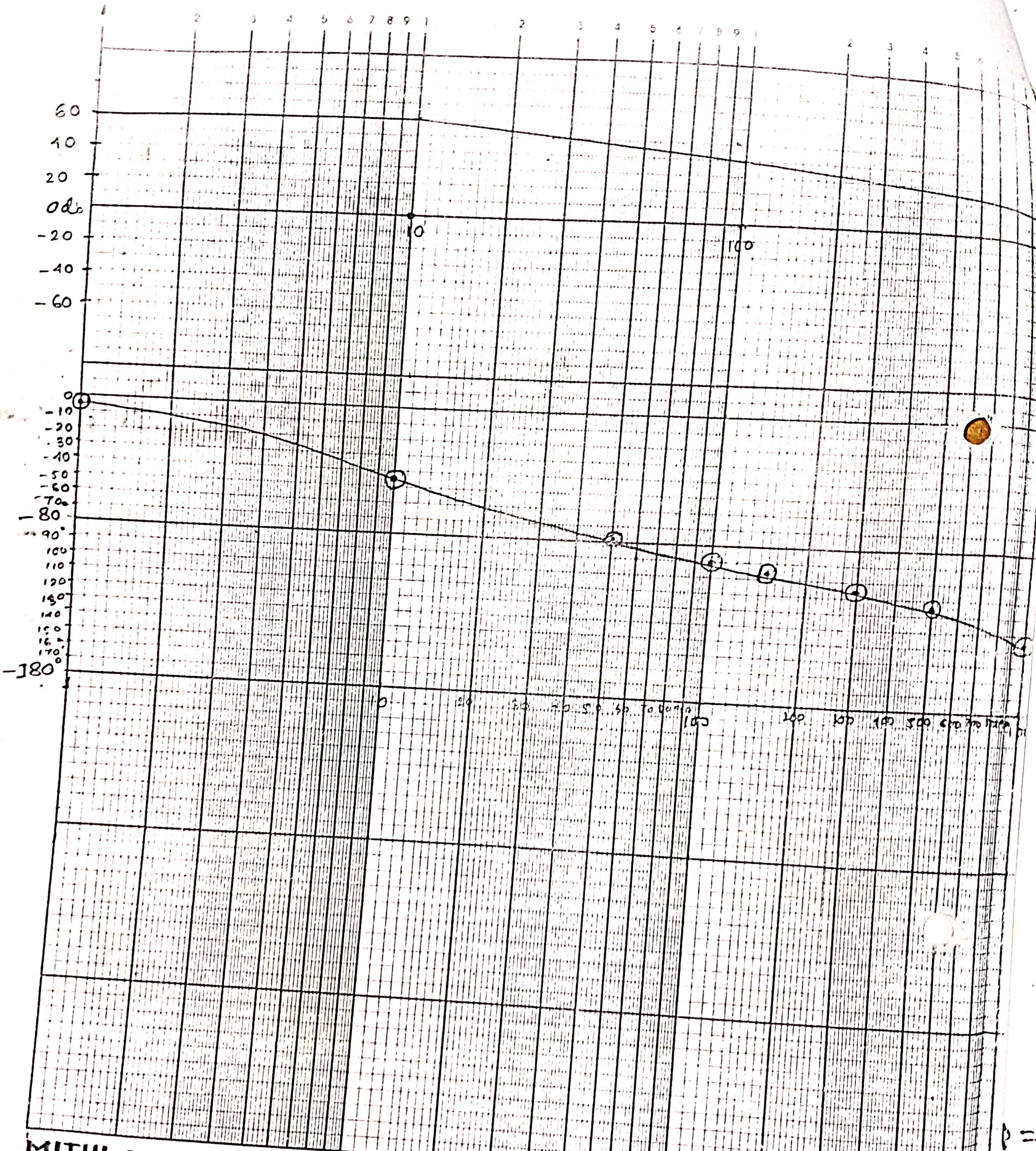
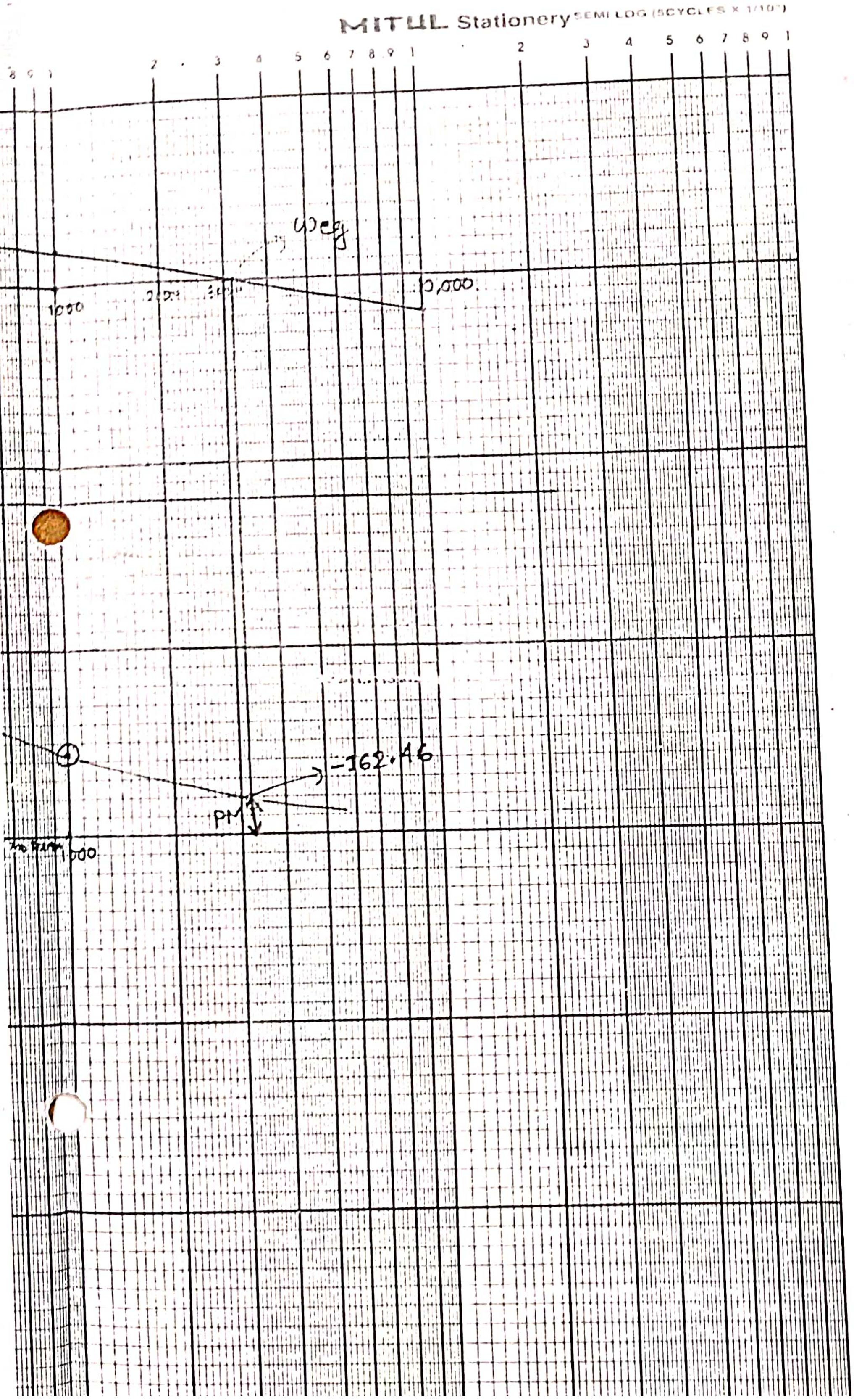


60
40
20
0db
-20
-40
-60
10
100
0
-10
-20
-30
-40
-50
-60
-70
-80
-90°
100
110
120
130
140
150
160
170
-180°



~~Slope contribution~~

~~20 log 10 = 20 db.~~

~~-20 db/decade.~~

~~-20 db/decade.~~

100

Ex (2) Draw the Bode plot for the transfer function

$$G(s) = \frac{50}{s(1+0.25s)(1+0.1s)}$$

From the graph determine

(1) Gain crossover frequency (ω_{cg}).

(2) Phase crossover frequency (ω_{cp})

(3) GM and PM.

(4) Stability of the system.

$$|G(j\omega)| = \frac{50}{\omega\sqrt{1+(0.25\omega)^2}\sqrt{1+(0.1\omega)^2}}$$

Soln

$$G(j\omega) = \frac{50}{(j\omega)(1+0.25j\omega)(1+0.1j\omega)}$$

Sl No	Factors	Corner frequency $\omega = 1/T$ (rad/sec)	Slope contribution (db/decade)	Overall Slope (db/decade)
1	50	—	$20\log 50 = 34$ db.	
2	$\frac{1}{j\omega}$	~~$\omega = \frac{1}{T} = 1$ rad/sec~~	-20 db/decade	-20
3	$\frac{1}{1+0.25j\omega}$	$\frac{1}{0.25} = 4$	-20	$-20+(-20) = -40$
4	$\frac{1}{1+0.1j\omega}$	$\frac{1}{0.1} = 10$	-20	$-40+(-20) = -60$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(0.25\omega) - \tan^{-1}(0.1\omega)$$

ω	$\angle G(j\omega)$
0.1	-92°
0.3	-96°
0.5	-100°
1	-109.74°
3	-143.56
5	-168°
10	-203.2°
20	-232.12°

ω	$\angle G(j\omega)$
50	-254.1
100	-262
6	-177.27
7	-185.24

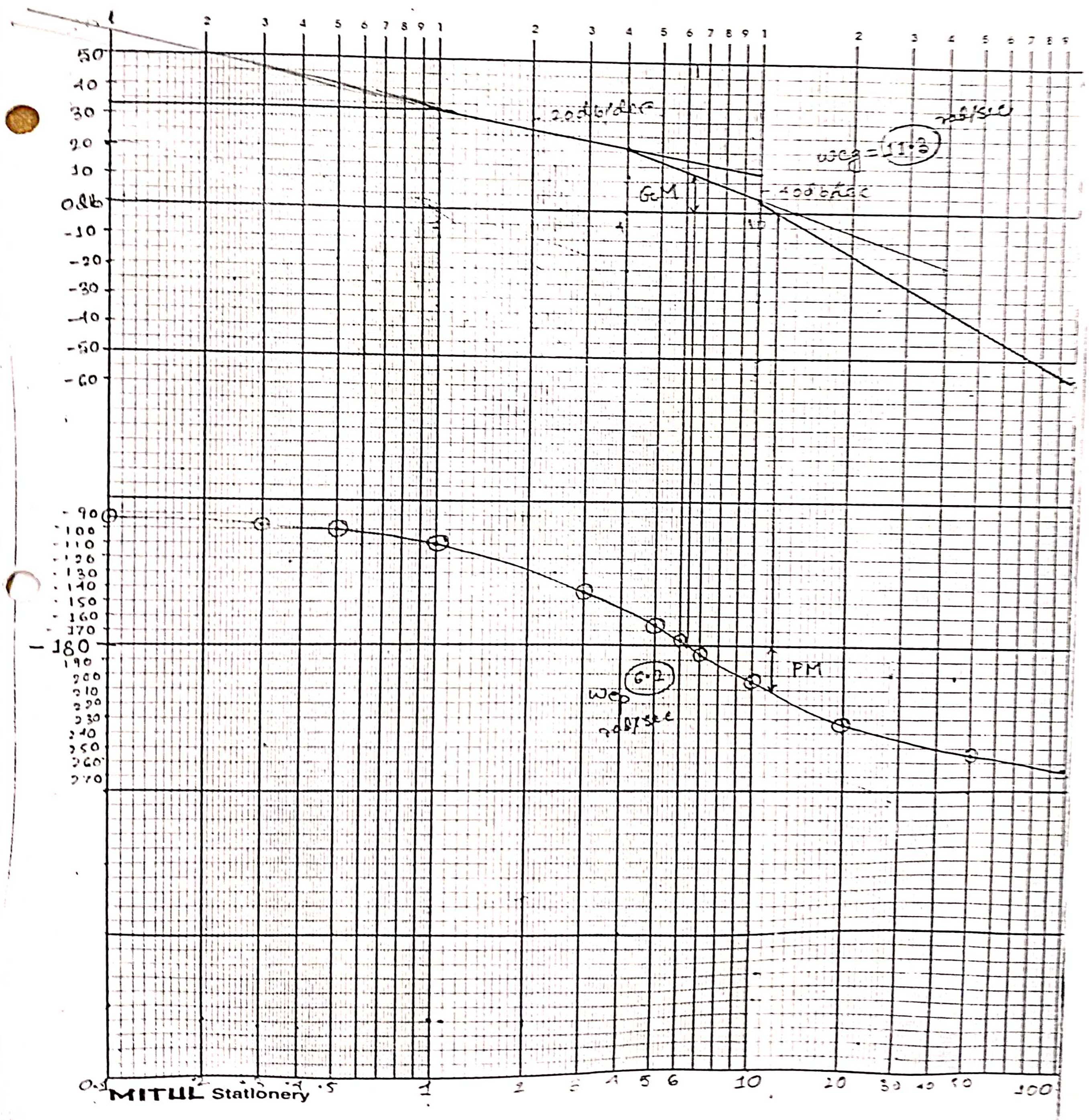
here ① gain crossover freq (ω_{cg}) = 11.3 rad/sec

② phase crossover freq (ω_{cp}) = 6.2 rad/sec.

③ $GM = -20 \log |G(j\omega)H(j\omega)|_{\omega=\omega_{cp}} = -11.4$ db.

$PM = \angle G(j\omega)H(j\omega)|_{\omega=\omega_{cg}} + 180° = -29°$

④ as both GM and PM are -ve so the system is unstable.



(102) Ex. 3 Sketch the bode plot for the transfer function,

$$G(S) = \frac{1000}{S(1+0.1S)(1+0.001S)}$$

determine: ① ω_{cg}. ② ω_{cp} ③ GM, PM ④ Stability.

Soln

$$G(j\omega) = \frac{1000}{j\omega(1+0.1j\omega)(1+0.001j\omega)}$$

$$\therefore |G(j\omega)| = \frac{1000}{\omega\sqrt{1+(0.1\omega)^2}\sqrt{1+(0.001\omega)^2}}$$

Sl. No	factors	corner freq $\omega = 1/T$ (rad/sec)	slope contribution (db/decade)	overall slope (db/decade)
1	1000	0	20 log 1000 = 60 db.	
2	$\frac{1}{j\omega}$	1	−20	−20
3	$\frac{1}{1+0.1j\omega}$	$\frac{1}{0.1} = 10$	−20	−20 + (−20) = −40
4	$\frac{1}{1+0.001j\omega}$	$\frac{1}{0.001} = 1000$	−20	−40 + (−20) = −60

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}(0.001\omega)$$

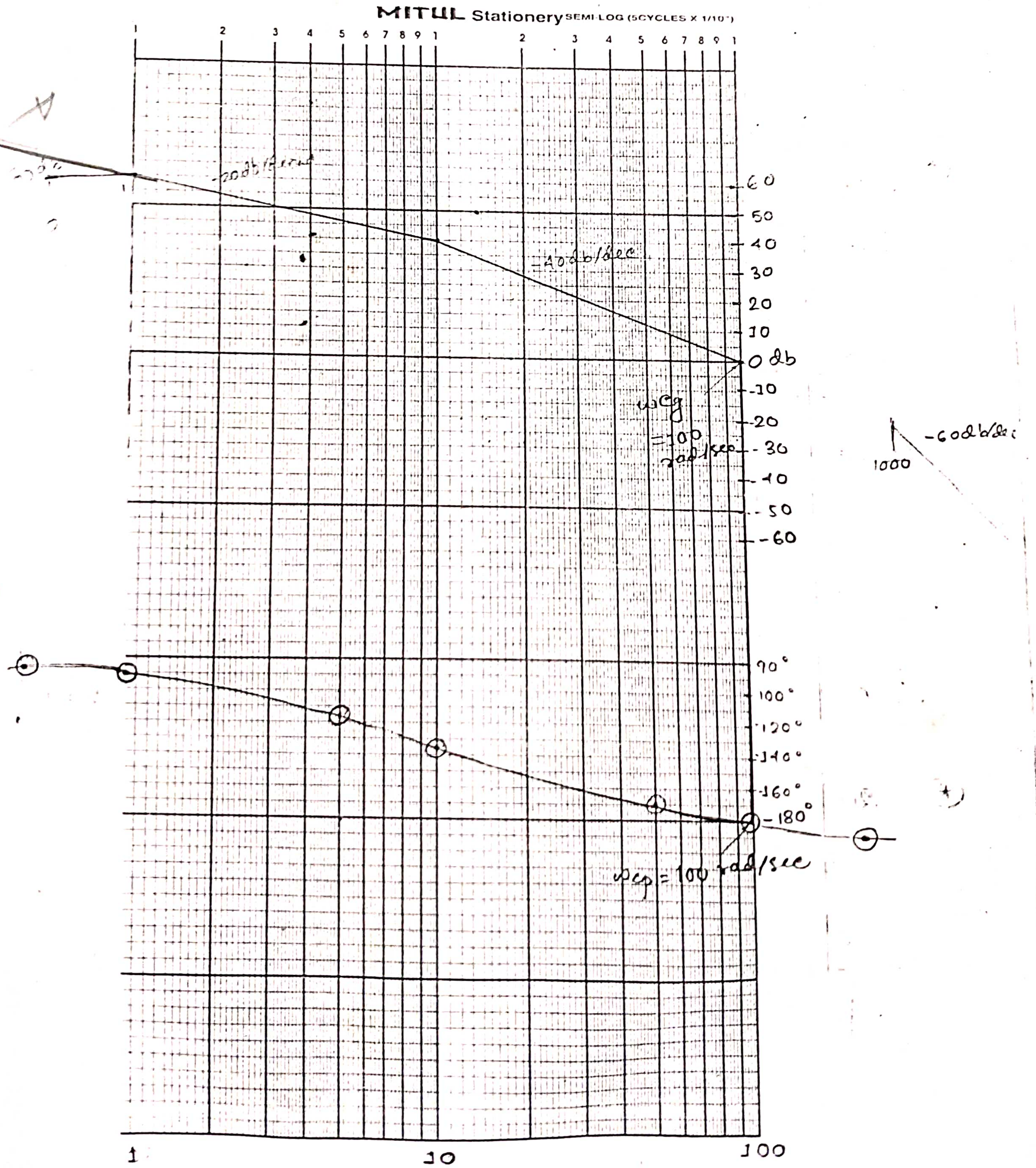
ω	$\angle G(j\omega)$
0.1	−90.57
0.5	−92.89
1	−95.76
5	−116.85
10	−135.57
50	−171.55
100	−180°
300	−194.79

Here $\omega_{cg} = \omega_{cp} = 100$ rad/sec.

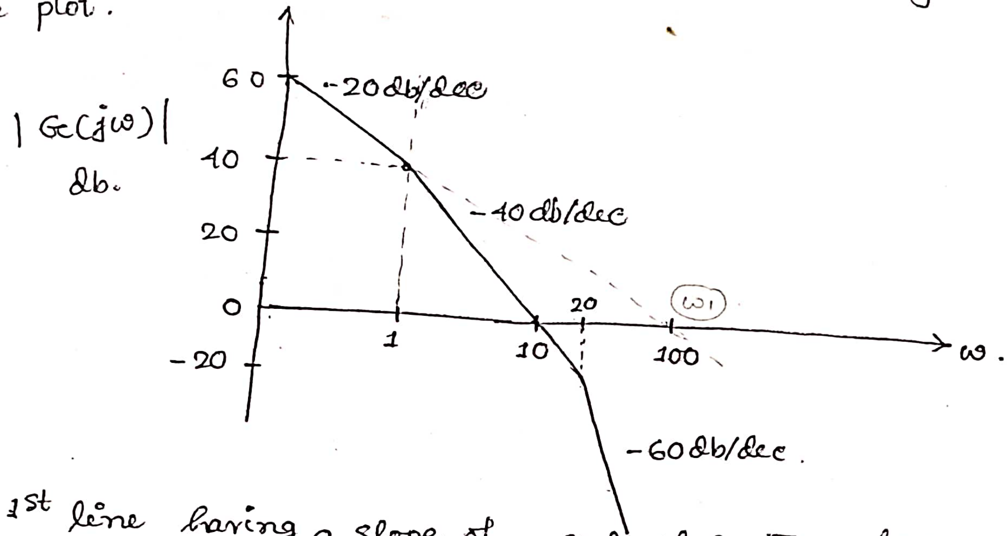
$GM = -20 \log |G(j\omega)H(j\omega)|_{\omega=\omega_{cp}} = -0.086$ db. $\simeq 0$ db.

$PM = \angle G(j\omega)H(j\omega) \Big|_{\omega=\omega_{cg}} + 180^\circ = 0^\circ$

$\therefore$ as $\omega_{cg} = \omega_{cp}$ and $GM = PM$; so the system is marginally stable.



Ex. (1) The asymptotic Bode plot of a T.F. is shown in fig. Determine the transfer function G(S) corresponding to this Bode plot.



Solⁿ (*)

① 1st line having a slope of $-20db/dec$. Therefore there is a $\frac{1}{S}$ term

② at $\omega = 1$ the slope changes to $-40db/dec$.

$\therefore$ corner freq $= \omega = 1$.

Now $\omega = 1/T \quad \therefore T = 1$.

$\therefore$ there is a $\frac{1}{1+ST} = \left(\frac{1}{1+S}\right)$ term.

③ at $\omega = 20$ the slope changes to -60 db/decade.

$\therefore T = \frac{1}{20} = 0.05$

$\therefore$ there is a $\left(\frac{1}{1+0.05S}\right)$ term.

$\therefore$ overall transfer function $\boxed{G(S) = \frac{K}{S(1+S)(1+0.05S)}}$

(*) $20 \log K = \cancel{60}\ 40$

$\therefore \log K = 2$.

$\therefore K = 10^2 = 1000$

$$20 = \frac{40}{\log \omega_1 - \log 1}$$

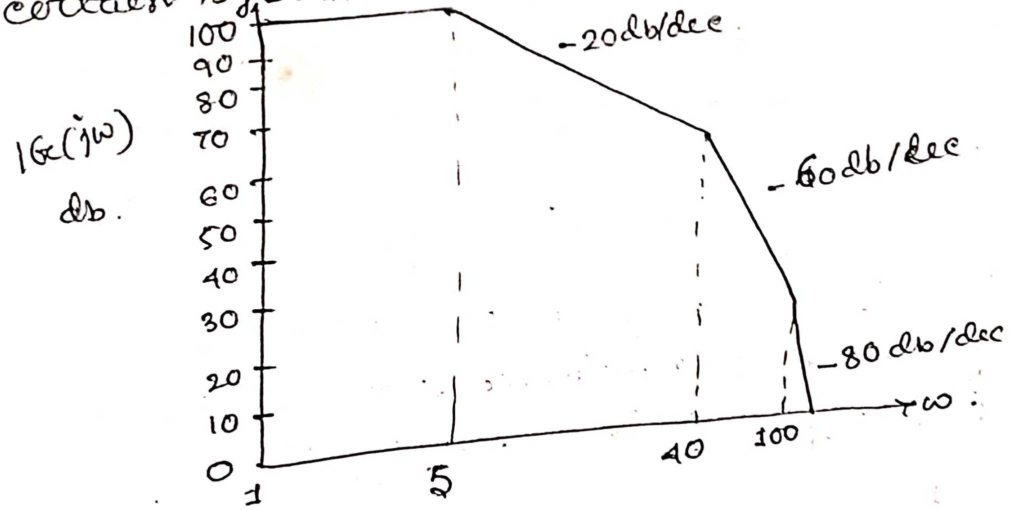
or, $\log(\omega_1/1) = 40/20 = 2$.

or, $\omega_1 = 100$

$K = 100$

$$\therefore \boxed{G(S) = \frac{1000}{S(1+S)(1+0.05S)}}$$

Ex 5 The magnitude plot of a open loop T.F G(S) of a certain System is shown in fig. Determine G(S)



Sol:

$20 \log K = 100$

$\log_{10} K = 5$ $\therefore K = 10^5$

(*) at $\omega = 5$, the slope changes to -20 db/dec

$T = \frac{1}{5} = 0.2$.

$\therefore$ There is a term $\left(\frac{1}{1+0.2S}\right)$

(*) at $\omega = 40$ the slope changes to -60 db/dec.

$T = \frac{1}{40} = 0.025$

$\therefore$ There is a term $\left(\frac{1}{1+0.025S}\right)^2$

(*) at $\omega = 100$ the slope change to -80 db/dec.

$T = \frac{1}{100} = 0.01$

$\therefore$ There is a term $\left(\frac{1}{1+0.01S}\right)$

$$\boxed{T.F = G(S) = \frac{10^5}{(1+0.2S)(1+0.025S)^2(1+0.01S)}}$$

Polar Plot

The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $|G(j\omega)|$ vs. the phase angle of $\angle G(j\omega)$ on a polar co-ordinate as ω is varied from 0 to infinity.

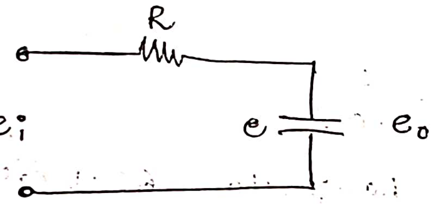
Advantage

The advantage of using a polar plot is; it depicts the frequency response characteristics of a system over the entire frequency range in a single plot.

disadvantage

These plot does not indicate the contribution of each individual factor of the open loop transfer function

Ex. 1 Prove that the polar plot of high pass RC circuit is a semicircle.



$$\frac{E_o(S)}{E_i(S)} = \frac{1/CS}{R + 1/CS} = \frac{1}{1 + RCS} = \frac{1}{1+TS}$$

$$\therefore \boxed{G(S) = \frac{1}{1+TS}}$$

put $s = j\omega$

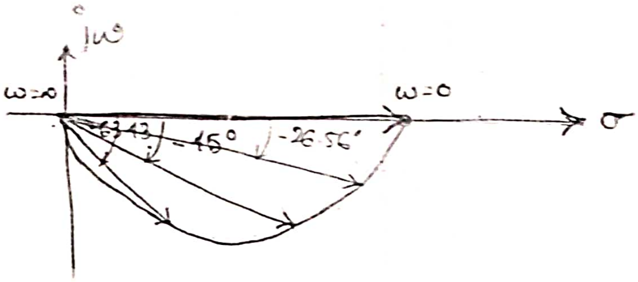
$$\therefore G(j\omega) = \frac{1}{1+Tj\omega}$$

$1 + 1/4$

$$\therefore |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2 T^2}} \qquad \angle G(j\omega) = -\tan^{-1}(\omega T)$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	1	0°
$1/T$	$1/\sqrt{2} = 0.707$	−45°
∞	0	−90°
$2/T$	0.447	−63.43°

$1/2T$	0.89	−26.56



Ex.(2) $G(s) = 1 + sT$. Find the polar plot.

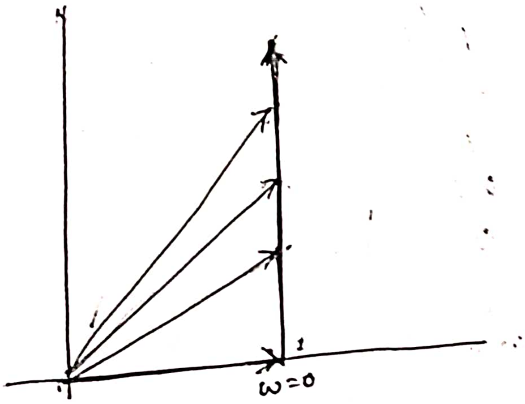
put $s = j\omega$.

$\therefore G(j\omega) = 1 + j\omega T$.

$$|G(j\omega)| = \sqrt{1 + (\omega T)^2}$$

$$\angle G(j\omega) = \tan^{-1}(\omega T)$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	1	0°
∞	∞	90°
$1/T$	$\sqrt{2} = 1.414$	45°
$1/2T$	1.11	26.5°
$2/T$	2.24	63.45°



(108)
Ex. (8) $G(s) = \frac{1}{s(1+sT)}$. Draw the polar plot.

Soln

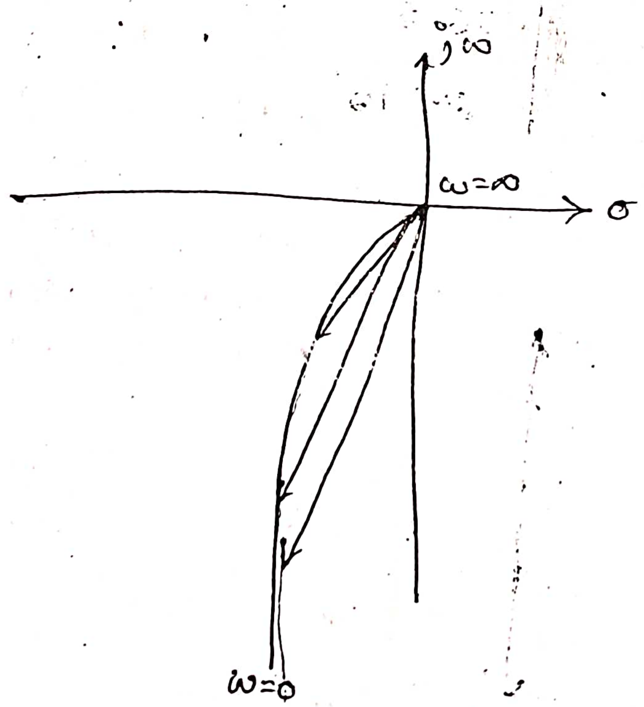
$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2T^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(\omega T)$$

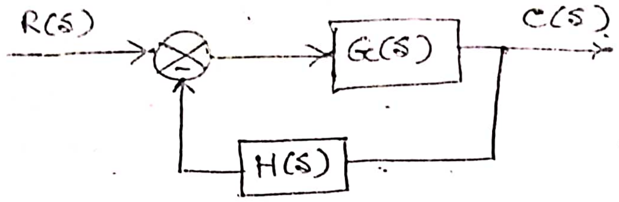
∴

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	-90°
$1/2T$	1.78T	-116.56°
$1/T$	0.707T	-135°
$2/T$	0.223T	-153.43°
∞	0	-180°



Nyquist Stability Criterion

Introduction



closed loop transfer function $= \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$

Now, if there is no point in the right half of S-plane, (including imaginary axis → here marginal stability region is taking as unstable region) ~~zero~~ for which $1+G(S)H(S)=0$, then the closed loop system is a stable system.

$\therefore$ $G(S)H(S)$ = open loop transfer function (known to me)

$$= \frac{N(S)}{\Delta(S)} \qquad [N(S) \leq \Delta(S)]$$

[hence zeros and poles of $G(S)H(S)$ is known to me]

if ~~some~~ poles are in the left half of S-plane then the system is open loop stable. If some poles are in the right half of S-plane then the system is open loop unstable.

(*) But here we want to study the stability of closed loop system. Because a feedback system has got the capability of stabilizing an open loop ~~to~~ unstable system.

$$\therefore \quad 1+G(S)H(S) = 1 + \frac{N(S)}{\Delta(S)} = \frac{\Delta(S)+N(S)}{\Delta(S)}$$

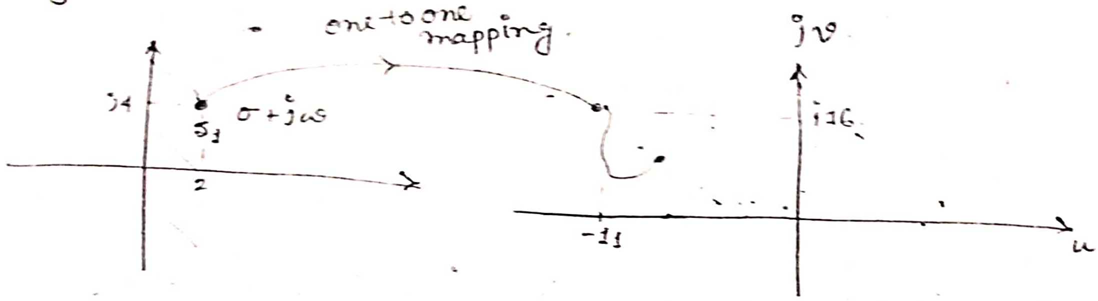
Suppose $\Delta(S)$ is the n^{th} order polynomial, then,

$$= \frac{\Delta(S)+N(S)}{(S-\alpha_1)(S-\alpha_2)\cdots\cdots(S-\alpha_n)}$$

[here order of $N(S) \leq \Delta(S)$]

$$\therefore 1+G(S)H(S) = \frac{(S-\beta_1)(S-\beta_2)\cdots\cdots(S-\beta_n)}{(S-\alpha_1)(S-\alpha_2)\cdots\cdots(S-\alpha_n)}$$

Mapping -



$s_1 = \sigma + j\omega$
$= 2 + j4$

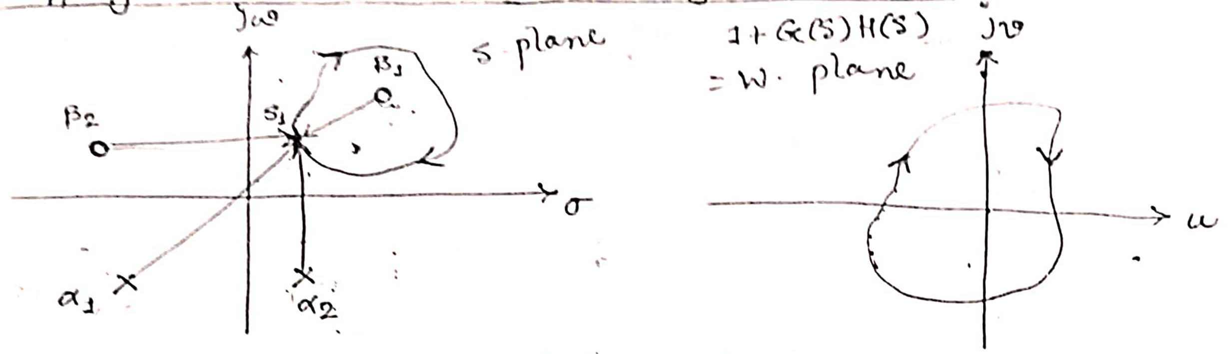
$1 + G(s)H(s)$ plane.
= W - plane.
Now, let $W(s) = s^2 + 1$.
$\therefore W(s_1) = (2+j4)^2 + 1$
$= -11 + j16$
$= u + jv$

- i.e. in every point in s-plane there is a point on w-plane
- we want to see the mapping of entire right half of s-plane under the function of $1 + G(s)H(s)$ to see wheter the origin is covered by the mapping or not.
- equivalently we can say, here we consider the imaginary axis and right half of s-plane completely and we want to map this onto the w-plane using the function $1+G(s)H(s)$ to see whether the origin of the w-plane is covered or not. If the origin of the w-plane is not covered Then the system is a ~~un~~stable system. if the origin is covered by the mapping then it is a unstable system.

[Now, connection of points / connected graph is called contour. Thus a contour in the s-plane will give us a contour in the w-plane since there is a one to one mapping.]

 → close contour.

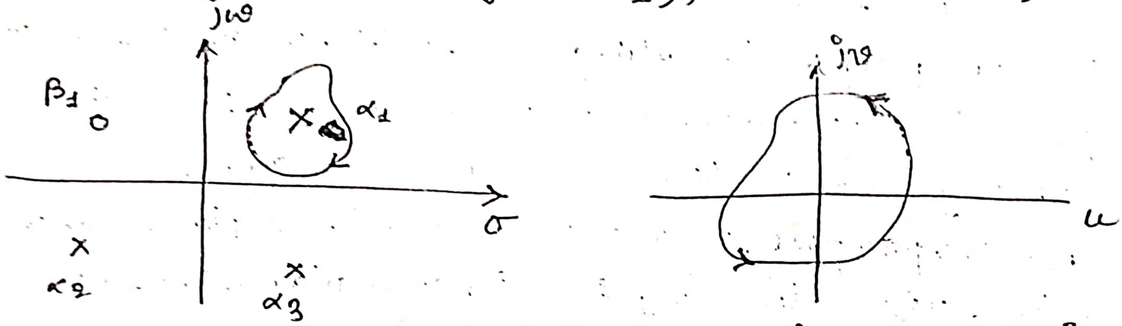
Mapping of close contour (Cauchy's Theorem) –



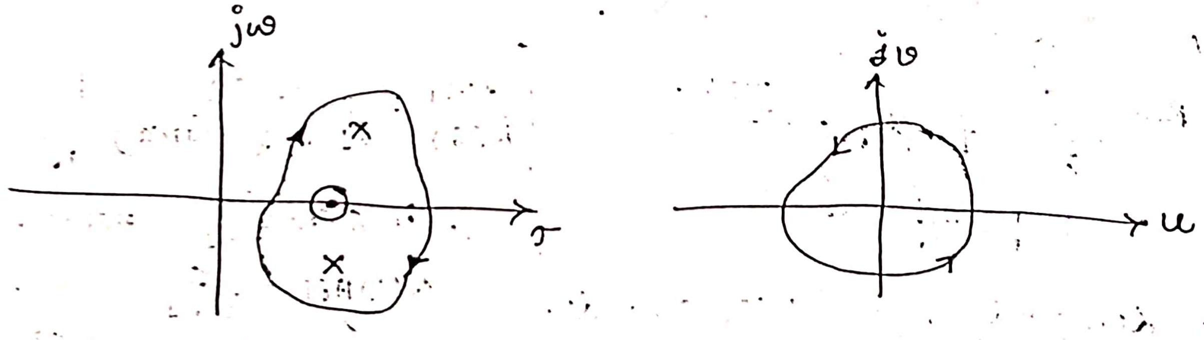
$$1 + G(S)H(S) = \frac{(S_1 - \beta_1)(S_1 - \beta_2)}{(S_1 - \alpha_1)(S_1 - \alpha_2)}$$

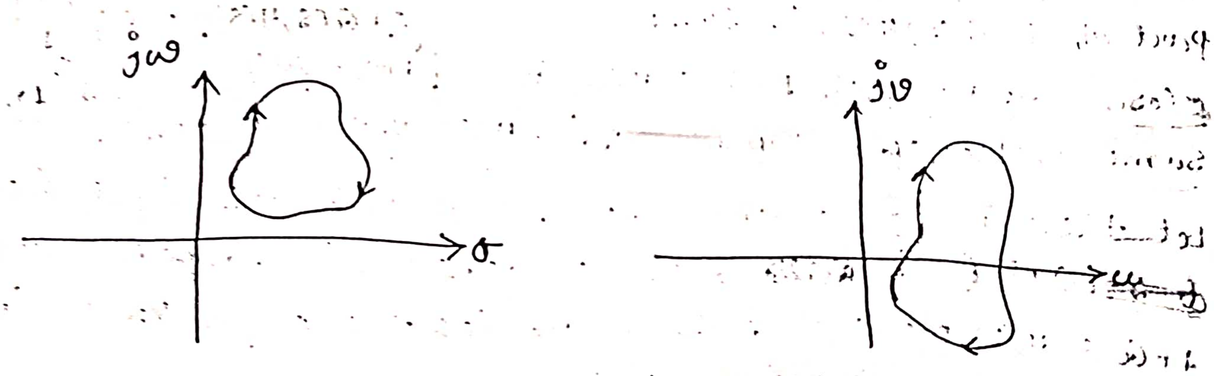
the angle contribution by $(S_1 - \beta_1) = -2\pi$

the angle contribution by $(S_1 - \beta_2), (S_1 - \alpha_1), (S_1 - \alpha_2)$ is 0°

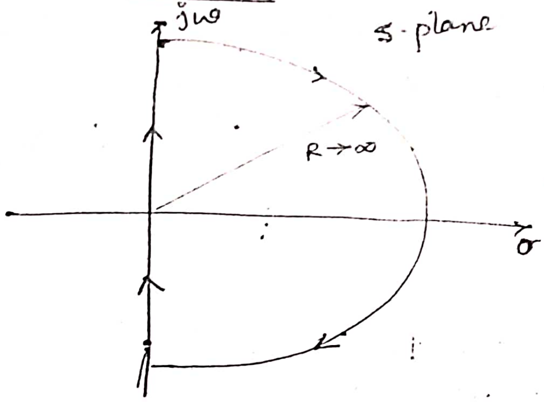


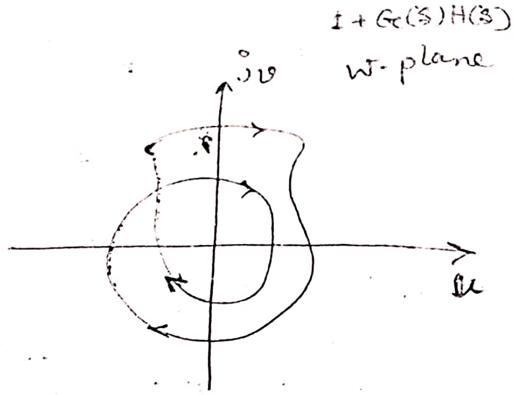
The angle contribution by this pole is -2π. Since the pole is in the denominator of $1 + G(S)H(S)$, thus the net angle contribution is $+2\pi$





Nyquist contour –





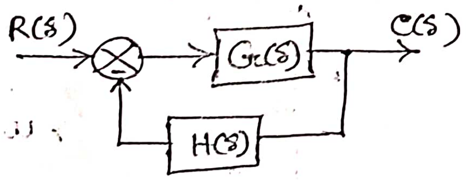
$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

∴ The ch. eq $1+G(S)H(S)=0$.

The main purpose in study the stability of closed loop system is to determine whether the ch. eq $1+G(S)H(S)=0$ has any roots in the right half of s-plane.

For this purpose, we use a contour in s-plane which encloses the entire right half of s-plane. This contour having the encirclement in clockwise direction and radius (R) → ∞. This path or contour is known as Nyquist contour.

Nyquist stability criterion: –



$$\boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}}$$

if $G(S) = \frac{S}{S+1}$, $H(S)=1$.

$G(S)H(S) = \frac{S}{S+1}$

$1+G(S)H(S) = \frac{2S+1}{S+1}$

closed loop T.F. $T(S) = \frac{G(S)}{1+G(S)H(S)} = \frac{S}{2S+1}$

The den part of $G(S)H(S)$ and den part of $1+G(S)H(S)$ is same. i.e.

(*) closed loop poles of $1+G(S)H(S)$ is same as the open loop ~~poles~~ of $G(S)H(S)$ of the system.

(*) ~~Let there is 'Z' zeroes and 'P' poles in the right half of s-plane~~ (If S-plane contour is mapped in the $1+G(S)H(S)$ plane and it encircles the origin 'N' times then

$$\boxed{N = P - Z}$$

N = +ve for anticlockwise direction
N = –ve for clockwise direction

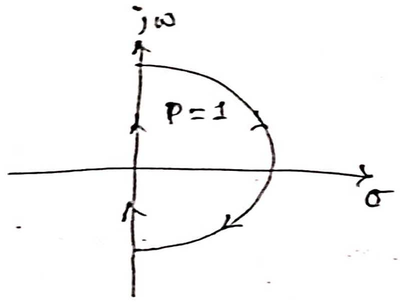
P = No of poles of $G(S)H(S)$ in the right half of s-plane.
Z = No of zeroes of $(1+G(S)H(S))$ in the right half of s-plane.

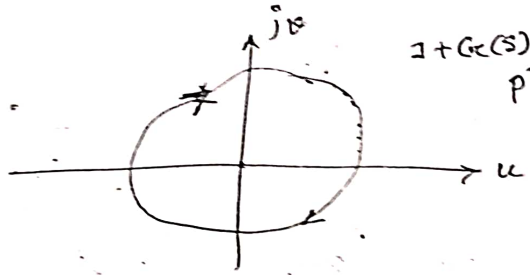
(*) A closed loop system is stable if and only if there is no zeroes of $1+G(S)H(S)$ in the right half of s-plane

i.e. $Z = 0$ as zeroes of $(1+G(S)H(S))$ = poles of the closed loop transfer function $T(S)$

$\therefore N = +P$

(*) Therefore for a closed loop system to be stable, the number of anticlockwise encirclement of origin (N) of $1+G(S)H(S)$ plane should be equal to the number of poles in right half of s-plane (P).



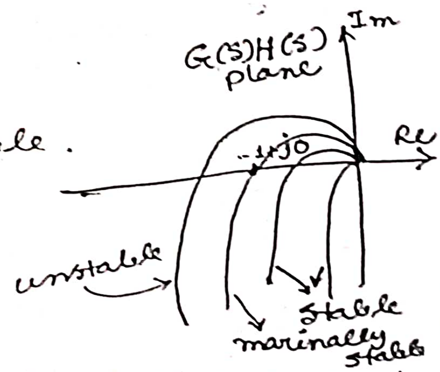


(*) $[1+G(S)H(S)]$ plot is difficult to plot. Thus instead of making $1+G(S)H(S)$ plot, we make only $G(S)H(S)$ plot instead of looking at the origin we look the encirclement of the point $(-1+j0)$ i.e. we shift the entire plot by 1 unit towards left.

$$G(S)H(S) = [1+G(S)H(S)] - 1$$

Conclusion –

An open loop stable system is stable under closed loop operation if and only if the plot of $G(S)H(S)$ does not encircle the $(-1+j0)$ point.



[let, $\dfrac{S+2}{(S+1)(S-1)}$ → open loop unstable.

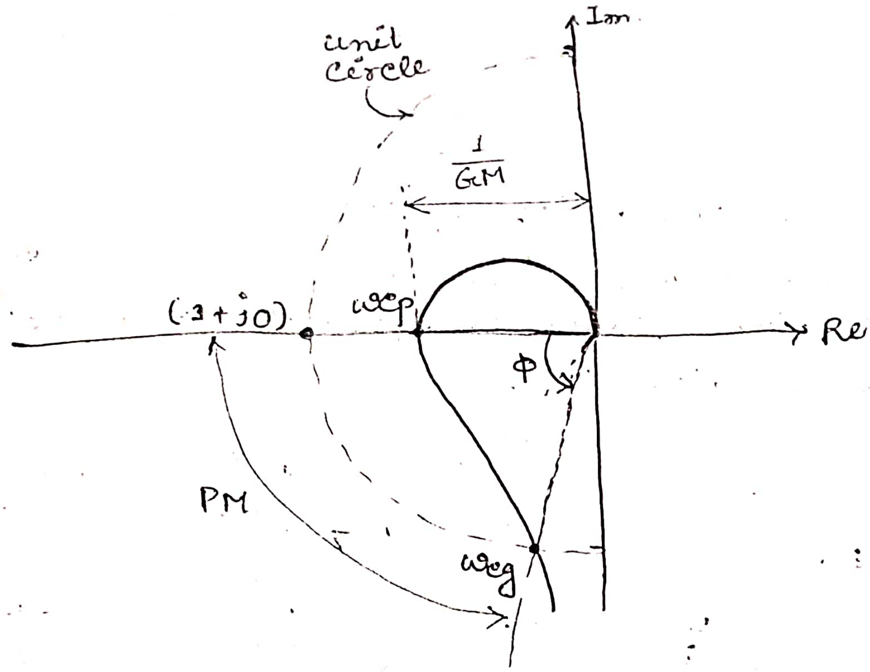
$P = 1$ and let $Z = 0$

$N = P - Z$

$\therefore N = 1$.

So encircles the $(-1+j0)$ point once in anticlockwise direction and it is a unstable system.]

Gain Margin and Phase Margin



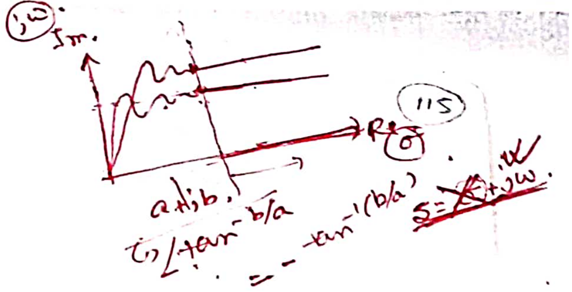
* Phase crossover freq (ωcp) - It is the frequency where system has a phase of $-180°$ or $+180°$.
* Gain crossover freq (ωcg) - It is the frequency where gain of the system become 0db.
* GM - It is the reciprocal of gain of the system at ωcp.

$$GM = 20 \log \frac{1}{|G(j\omega) H(j\omega)|}\Bigg|_{\omega = \omega_{cp}}$$

$$= -20 \log |G(j\omega) H(j\omega)|\Big|_{\omega = \omega_{cp}}$$

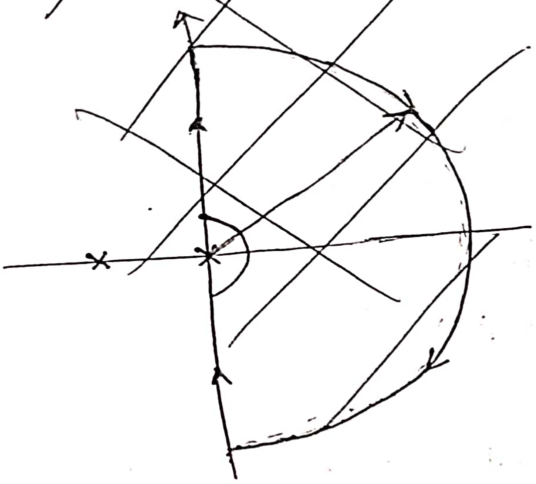
* PM - It is the additional phase angle introduce at ωcg to make the system in verge of instability.

$$PM = \angle G(j\omega) H(j\omega)\Big|_{\omega = \omega_{cg}} + 180°$$



Prob ① $G(s)H(s) = \dfrac{1}{s(s+1)}$

Soln $G(j\omega)H(j\omega) = \dfrac{1}{j\omega(j\omega+1)}$



① $|G|$, $\angle G$, $\omega = 0$

② $|G|$, $\angle G$, $\omega = \infty$

③ Intersection with the img. axis

④ " " " real axis

Prob ① Sketch the Nyquist plot for the open loop transfer function

$$G(s)H(s) = \frac{10}{(s+2)(s+4)}$$

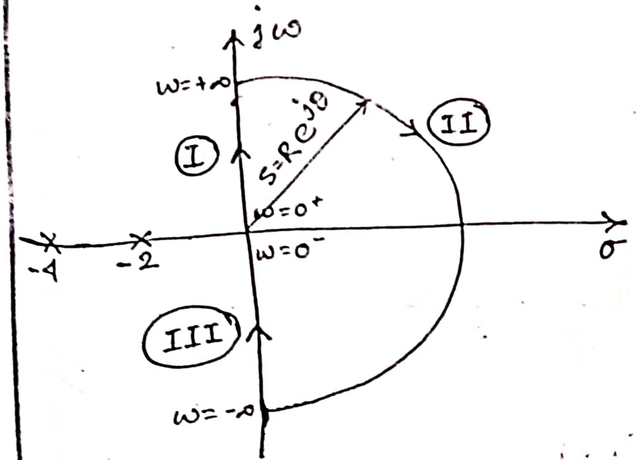
Determine the stability of the closed-loop system by Nyquist criterion.

Soln
$$G(j\omega)H(j\omega) = \frac{10}{(j\omega+2)(j\omega+4)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{10}{\sqrt{\omega^2+4}\,\sqrt{\omega^2+16}}$$

$$\phi = \angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$$

* The Nyquist contour will be



(118)

① Mapping for section (I)

ⓐ at $\omega = 0^+$, $M = \dfrac{10}{2\times 4} = 1.25$

$\phi = 0°$

ⓑ at $\omega = +\infty$, $M = \dfrac{10}{\infty} = 0$

$\phi = -\tan^{-1}(\infty) - \tan^{-1}(\infty) = -180°$

ⓒ

$$\frac{10}{(j\omega+2)(j\omega+4)} = \frac{10}{(2+j\omega)(4+j\omega)}$$

$$= \frac{10(2-j\omega)(4-j\omega)}{(4+\omega^2)(16+\omega^2)}$$

$$= \frac{10(8-2j\omega-4j\omega-\omega^2)}{(4+\omega^2)(16+\omega^2)}$$

$$= \frac{10(8-\omega^2)}{(4+\omega^2)(16+\omega^2)} + j\,\frac{-6\omega\times 10}{(4+\omega^2)(16+\omega^2)}$$

put, $\dfrac{10(8-\omega^2)}{(4+\omega^2)(16+\omega^2)} = 0$.

$\therefore\ 8-\omega^2 = 0$.

$\therefore\ \omega = \sqrt{8} = 2.828$.

$\therefore\ M\big|_{\omega=2.828} = 0.589$ $\quad \phi\big|_{\omega=2.828} = -90°$

put $\dfrac{-6\omega\times 10}{(4+\omega^2)(16+\omega^2)} = 0$

$\therefore\ \omega = 0$

$\therefore\ M\big|_{\omega=0} = \cancel{\infty}\ 1.25$

② Mapping for Section (II)

$$s = \lim_{R\to\infty} Re^{j\theta}$$

θ is varying from $+90°$ to $-90°$ through $0°$.

$$\therefore\ G(s)H(s) = \frac{10}{(Re^{j\theta}+2)(Re^{j\theta}+4)}$$

as $R \to \infty$ and $Re^{j\theta} >> 2$

$Re^{j\theta} >> 1$.

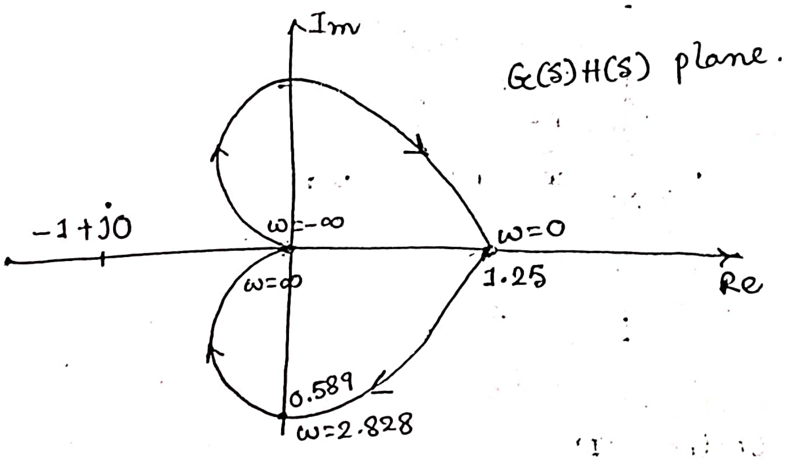
$$\therefore G(S)H(S) = \lim_{R\to\infty} \frac{10}{Re^{j\theta} \cdot Re^{j\theta}}$$

$$= \lim_{R\to\infty} \frac{10}{R^2 e^{2j\theta}} = 0$$

Thus the entire semicircle (II) in s plane is mapped into a circular arc of radius zero at the origin in the GH plane.

③ Mapping for section (III).

* here ω varies from $-\infty$ to 0^-
* This mapping will be mirror image about the real axis corrosponding to section (I)



Stability

* No. of poles in the right half of s plane (P) = 0
* It is seen that Nyquist plot does not encircle the point (-1+j0) ∴ N = 0.
* $\therefore N = P - Z$

  $\therefore Z = 0$

Thus there are no zeroes of $1 + G(S)H(S)$

Hence the closed loop system is stable.

(118)
Prob (2) Sketch the Nyquist plot for the open-loop transfer function

$$G(S)H(S) = \frac{(S+2)}{(S+1)(S-1)}$$

Determine the stability of the closed-loop system by Nyquist criterion.

Solⁿ

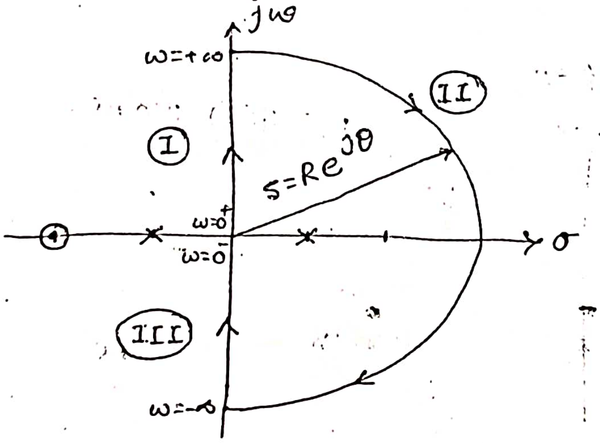
$$G(j\omega)H(j\omega) = \frac{j\omega+2}{(j\omega+1)(j\omega-1)} = \frac{2+j\omega}{j^2\omega^2-1} = -\frac{2+j\omega}{1+\omega^2}$$

$$M = |G(j\omega)H(j\omega)| = \frac{\sqrt{\omega^2+4}}{1+\omega^2}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\omega - (\pi - \tan^{-1}\omega)$$

$$= \tan^{-1}\left(\frac{\omega}{2}\right) - \pi$$

(*) The Nyquist contour will be.



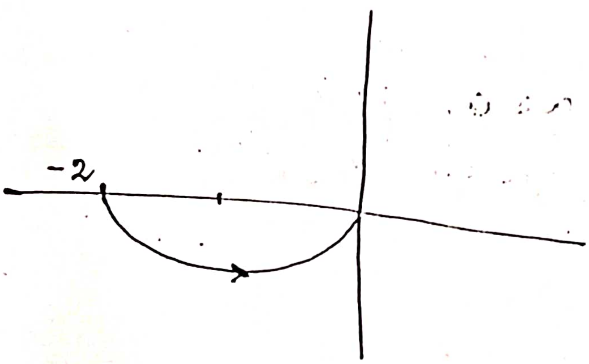
(1) Mapping of section (I)

at $\omega = 0^+$, $M = 2$, $\phi = -180°$

at $\omega = +\infty$, $M = 0$, $\phi = -90°$

During $G(j\omega)H(j\omega) = \frac{-2}{1+\omega^2} - j\frac{\omega}{1+\omega^2}$

Put, $\omega = 2$, $M = 0.565$, $\phi = -135°$



② Mapping of section (II)

$$s = \lim_{R \to \infty} Re^{j\theta}$$

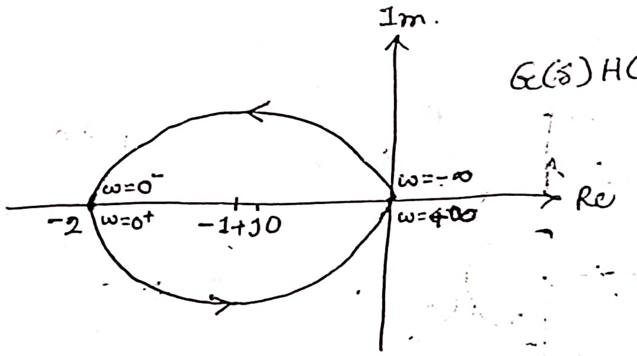
θ is varying from $+90°$ to $-90°$ through $0°$

$$G(s)H(s) = \lim_{R\to\infty} \frac{Re^{j\theta}+2}{(Re^{j\theta}+1)(Re^{j\theta}-1)} = \lim_{R\to\infty} \frac{Re^{j\theta}}{R^2 e^{j2\theta}}$$

$$= \lim_{R\to\infty} \frac{1}{R} e^{-j\theta} \qquad = 0\angle -90° \text{ to } 0\angle 90°$$

③ Mapping of Section (III)

In this section ω varies from $-\infty$ to 0^-. The mapping will be mirror image about the real axis corrosponding to the section (I).



Stability -

Here $N = 1$, $P = 1$

$N = P - Z$

$\therefore\ 1 = 1 - Z$

$\therefore\ Z = 0$

⊛ Thus there is No zeroes of $1 + G(s)H(s)$ in the right half of s-plane .. i.e. No pole of the closed loop system that lies in the right half of s-plane.

⊛ Hence the closed loop system is stable.

(125)

Prob (3)

$$G(S)H(S) = \frac{1}{S(1+S)(1+2S)}$$

(a) Investigate the stability using Nyquist stability criterion

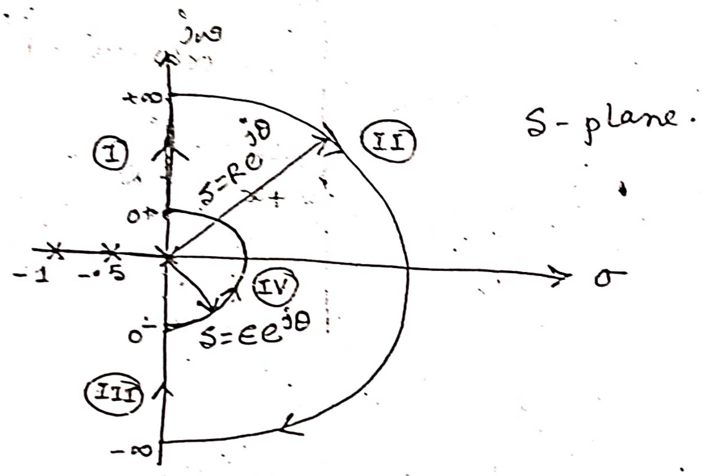
(b) Determine ω_{cg}, ω_{cp}, GM, PM.

Solu^n $$G(j\omega)H(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$$\therefore M = \frac{1}{\omega\sqrt{\omega^2+1}\sqrt{4\omega^2+1}}$$

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

(*) There is one pole at the origin. So the Nyquist contour will be..



(1) Mapping of section (I)

at $\omega = 0^+$, $M = \infty$, $\phi = -90^\circ$

at $\omega = +\infty$, $M = 0$, $\phi = -270^\circ$

$$\frac{1}{j\omega(1+j\omega)(1+2j\omega)} = \frac{-j(1-j\omega)(1-2j\omega)}{\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-j[1 - 2j\omega - j\omega - 2\omega^2]}{\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-j + 2\omega^2 + \omega^2 + 2j\omega^2}{\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{3}{(1+\omega^2)(1+4\omega^2)} - j\,\frac{1-2\omega^2}{\omega(1+\omega^2)(1+4\omega^2)}$$

Put $\dfrac{1-2\omega^2}{\omega(1+\omega^2)(1+4\omega^2)} = 0$

$$2\omega^2 = +1$$

$$\therefore \omega^2 = +\frac{1}{2}$$

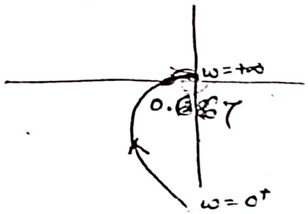
$$\therefore \omega = 0.707$$

$$M\Big|_{\omega=0.707} = \frac{1}{0.707\sqrt{(0.707)^2+1}\,\sqrt{(0.707^2\times 4)+1}}$$

1.224

$$= 0.667$$

$$\phi\Big|_{\omega=0.707} = -180^\circ$$



Mapping for Section (II).

$s = Re^{j\theta}$

θ is varying from $+90^\circ$ to -90° through 0°

$$G(s)H(s) = \lim_{R\to\infty} \frac{1}{Re^{j\theta}(1+Re^{j\theta})(1+R2.e^{j\theta})}$$

$$= \lim_{R\to\infty} \frac{1}{2R^3 e^{3j\theta}}$$

$$= 0$$

Mapping of Section (III)

The plot of $G(j\omega)H(j\omega)$ will be symmetrical about the real axis of GH plane and mirror image of section (I).

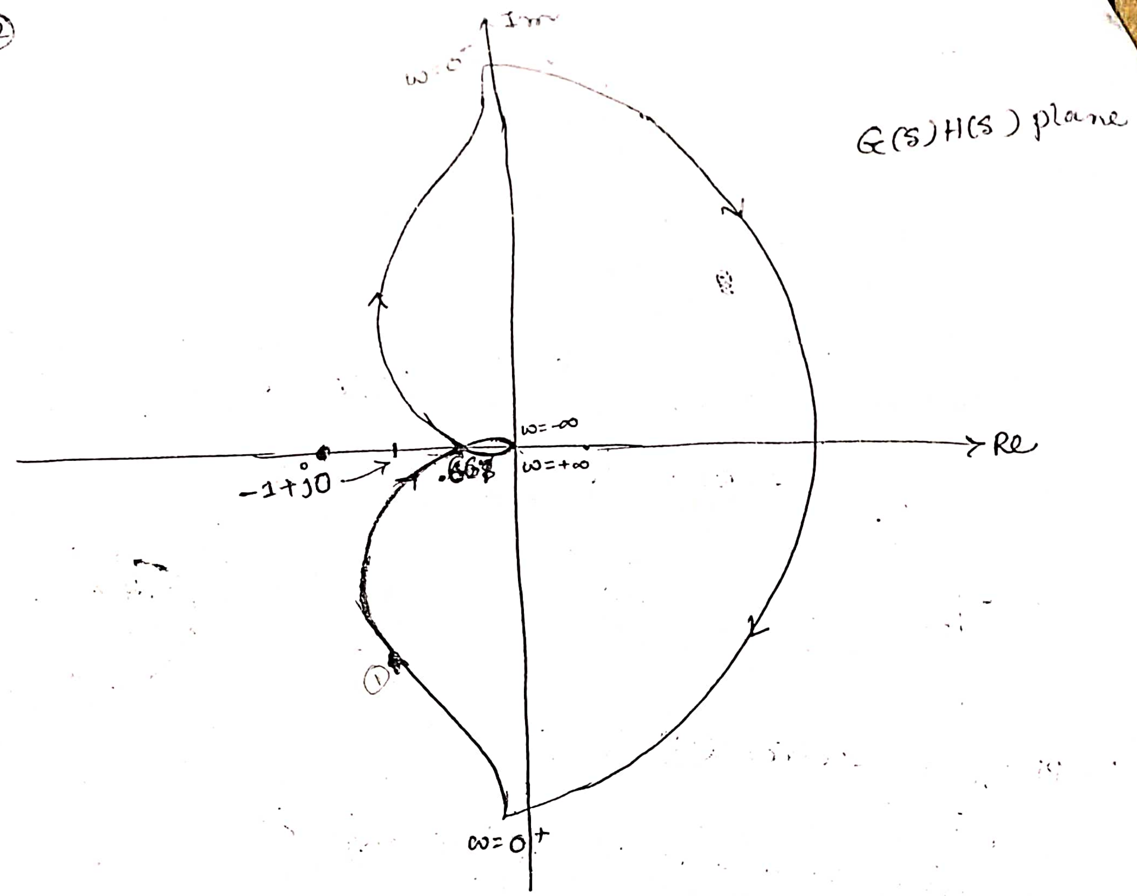
Mapping of Section (IV).

, θ varies from $+90^\circ$ to -90°

$s = \epsilon e^{j\theta}$

$$G(s)H(s) = \lim_{\epsilon\to 0} \frac{1}{\epsilon e^{j\theta}(1+\epsilon e^{j\theta})(1+2\epsilon e^{j\theta})} = \lim_{\epsilon\to 0} \frac{1}{\epsilon} e^{-j\theta}$$

$$= \infty\, e^{-j\theta}$$



<u>Stability</u>.

Here, $N = 0$, $P = 0$.

$\therefore Z = 0$

$\therefore$ The system is a stable system.

(*) $G(j\omega) \cdot H(j\omega) = \dfrac{1}{j\omega(1+j\omega)(1+2j\omega)}$

(I)

$$= \cancel{\frac{1}{j\omega(1+2\omega^2+j3\omega)}}$$

$$= \cancel{\frac{1}{-3\omega^2 + j\omega(1-2\omega^2)}}$$

$$= \frac{3}{(1+\omega^2)(1+4\omega^2)} - j\,\frac{1-2\omega^2}{\omega(1+\omega^2)(1+4\omega^2)}$$

(*) At phase crossover frequency (ω_{cp}), the imaginary part is zero.

$\therefore 1 - 2\omega^2 = 0$.

$\therefore \boxed{\omega = 0.707 \text{ rad/sec}} \rightarrow \omega_{cp}$

$$GM = \frac{1}{M}\Big|_{\omega=\omega_{cp}}$$

$$= 0.707\sqrt{(0.707)^2+1}\sqrt{(0.707)^2\times 4+1}$$

$$= 1.5$$

in db $= 20\log 1.5 = 3.5$ db.

(*) at gain crossover freq (ω_{cg}) the magnitude of $G(j\omega)H(j\omega)$ is unity (1)

$$\therefore \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} = 1.$$

$$\therefore \omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2} = 1$$

$$\omega^2(1+\omega^2)(1+4\omega^2) = 1.$$

let, $\omega^2 = x$.

$$\therefore x(1+x)(1+4x) = 1.$$

by trit and trial, $x = 0.33$.

$$\therefore \omega = \sqrt{x} = \sqrt{0.33} = 0.574 \text{ rad/sec} \rightarrow \omega_{cg}.$$

$$\therefore PM = \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{cg}} + 180^\circ$$

$$11.2^\circ.$$

as GM and PM both are +ve, so the system is a ~~stable~~ system.

(124)
Prob (4) The open loop T.F.

$$G_c(s)H(s) = \frac{K(s+1)}{s^2(s+2)(s+4)}$$

using nyquist criterion, determine the range of values of k for which the system is stable.

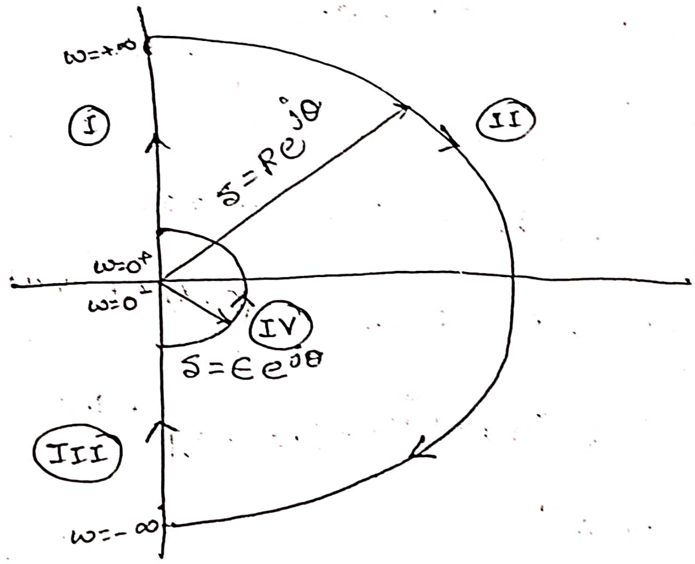
soln

$$G_c(j\omega)H(j\omega) = \frac{K(j\omega+1)}{(j\omega)^2(j\omega+2)(j\omega+4)}$$

$$M = \frac{K\sqrt{\omega^2+1}}{\omega^2\sqrt{\omega^2+4}\,\sqrt{\omega^2+16}}$$

$$\phi = \tan^{-1}\omega - 180^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

(*) Nyquist contour will be



(1) Mapping of Section (I)

$\omega = 0^+$, $M = \infty$, $\phi = -180^\circ$

$\omega = +\infty$, $M = 0$, $\phi = -270^\circ$

$\omega = 3$
$M = 0.08$
$\phi = -201.61^\circ$

$$G_c(j\omega)H(j\omega) = \frac{-K(j\omega+1)(2-j\omega)(4-j\omega)}{\omega^2(\omega^2+4)(\omega^2+16)}$$

$$= \frac{-K(j\omega+1)(8-6j\omega-\omega^2)}{\omega^2(\omega^2+4)(\omega^2+16)}$$

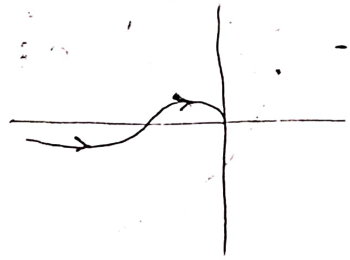
$$= \frac{-K(8j\omega+6\omega^2-j\omega^3+8-6j\omega+\omega^2)}{\omega^2(\omega^2+4)(\omega^2+16)}$$

$$= \frac{-K(2j\omega + 7\omega^2 - j\omega^3 + 8)}{\omega^2(\omega^2+4)(\omega^2+16)}$$

$$= \frac{-K(7\omega^2+8)}{\omega^2(\omega^2+4)(\omega^2+16)} - Kj\,\frac{2\omega - \omega^3}{\omega^2(\omega^2+4)(\omega^2+16)}$$

$$2 - \omega^2 = 0.$$

$$\therefore \omega = \sqrt{2} = 1.414.$$



$$M\big|_{\omega=1.414} = 0.0833K$$

$$\phi\big|_{\omega=1.414} = -180^\circ$$

Mapping for Section (II)

Mapping for Section (III)

Mapping for Section (IV)

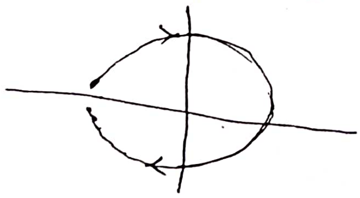
$$s = \epsilon e^{j\theta}$$

$$G(s)H(s) = \frac{K(\epsilon e^{j\theta})}{(\epsilon e^{j\theta})^2(\epsilon e^{j\theta}+2)(\epsilon e^{j\theta}+4)}$$

$$= \lim_{\epsilon \to 0} \frac{K}{8\epsilon^2 e^{2j\theta}}$$

$$= \infty\, e^{-2j\theta}$$

$\therefore$ Nyquist plot in GH plane is moving from $+180^\circ$ to -180°



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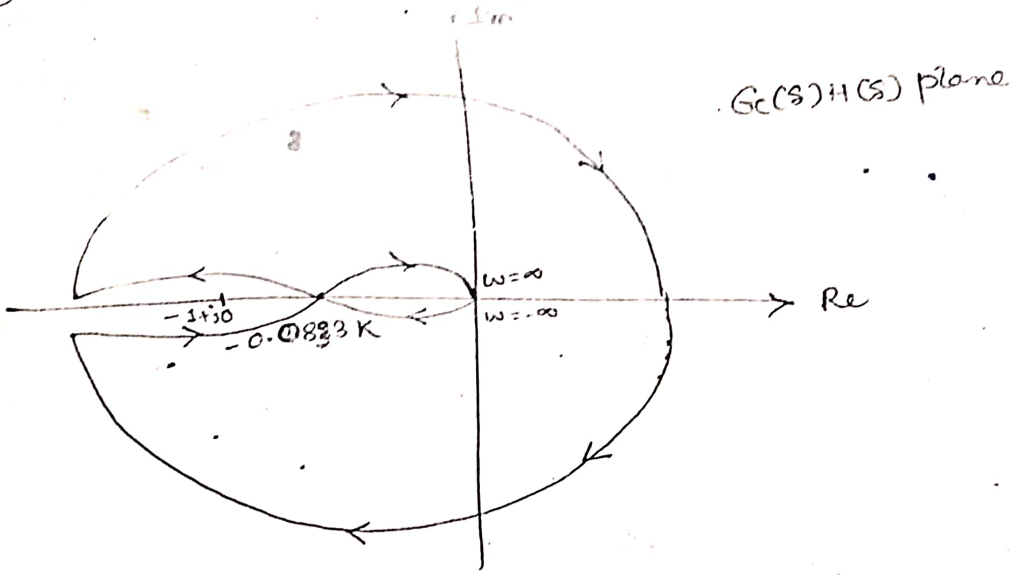


$-0.0833K = -1$.

$K = 12 \rightarrow$ marginally stable.

$K > 12 \rightarrow$ ~~stable~~ unstable.

$K < 12 \rightarrow$ stable.

Prob The open loop T.F. of a feedback control system is $G(S)H(S) = \dfrac{K(1+2S)}{S(1+S)(1+S+S^2)}$. Sketch complete Nyquist plot and hence find the range of K for stability using Nyquist stability criterion.

Ans. $G(j\omega)H(j\omega) = \dfrac{K(1+2j\omega)}{j\omega(1+j\omega)(1+j\omega-\omega^2)}$ $\quad M = \dfrac{K\sqrt{1+4\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{(1-\omega^2)^2+\omega^2}}$

$\phi = \tan^{-1}2\omega - 90^\circ - \tan^{-1}\omega - \tan^{-1}\dfrac{\omega}{1-\omega^2}$ $\qquad 0 < K < 0.865$

$\omega = 0 \rightarrow M = \infty$, $\phi = -90^\circ$, $\quad \omega = \infty$, $M = 0$, $\phi = -270^\circ$

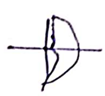
$G(j\omega)H(j\omega) = \dfrac{K[-3\omega^3 - j(1+2\omega^2-2\omega^4)]}{\omega(1+\omega^2)((1-\omega^2)^2+\omega^2)}$ $\quad Im = 0$, $\omega = 1.168$.
$M = 1.155K$.

Prob. $G(S)H(S) = \dfrac{2}{S(1-2S)}$ $\quad M = \dfrac{2}{\omega\sqrt{1+4\omega^2}}$ $\quad \phi = -90^\circ + \tan^{-1}(2\omega)$

$\omega = 0$ $\quad M = \infty$, $\phi = 90^\circ$, $\quad \omega = \infty$ $\quad M = 0$, $\phi = 0^\circ$

$G(j\omega)H(j\omega) = \dfrac{2(1+2j\omega)}{j\omega(1-2j\omega)} = \dfrac{2}{j\omega(1-2j\omega)} + \dfrac{4j\omega}{j\omega(1-2j\omega)}$

$\omega = 1$, $M = 0.89$, $\phi = -26.56^\circ$



$N = 0$, $P = 1$ $\quad Z = 1$. unstable.

by observation. By contrast a car's cruise control uses closed loop feedback, ... servomechanism.

Uses

Position control

A common type of servo provides position control. Servos are commonly electrical or partially electronic in nature, using an electric motor as the primary means of creating mechanical force. Other types of servos use hydraulics, pneumatics, or magnetic principles. Servos operate on the principle of negative feedback, where the control input is compared to the actual position of the mechanical system as measured by some sort of transducer at the output. Any difference between the actual and wanted values (an "error signal") is amplified (and converted) and used to drive the system in the direction necessary to reduce or eliminate the error. This procedure is one widely used application of control theory.

Speed control

Speed control via a governor is another type of servomechanism. The steam engine uses mechanical governors; another early application was to govern the speed of water wheels. Prior to World War II the constant speed propeller was developed to control engine speed for maneuvering aircraft. Fuel controls for gas turbine engines employ either hydromechanical or electronic governing.

Other

Positioning servomechanisms were first used in military fire-control and marine navigation equipment. Today servomechanisms are used in automatic machine tools, satellite-tracking antennas, remote control airplanes, automatic navigation systems on boats and planes, and antiaircraft-gun control systems. Other examples are fly-by-wire systems in aircraft which use servos to actuate the aircraft's control surfaces, and radio-controlled models which use RC servos for the same purpose. Many autofocus cameras also use a servomechanism to accurately move the lens, and thus adjust the focus. A modern hard disk drive has a magnetic servo system with sub-micrometre positioning accuracy. In industrial machines, servos are used to perform complex motion, in many applications.

This is the control synchro equivalent of the Torque Differential Transmitter previously described, and is used to add or subtract a mechanical shaft angle into the control chain.

Before describing how Resolvers differ from Synchros, it is worth mentioning that most synchros fall into one of three voltage and reference frequency categories, viz.

(1) 60 Hz reference frequency with a 115 volt r.m.s. reference and a 90 volt r.m.s. line to line signal voltage.

(2) 400 Hz reference frequency with a 115 volt r.m.s. reference and a 90 volt line to line signal voltage.

(3) 400 Hz reference frequency with 26 volts r.m.s. reference and an 11.8 volt r.m.s. line to line signal voltage.

Resolvers

The Resolver is a form of synchro (Resolvers are very often called Synchro Resolvers) in which the windings on the stator and rotor are displaced mechanically at 90° to each other instead of 120° as in the case of synchros. The Resolver therefore exploits the sinusoidal relationship betwen the shaft angle and the output voltage.

In outward appearance, Resolvers are very similar to Synchros and are produced in the standard Synchro frame diameters.

Internally, Resolvers come in many forms with a wide variety of winding configurations and transformation ratios. The simplest Resolver would have a rotor with a single winding and a stator with 2 windings at 90 degrees to each other. It would be represented as shown in Fig. 1-11.

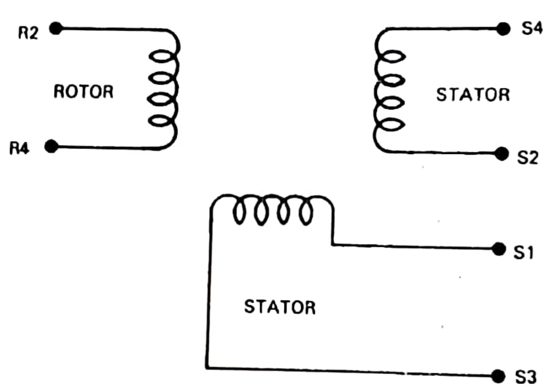


Fig. 1-11 Electrical Representation of a simple Resolver

If we assume that the rotor is excited by an AC reference voltage:

$$A \sin \omega t$$

Then the voltages appearing on the stator terminals will be:

$$S1 \text{ to } S3 = V\sin \omega t \sin \theta$$

$$\text{and} \quad S4 \text{ to } S2 = V\sin \omega t \cos \theta$$

where θ is the Resolver shaft angle.

These voltages are known as Resolver format voltages and will be referred to extensively during the rest of this book.

Such a Resolver could be used simply as an angular transducer in much the same way as a synchro transmitter.

A more complex Resolver would have two rotor windings at 90° to each other and two stator windings also at 90° to each other. This would be represented as shown in Fig. 1-12.

winding excited with the reference voltage, causing the Resolver Format signals to be present on the rotor winding terminals R1, R2, R3 and R4.

Resolvers used for angular measurement as described above are known as Data Transmission Resolvers, and a control chain can be set up using:

	a Resolver Transmitter	(Symbol TX)
and	a Resolver Control Transformer	(Symbol RC)

These control chains are analogous to the Synchro control chains described earlier. Such a control chain is shown in Fig. 1-13.

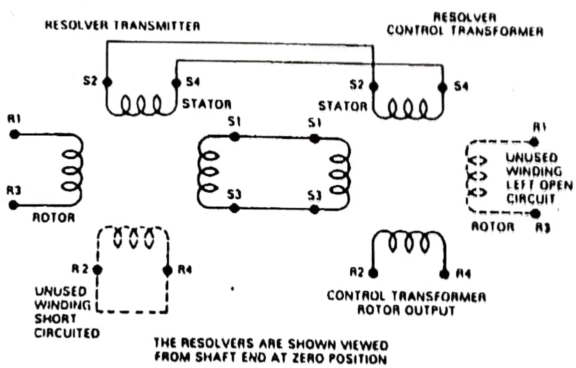


Fig. 1-13 A Resolver control chain

Resolvers are not available as Torque components.

Another application of Resolvers is in a computing mode, and when used for this purpose, they are known as Computing Resolvers. For example, a Computing Resolver has two stator windings and two rotor windings and can be used to perform a polar to rectangular or Cartesian Coordinate conversion.

eg. Assume that the polar coordinates of a point are represented by a voltage E Sin ωt and an angle θ. If θ is the angle applied to the Resolver shaft and E Sin ωt is applied to one of the stator windings as the reference voltage V_{S1-S3}, (the other stator winding being short circuited), then the Resolver Format voltages appearing on the rotor will be:

$$V_{R1-R3} = E \sin \omega t \sin \theta$$

$$\text{and} \quad V_{R4-R2} = E \sin \omega t \cos \theta$$

These voltages will represent the Cartesian or Rectangular coordinates of the point.

SUMAN GHOSH.

Compensation Technique

The alteration or adjustment of a control system in order to provide a suitable performence is called compensation i.e compensation is the adjustment of a system in order to make up for deficiencies or inadequacies.

An additional component or circuit called compensator is inserted into the system to compensate for the deficient performence.

Different types of compensator –

1. Lead Network or Lead compensator.
2. Lag Network or Lag compensator.
3. Lag-lead Network or Lag-lead compensator

* If the steady state output leads the input, the network is called Lead network.
* If the steady state output lags the input, the network is called lag network.
* Both phase lag and lead occur in the lag-lead network in different frequency regions. The phase lag occurs in the low frequency region and phase lead occur in high-frequency region.

Phase-Lead Compensator –

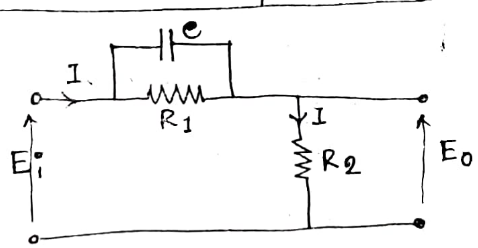


Fig. shows a phase-lead compensator. Here the sinusoidal o/p E_o leads the sinusoidal i/p E_i. The angle of lead is a function of frequency.

using KVL

$$E_i(s) = \left(\frac{1}{Cs} \parallel R_1\right) I(s) + R_2 I(s) \quad \ldots (1)$$

and $E_o(s) = R_2 I(s) \quad \ldots (2)$

From (1) ⇒

$$E_i(s) = \frac{R_1/Cs}{1/Cs + R_1} I(s) + R_2 I(s)$$

$$E_i(s) = \left[\frac{R_1}{Cs} + \frac{R_2}{Cs} + R_1R_2 \right] I(s) \cdot \frac{1}{1/Cs + R_1}$$

∴ Transfer function is given by.

$$\frac{E_0(s)}{E_i(s)} = \frac{R_2\left(\frac{1}{Cs} + R_1\right)}{\frac{R_1}{Cs} + \frac{R_2}{Cs} + R_1R_2}$$

$$= \frac{R_2 + R_1R_2Cs}{R_1 + R_2 + R_1R_2Cs}$$

$$= \frac{R_2(1 + R_1Cs)}{R_1 + R_2(1 + R_1Cs)}$$

$$= \frac{R_2}{R_1 + R_2} \cdot \frac{1 + R_1Cs}{\left(\frac{R_2}{R_1+R_2}\right)R_1Cs + 1}$$

let, $T = R_1C$.

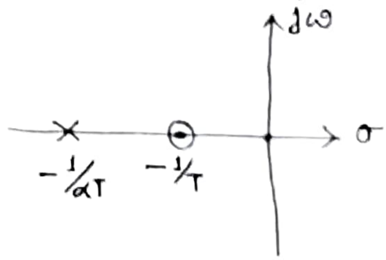
$$\alpha = \frac{R_2}{R_1 + R_2} < 1$$

$$\therefore \quad G(s) = \frac{E_0(s)}{E_i(s)} = \alpha\frac{1 + Ts}{1 + \alpha Ts}$$

$$\boxed{G(s) = \frac{s + 1/T}{s + 1/\alpha T}} \quad \cdots (3)$$

From eq (3) It is seen that lead network has a zero at $s = -1/T$ and a pole at $s = -1/\alpha T$

Since $0 < \alpha < T$, thus zero is always located to the right of the pole in the complex s-plane.



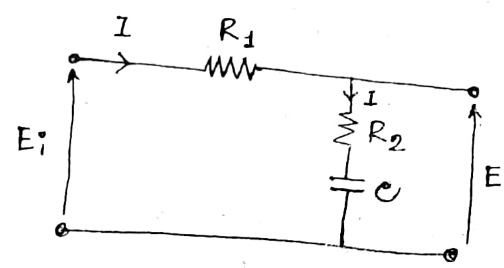
Since zero is nearer to the origin compared to pole, therefore the effect of zero is dominant. Hence the phase lead network, when introduced in the cascade with the forward path of a transfer function, the phase shift is increased. For this reason, it is called a phase-lead network.

The phase lead compensation network alters the frequency response of a network by adding a positive (leading) phase angle and, therefore increases the phase margin at the gain crossover frequency.

Effect of phase-lead compensation.

1. The damping of the closed-loop system is increased.
2. The overshoot is reduced and the transient response improved.
3. The bandwidth of the system is increased. This corresponds to faster time response.
4. The slope of the magnitude curve of the Bode plot is reduced at the gain crossover frequency. Therefore the relative stability is improved:
5. The steady state error of the system is not effected.
6. The phase margin of the closed loop system is increased.

Phase-lag Compensator



The o/p voltage E_o lags the i/p voltage E_i by an angle.

using KVL

$$E_i(S) = \left(R_1 + R_2 + \frac{1}{CS}\right) I(S) \quad \ldots (1)$$

$$E_o(S) = \left(R_2 + \frac{1}{CS}\right) I(S) \quad \ldots (2)$$

$$\therefore \quad \frac{E_o(S)}{E_i(S)} = \frac{R_2 + 1/CS}{R_1 + R_2 + 1/CS} = \frac{1 + R_2 CS}{1 + (R_1 + R_2)CS}$$

let, $R_2 C = T$

and, $\frac{R_1 + R_2}{R_2} = \beta > 1$

or, $\beta T = (R_1 + R_2) C$

$$\frac{E_o(s)}{E_i(s)} = \frac{1+sT}{1+\beta sT}$$

$$\text{or,}\quad \frac{E_o(s)}{E_i(s)} = \frac{1}{\beta}\left[\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right] \quad (1)$$

From eq (1) it is seen that a lag compensator has a zero at $s = -1/T$ and a pole at $s = -1/\beta T$.

Since $\beta > 1$, the pole is always located to the right of zero in the complex plane

Effect of phase lag compensation

1. Lag compensator has a high gain at low frequencies and low gain at higher frequencies. Therefore, a lag compensator acts as a low pass filter.
2. It attenuates high frequencies, so it acts as a high frequency noise filter.
3. Lag compensator increases the order of the system by one. Therefore the transient response becomes sluggish.
4. The phase lag characteristics of a lag compensator is of no use for compensation purposes.
5. Since the attenuation due to lag compensation decreases, the gain crossover frequency, the bandwidth of the system is reduced. The rise time, settling time are increased and the response becomes slower.
6. High gain at low frequencies improves steady-state performance and low gain at high frequencies improves phase margin.
7. For a given relative stability, the value of error constant is increased.

Lag-lead compensator

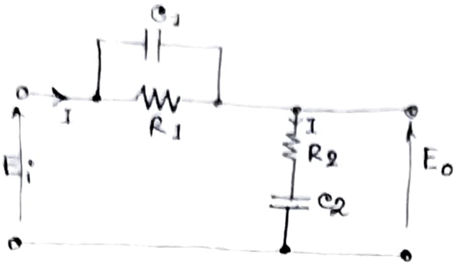


Fig. shows a single lag-lead compensator, which combines the characteristics of the lag and lead compensators. This is called a lag-lead network because the phase of the E_o compared to E_i varies from a lag to a lead angle as the reference frequency is increased from zero to infinity. For frequency from zero to ω_x, the output voltage lags the i/p voltage. For frequency above ω_x, the output voltage leads the input voltage.

$$E_i(s) = \left[\frac{R_1/C_1 s}{R_1 + 1/C_1 s} + R_2 + \frac{1}{C_2 s}\right] I(s)$$

$$= \left[\frac{R_1}{1+R_1C_1 s} + \frac{1+R_2C_2 s}{C_2 s}\right] I(s) \quad \ldots (1)$$

$$E_o(s) = \left[R_2 + \frac{1}{C_2 s}\right] I(s)$$

$$= \frac{1+R_2C_2 s}{C_2 s} I(s) \quad \ldots (2)$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{\dfrac{1+R_2C_2 s}{C_2 s}}{\dfrac{R_1}{1+R_1C_1 s} + \dfrac{1+R_2C_2 s}{C_2 s}}$$

$$= \frac{(1+R_1C_1 s)(1+R_2C_2 s)}{(1+R_1C_1 s)(1+R_2C_2 s) + R_1C_2 s}$$

Let, $R_1C_1 = T_1$, $R_2C_2 = T_2$

$$\frac{R_1+R_2}{R_2} = \beta$$

$$\text{or, } \frac{R_1}{R_2} + 1 = \beta$$

$$R_1C_2 = \frac{R_1}{R_2} T_2 = (\beta - 1) T_2$$

$$(1+R_1C_1 s)(1+R_2C_2 s) + R_1C_2 s$$

$$= 1 + R_2C_2 s + R_1C_1 s + R_1R_2C_1C_2 s^2 + R_1C_2 s$$

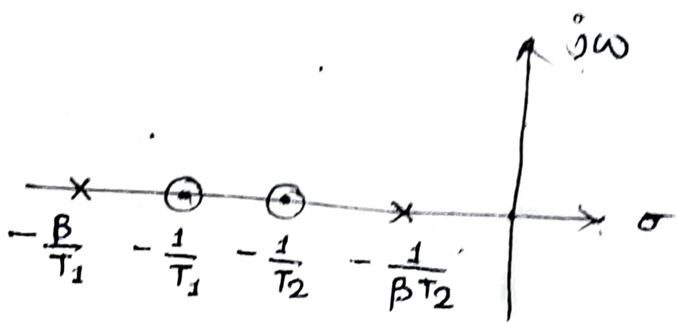
$$= 1 + T_1 s + T_2 s + T_1T_2 s^2 + (\beta - 1)T_2 s$$

$$= 1 + T_1 s + \cancel{T_2 s} + T_1T_2 s^2 + \beta T_2 s - \cancel{T_2 s}$$

$$= (1+\beta T_2 s) + \frac{T_1}{\beta} s (1+\beta T_2 s) = (1+\beta T_2 s)\left(1+\frac{T_1}{\beta} s\right)$$

$$\therefore \quad \frac{E_0(s)}{E_i(s)} = \frac{(1+T_1 s)(1+T_2 s)}{\left(1+\frac{T_1}{\beta} s\right)(1+\beta T_2 s)}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\beta/T_1\right)\left(s+1/\beta T_2\right)}$$



Since most control systems in practice are of orders higher than two, they can be approximated by second-order systems maintaining the same transient response. If the system dynamics of a higher order system can be accurately represented by a pair of complex conjugate dominant poles, then the damping ratio ζ is called the **relative damping ratio** of the system.

If the insignificant poles are neglected, the transient response is not changed, but the steady-state response is affected. The proper way of neglecting insignificant poles with the consideration of steady-state response is to write the transfer function in time constant form and then to neglect the insignificant poles so that the steady-state response is not affected.

6.21 METHODS TO IMPROVE TIME RESPONSE

It is necessary for a control system to meet certain specifications regarding its performance. This depends upon job the control system is expected to do. Generally system performance can be improved by using any of the following linear control methods :

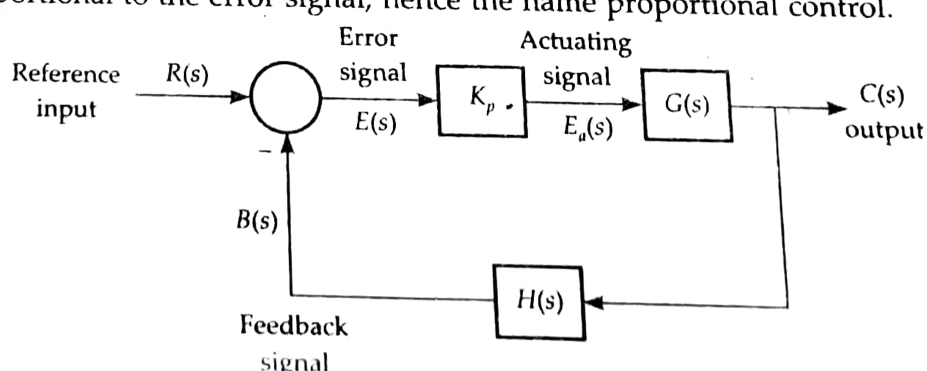
1. Proportional control.
2. Proportional Derivative (PD) control.
3. Proportional plus Integral (PI) control.
4. Proportional plus Integral plus Derivative (PID) control.

A controller is device which when introduced in feedback or forward path of a system, controls the steady-state and transient response according to the requirement.

6.22 PROPORTIONAL CONTROLLER

The proportional controller is a device that produces a control signal which is proportional to the input error signal $e(t)$. The error signal is the difference between the reference input signal and the feedback signal obtained from the output.

Figure 6.20 shows the block diagram of a proportional control system. The actuating signal is proportional to the error signal, hence the name proportional control.



The equation representing the proportional controller in time domain is

$$e_a(t) = K_p e(t) \qquad \text{...(6.22.1)}$$

Taking Laplace transform of Eq. (6.22.1), we get

$$E_a(s) = K_p E(s) \qquad \text{...(6.22.2)}$$

The transform function of the proportional controller is

$$K(s) = \frac{E(s)}{E_a(s)} = K_p \qquad \text{...(6.22.3)}$$

The proportional (P) controller amplifies the error signal by an amount K_p. Also, the introduction of P controller in the system increase the forward path gain by amount K_p. If the forward path gain is increased the peak overshoot increases while the steady-state error is reduced. In actual systems both peak overshoot and steady-state errors are desired to be small. Hence a compromised value of the forward path gain (K_p) is selected for which the peak overshoot and the steady-state errors are within specified values. In practice, these selected values, of forward path gain often cannot be used, because for these values the system become, unstable.

6.23 PROPORTIONAL PLUS DERIVATIVE (PD) CONTROL

In Proportional plus Derivative (PD) controllers, the actuating signal $e_a(t)$ is proportional to the error signal $e(t)$ and also proportional derivative of the error signal. Thus, the actuating signal for proportional plus derivative control is given by

$$e_a(t) = K_p e(t) + K_D \frac{d}{dt} e(t) \qquad \text{...(6.23.1)}$$

Taking the Laplace transform of both the sides of Eq. (6.23.1), we get

$$E_a(s) = K_p E(s) + K_D s E(s)$$

or

$$E_a(s) = (K_p + sK_D)\, E(s) \qquad \text{...(6.23.2)}$$

Fig. 6.21 shows the block diagram of a PD control for a second-order system.

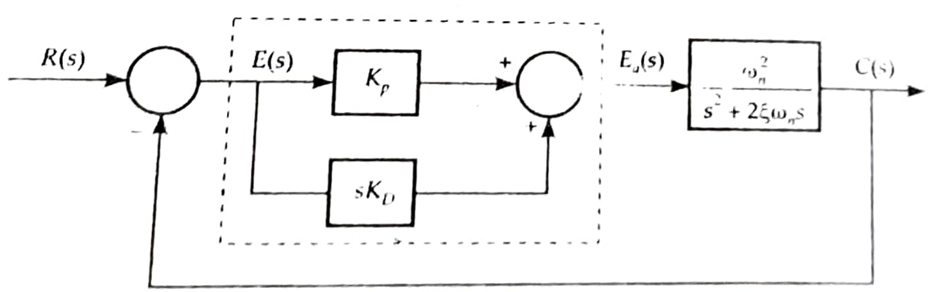


Fig. 6.21 Block diagram of a PD control system.

From Fig. 6.21, open-loop transfer function is

$$G(s) = \frac{C(s)}{E(s)}$$

$$= (K_p + sK_D) \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} \qquad \text{...(6.23.3)}$$

and $$H(s) = 1 \qquad \text{...(6.23.4)}$$

Therefore, closed-loop transfer function of the system is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{(K_p + sK_D)\omega_n^2}{s^2 + (2\zeta\omega_n + K_D\omega_n^2)s + \omega_n^2 K_p} \qquad \text{...(6.23.5)}$$

The characteristic equation of the system given by the denominator of Eq. (6.23.5) is

$$s^2 + (2\zeta\omega_n + K_D\omega_n^2)s + K_p\omega_n^2 = 0 \qquad \text{...(6.23.6)}$$

The standard equation of a second-order system is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad \text{...(6.23.7)}$$

We shall compare (Eq. 6.23.6) with standard Eq. (6.23.7).

Now, $$2\zeta\omega_n + K_D\omega_n^2 = 2\left(\zeta + \frac{1}{2}K_D\omega_n\right)\omega_n = 2\zeta'\omega_n$$

where $$\zeta' = \left(\zeta + \frac{1}{2}k_D\omega_n\right) \qquad \text{...(6.23.8)}$$

Equation (6.23.8) shows that effective damping has increased using PD control. This makes the system response slower with less overshoots increasing delay time. Proportional derivative control will not affect the steady-state error of the system.

6.24 PROPORTIONAL PLUS INTEGRAL (PI) CONTROL

In proportional plus Integral controllers, the actuating signal consists of proportional error signal added to the integral of the error signal. The actuating signal in time domain is given by

$$e_a(t) = K_p e(t) + K_I \int_0^t e(\tau)\, d\tau \qquad \text{...(6.24.1)}$$

Here the constants K_p and K_I are proportional and integral gains known as **controller parameters**.

By taking the Laplace transform of both sides of Eq. (6.24.1), we get

$$E_a(s) = K_p E(s) + K_I \frac{E(s)}{s}$$

or $$E_a(s) = \left(K_p + \frac{K_I}{s}\right)E(s) \qquad \text{...(6.24.2)}$$

Figure 6.22 shows the block diagram of a proportional plus integral (or PI) control system of a second-order system.

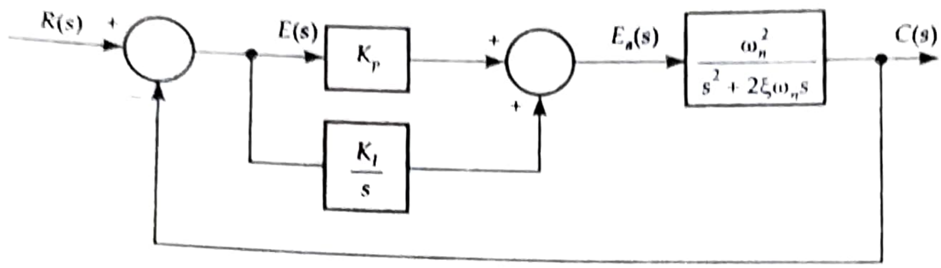


Fig. 6.22 Block diagram of a PI control system.

From Fig. 6.22, the open-loop transfer function is

$$G(s) = \frac{C(s)}{E(s)} = \frac{\omega_n^2\left(K_P + \frac{K_I}{s}\right)}{s^2 + 2\zeta\omega_n s}$$

or

$$G(s) = \frac{(K_P s + K_I)\omega_n^2}{s^2(s+2\zeta\omega_n)} \quad \text{...(6.24.3)}$$

and

$$H(s) = 1$$

Therefore, closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{(K_P s + K_I)\omega_n^2}{s^2(s+2\zeta\omega_n)}}{1+\frac{(K_P s + K_I)\omega_n^2}{s^2(s+2\zeta\omega_n)}}$$

or

$$\frac{C(s)}{R(s)} = \frac{(K_P s + K_I)\omega_n^2}{s^3 + 2\zeta\omega_n s^2 + K_P\omega_n^2 s + K_I\omega_n^2} \quad \text{...(6.24.4)}$$

The characteristic equation is

$$s^3 + 2\zeta\omega_n s^2 + K_P\omega_n^2 s + K_I\omega_n^2 = 0 \quad \text{...(6.24.5)}$$

Equation (6.24.5) is third-order equation.

Thus, a second-order system has been changed to a third-order system by adding an integral control in the system. Therefore, the effect of PI controller on the system performance is that it increases the order of the system by one, which results in the reduction of the steady-state error. The system relatively becomes less stable. Therefore, K_I should be designed properly to maintain stability of the system.

6.25 PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) CONTROL

A PID controller (Proportional plus Integral plus Derivative Controller) produces an output signal consisting of three terms – one proportional to error signal, another one proportional to integral of error signal and third one proportional to derivative of error signal.

The combination of proportional control action, integral control action and derivative control action is called PID control action. The combined action has the advantage of each of the three individual control actions.

The proportional controller stabilizes the gain but produces a steady-state error. The integral controller reduces or eliminates the steady-state error. The derivative controller reduces the rate of change of error. The main advantages of PID controllers are higher stability, no offset and reduced overshoot.

PID controllers are commonly used in process control industries.

The actuating signal or output signal from a PID controller in time domain is given by

$$e_a(t) = K_P e(t) + K_I \int_0^t e(\tau)\,d\tau + K_D \frac{de}{dt} \qquad \text{...(6.25.1)}$$

In the s-domain, the output signal from the controller is

$$E_a(s) = \left(K_P + \frac{K_I}{s} + K_D s \right) E(s) \qquad \text{...(6.25.2)}$$

Figure 6.23 shows a PID controller for second-order system. The closed-loop transfer function of a PLD controller for a second-order system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 (K_D s^2 + K_P s + K_I)}{[s^3 + (2\zeta\omega_n + K_D\omega_n^2)s^2 + K_P\omega_n^2 s + K_I\omega_n^2]} \qquad \text{...(6.25.3)}$$

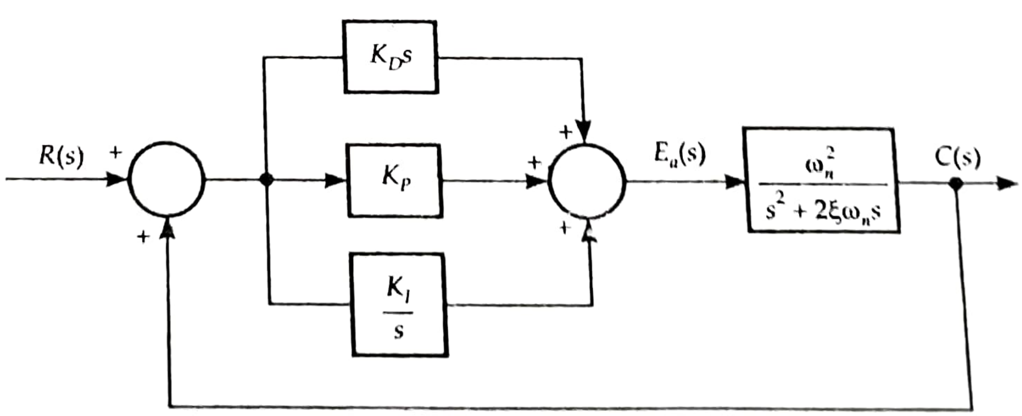


Fig. 6.23 Block diagram of a PID controller for second-order system

Example 6.1 *For the system with the following transfer function, determine the type and order of the system :*

(a) $G(s)H(s) = \dfrac{40(s+2)}{s(s+1)(s+4)}$ (b) $G(s)H(s) = \dfrac{K(s+2)}{s^2(s+4)(s+3)}$

(c) $G(s)H(s) = \dfrac{s+3}{(s-4)(s+0.2)}$ (d) $G(s)H(s) = \dfrac{30}{s^3(s^4+8s^2+16)}$

Solution.

(a) The open-loop transfer function has one pole at the origin of the *s*-plane. Therefore, it is a type-1 system. The highest power of s present in the denominator is 3, therefore, it is a third-order system.

(b) The open-loop transfer function has two poles at the origin of the *s*-plane, therefore, it is a type-2 system. The highest power of s present in the denominator is 4, so it is a fourth-order system.

(c) The open-loop transfer function has no pole at the origin of the *s*-plane, so it is a type-0 system. The highest power present in the denominator is 2, so it is a second-order system.

(d) The open-loop transfer function has three poles at the origin of the *s*-plane, therefore, it is a type-3 system. The highest power of s present in the denominator is 7, so it is a seventh-order system.

Example 6.2 *The closed-loop transfer functions of certain second-order unity feedback control systems are given below. Determine the type of damping in the systems :*

(a) $\dfrac{C(s)}{R(s)} = \dfrac{8}{s^2+3s+8}$ (b) $\dfrac{C(s)}{R(s)} = \dfrac{2}{s^2+4s+2}$

(c) $\dfrac{C(s)}{R(s)} = \dfrac{2}{s^2+2s+1}$ (d) $\dfrac{C(s)}{R(s)} = \dfrac{4}{s^2+16}$

Solution. The standard form of the transfer function of a second-order system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

We shall compare the given transfer functions with this standard form.

(a) $$\frac{C(s)}{R(s)} = \frac{8}{s^2+3s+8} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+8}$$

Therefore, $\omega_n^2 = 8$

Hence, $\omega_n = \sqrt{8} = 2.82$ rad/s

$2\zeta\omega_n = 3$

$\zeta = \dfrac{3}{2\omega_n} = \dfrac{3}{2\times 2.82} = 0.53.$ Since $\zeta < 1$, the given system is undamped.

The concept of analogy can be extended to thermal and hydraulic systems also. For the purpose of comparison, Table 1.11 illustrates the analogy between various physical systems.

Table 1.11 Analogous Quantities

Mechanical (Linear)	Mechanical (Rotational)	Thermal	Hydraulic	Electrical (Force-current)
Force	Torque	Heat flow	Flow	Current
Linear velocity	Angular velocity	Temperature	Pressure	Voltage
Spring	Spring	-	Inertia	Inductance
Mass	Inertia	Capacitance	Compression	Capacitance
Dash-pot	Dash-pot	Resistance	Resistance	Resistance

1.9 Mathematical Model of a Liquid Level System

Fluids can be divided into two categories. 1. Incompressible 2. Compressible. If the density of a fluid remains constant despite changes in pressure, then the fluid is known as incompressible. On the otherhand, if the density changes with pressure, the fluid is compressible. Practically fluids are compressible to some extent. As far as the level process is concerned, the liquid is assumed to be incompressible.

Let us consider a liquid level system as shown in Fig.1.41. Let A be the cross sectional area of the tank and let the level of the liquid in the tank be h. Then the total mass in the tank

$$m = \rho A h \tag{1.106}$$

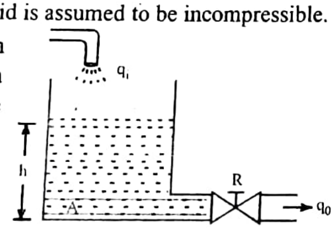


Fig. 1.41 A level process.

where ρ is the fluid density.

In a liquid level process h is the controlled variable. The inflow rate q_i is adjusted by a valve to control liquid level h. In a tank having a mass of fluid m, the rate of change of mass $\dot{m}$ is equal to the total mass inflow rate minus the total mass outflow rate.

$$\text{i.e.,} \quad \dot{m} = \rho q_i - \rho q_0 \tag{1.107}$$

Where q_i and q_0 are volume inflow rate and volume outflow rate respectively.

From Eq.(1.106), we have $\dot{m} = \frac{d}{dt}(\rho A h)$

$$\dot{m} = \rho A \frac{dh}{dt} \tag{1.108}$$

Substituting Eq.(1.108) in Eq.(1.107) we get

$$\rho A \frac{dh}{dt} = \rho q_i - \rho q_0$$

$$\Rightarrow \quad A \frac{dh}{dt} = q_i - q_0 \tag{1.109}$$

Let us assume that the outflow rate is proportional to the liquid level

$$\therefore \text{Outflow rate} \quad q_0 = \frac{h}{R} \tag{1.110}$$

where h is the height of the liquid level and R is the resistance to flow through outflow valve.

Substituting Eq. (1.110) in Eq. (1.109) we have

$$A \frac{dh}{dt} = q_i - \frac{h}{R}$$

$$RA \frac{dh}{dt} = Rq_i - h \tag{1.111}$$

where RA is the hydraulic capacitance of the system. Equation (1.111) is the mathematical model of the liquid level system.

Example 1.1:

Develop a mathematical model for the two coupled tank system shown in Fig. 1.42.

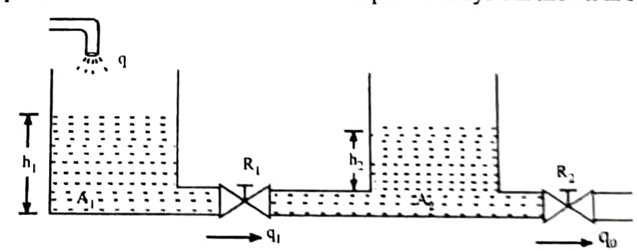


Fig. 1.42 Interacting level process.

Let h_1 and h_2 are liquid levels in the tank 1 and tank 2 respectively. Let us further assume that $h_1 > h_2$. The mass flow rate q_1 is flowing from tank 1 to tank 2. Total mass m_1 in the tank 1 = $\rho A_1 h_1$ and q is the mass inflow flow rate to the tank 1.

To consider T_x and T_y separately, we have

$$\text{when } T_y(s) = 0,\ G_1 = \frac{\phi(s)}{T_x(s)} = \frac{s^2 I_y + Bs}{s^2 I_x(s^2 I_y + Bs) + s^2 H_s^2} \tag{1.287}$$

Similarly when $T_x = 0$,

$$G_2 = \frac{\phi(s)}{T_y(s)} = \frac{-sH_s}{s^2 I_x(s^2 I_y + Bs) + s^2 H_s^2} \tag{1.288}$$

Similarly eliminating $\phi(s)$ from Eq. (1.283) and Eq. (1.284), we have

$$\text{when } T_y = 0,\ G_3 = \frac{\theta(s)}{T_x(s)} = \frac{sH_s}{s^2 I_x(s^2 I_y + Bs) + s^2 H_s^2} \tag{1.289}$$

Similarly when $T_x = 0$

$$G_4 = \frac{\theta(s)}{T_y(s)} = \frac{sI_x}{s^2 I_x(s^2 Iy + Bs) + s^2 H_s^2} \tag{1.290}$$

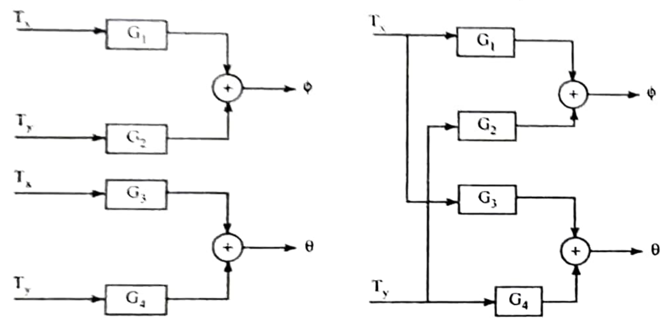


Fig. 1.304 Block diagram representation of Free Gyro.

In a practical situation there is no outer gimbal. The body of a ship or an aircraft acts as outer gimbal. Therefore the two axes (T_x and T_y) of gyros are replaced by a single axis gyros. The torque T_x is input, the angle ϕ is the displacement due to T_x. θ is the angle proportional to ϕ. There is no torque applied along T_y. Therefore we can neglect T_y. Considering (1.287) and (1.289) we have

$$\frac{\theta(s)}{T_x(s)} \cdot \frac{T_x(s)}{\phi(s)} = \frac{sH_s}{s^2 I_y + sB} = \frac{H_s}{sI_y + B}$$

$$\frac{\theta(s)}{\phi(s)} = \frac{H_s}{sI_y + B} \tag{1.291}$$

It is a first order system. In this case the input is $\phi(s)$(angular displacement of outer gimbal or body of ship or aircraft) and the output is $\theta(s)$. In otherwords $\theta(s)$ is measure of $\phi(s)$. To measure the position of an aircraft or a ship, we need three angular measurements with respect to x axis, y-axis and z-axis.

Three single axis gyros fitted as shown in Fig. 1.305 will give three angular displacements. We know that $\frac{d\phi}{dt} = s\phi(s)$, that is the angular velocity of vehicle or outer gimbal.

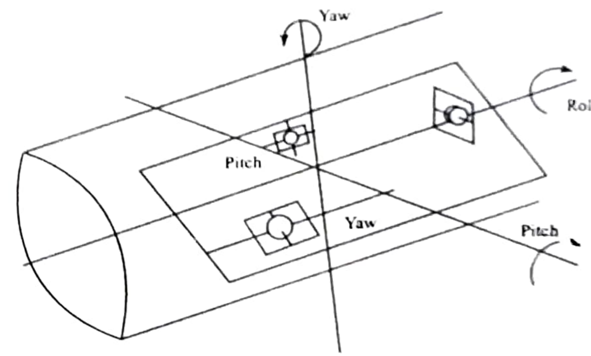


Fig. 1.305 Three Single Gyros

Dividing by s on both sides of Eq. (1.280), we get

$$\frac{\theta(s)}{s\phi(s)} = \frac{H_s}{s^2 I_y + sB} \tag{1.292}$$

In this case the output is proportional to $s\phi(s)$, that is **the angular velocity of the system. By integrating each of the three gyros, we can measure the three components of the angular velocity along x-axis (Roll); y-axis (Yaw) and z-axis (Pitch)**

1.18.2 *Synchros*

AU April / May 2010

The other names for synchros are selsyn and autosyn. It is an **electromagnetic** transducer that produces an output voltage depending upon the **angular displacement**. It consists of two devices called synchro transmitter and synchro **receiver**. It is mostly used as an error detector in control systems. The synchro **pair measures** and compares two angular displacements and produces an output voltage which is approximately linear with the angular difference.

Syncro transmitter is a basic system similar to a Y connected 3 phase alternator. The stator windings are concentric coils displaced 120°apart. The rotor is a salient pole type wound with concentric coils but excited with single phase ac through sliprings. A schematic diagram and assembly of a typical synchro transmitter are shown Fig. 1.306 and Fig. 1.307 respectively.

The synchro transmitter acts as a transformer with single primary windings (rotor) and three secondary windings displaced 120° apart from each other. The flux produced by the rotor is directed along its axis and distributed sinusoidally in the airgaps depending upon its angular position with the rotor. Therefore the flux linked with the stator winding will induce an emf proportional to the cosine of the angle between the rotor and stator windings. Let us assume that the ac voltage applied across the rotor is

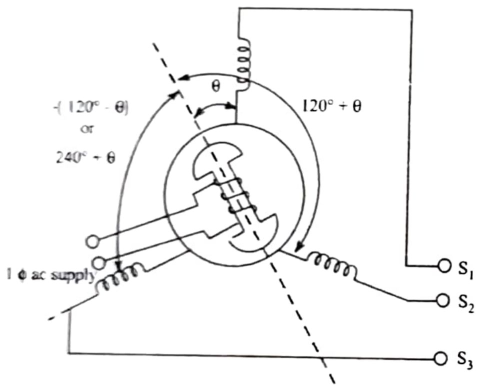


Fig. 1.306 Schematic diagram of synchro transmitter.

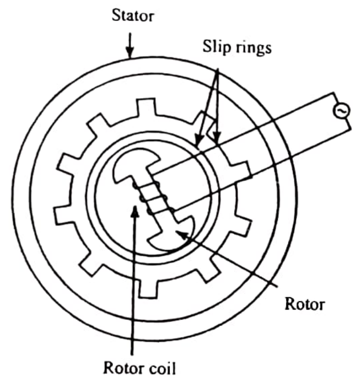


Fig. 1.307 Assembly of typical synchro transmitter.

$$v_r(t) = A \sin \omega t$$

Phase voltages induced in the stator coils s_1,$s_2$and s_3 are

$$V_{s1} = KA \sin \omega t \cos \theta \quad (1.293)$$
$$V_{s2} = KA \sin \omega t \cos(120^\circ + \theta) \quad (1.294)$$
$$V_{s3} = KA \sin \omega t \cos(240^\circ + \theta) \quad (1.295)$$

The corresponding line voltages are

$$\boxed{Note : \cos A - \cos B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}$$

$$\begin{aligned}
V_{L1} = V_{s1,s2} &= V_{s2} - V_{s1} \\
&= KA \sin \omega t(\cos(120^\circ + \theta) - \cos \theta) \\
&= KA \sin \omega t[2 \sin(60^\circ + \theta) \sin 60^\circ] \\
&= KA \sin \omega t[\sqrt{3} \sin(60^\circ + \theta)] && (1.296) \\
V_{L2} = V_{s2,s3} &= V_{s3} - V_{s2} \\
&= KA \sin \omega t[\cos(240^\circ + \theta) - \cos(120^\circ + \theta)] \\
&= 2KA \sin \omega t[\sin(180^\circ + \theta) \sin 60^\circ] \\
&= \sqrt{3}KA \sin \omega t \sin(180^\circ + \theta) && (1.297) \\
V_{L3} = V_{s3,s1} &= V_{s1} - V_{s3} \\
&= KA \sin \omega t[\cos \theta - \cos(240^\circ + \theta)] \\
&= -2KA \sin \omega t[\sin(120^\circ + \theta) \sin(120^\circ)] \\
&= \sqrt{3}KA \sin \omega t \sin(300^\circ + \theta) && (1.298)
\end{aligned}$$

When $\theta = 0$, V_{s1}has the maximum value of voltage $V_{s1} = KA \sin \omega t$. But when $\theta = 0$ the line voltage V_{L2} is zero.

The position at which V_{s1} has maximum value and $V_{L2} = 0$ is known as "electrical zero" or reference position of the transmitter. For an input of angular position of the rotor shaft, the synchro transmitter gives a set of three line voltages given by the Eq. (1.296), Eq. (1.297) and Eq. (1.298) respectively.

Two angular positions can be compared if a synchro control transformer is connected to the output of a synchro transmitter. The output of the synchro control transformer is the error signal which is proportional to the angular displacement between the two rotors of synchro control transformer and synchro transmitter. The control transformer is similar in construction to a syncho transmitter except the construction of the rotor. The rotor in synchro control transformer is cylindrical in shape so that the airgap is practically uniform. Stators of both synchro control transformer and synchro transmitter are identical and the output signal of transmitters is given as input to the stator of the control transformer. Therefore flux patterns are identical in both systems. A voltage will be induced in the rotor of control transformer by transformer action. This voltage is proportional to the cosine of the angle between the two rotors

$$\therefore \quad e(t) = K' A \sin \omega t \cos \varphi \quad (1.299)$$

where ϕ is the angular displacement between the two rotors. When $\varphi = 90^\circ$, the error voltage is equal to zero. This position is known as electrical zero or reference.

An assembly of a synchro error detector is shown in Fig 1.308.

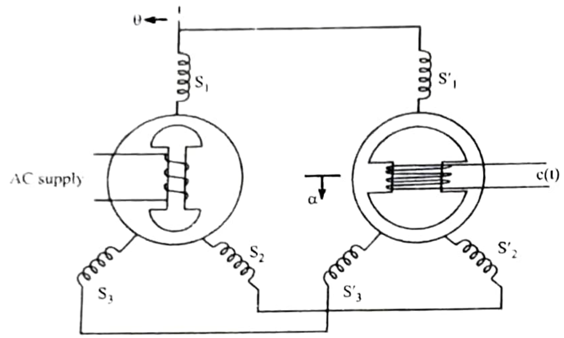


Fig. 1.308 Synchro Error Detector.

Let the initial position of rotors is 90° out of phase as shown in Fig. 1.308. Now, the error signal is $e(t) = K'A\sin\omega t\cos 90 = 0$. Let us assume that the transmitter rotor is displaced by an angle θ and control transformer rotor is displaced by an angle α. Therefore the net angle displacement between the rotor is $(90° + \theta - \alpha)$

$$\text{The error signal is } e(t) = K'A\sin\omega t\cos(90° + \theta - \alpha)$$

$$= K'A\sin\omega t\sin(\theta - \alpha) \qquad (1.300)$$

For a small angular displacement

$$e(t) = K'A(\theta - \alpha)\sin\omega t \qquad (1.301)$$

This shows that the synchro transmitter - control transformer pair thus acts as an error detector by giving an error signal proportional to the angular difference between the transmitter and control transformer shaft positions.

The plots of AC input to the transmitter, error and error signals are shown in Fig. 1.309.

It is evident from the plots that input to the transmitter is a carrier signal, the error $(\theta - \alpha)$ acts as a modulating signal. The error signal $e(t)$ is a modulated signal. This type of modulated error signals are called suppressed carrier modulated signals.

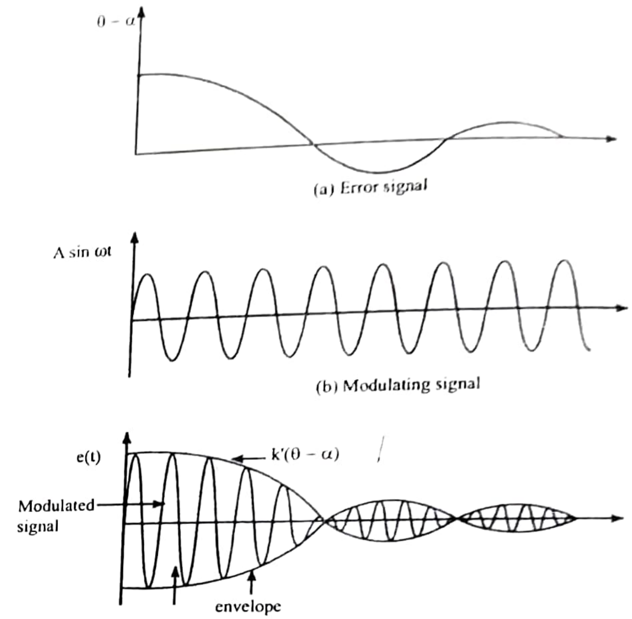


Fig. 1.309 Signals of Synchros

1.18.3 *Tachometer*

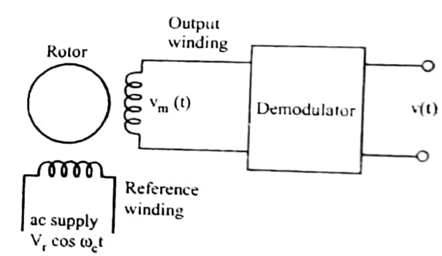


Fig. 1.310 Schematic diagram of AC Tachometer.

A tachometer is an electromechanical device that converts angular **velocity** into output voltage. Tachometers are of two types: ac tachometer and dc **tachometer**.

AC tachometer

The schematic diagram of an ac tachometer is shown in Fig. 1.310.

It consists of two stator windings at right angles to each other. A sinusoidal voltage $V_r \cos\omega_c t$ is applied to the reference winding. When the rotor shaft is stationary the voltage on the secondary windings is zero. When the rotor shaft rotates at θ radians / sec, the output voltage is proportional to the speed. When the output voltage is in phase with reference, the direction of rotation is said to be positive and when the output voltage is 180° out of phase the direction is said to be negative. Thus the output of the ac tachometer is in a modulated form and can be represented as

$$v_m(t) = r(t)\sin\omega_c(t)$$

where $r(t)$ is the varying magnitude of the modulated signal.

The envelope of $v_m(t)$ has information about the speed of rotor. Let $v(t)$ be proportional to speed. To obtain $v(t)$, the output of the ac tachometer is applied to a demodulator. Since the output of the tachometer is proportional to angular velocity, we can write

$$v(t) = K_t\frac{d\theta(t)}{dt} \tag{1.302}$$

where K_t is the tachometer constant in $v/rad/sec$.

The transfer function of an ac tachometer is of the form

$$\frac{V(s)}{s\theta(s)} = K_t \tag{1.303}$$

DC tachometer

A schematic diagram of a dc tachometer is shown in Fig. 1.311.

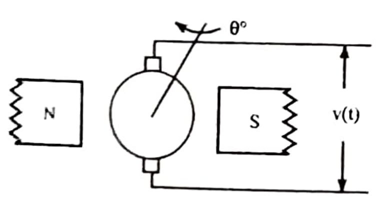


Fig. 1.311 Schematic diagram of DC Tachometer.

A dc tachometer comprises a fixed permanent magnet field, a rotating armature circuit, a commutator and a brush assembly. The rotor is connected to the shaft whose speed is to be measured. The output voltage of the tachometer is proportional to the angular velocity of the shaft. Thus

$$v(t) = K_t\frac{d\theta(t)}{dt} \tag{1.304}$$

where $v(t)$ = Output voltage of the tachometer

θ = Angular displacement

K_t = Tachometer constant $(volts/rad/sec)$

The transfer function of a dc tachometer is

$$\frac{V(s)}{s\theta(s)} = K_t \tag{1.305}$$

DC tachometers have many applications in control systems

1. It can be used as angular velocity indicators.
2. It can be used to provide velocity feedback.

1.18.4 DC Servomotors

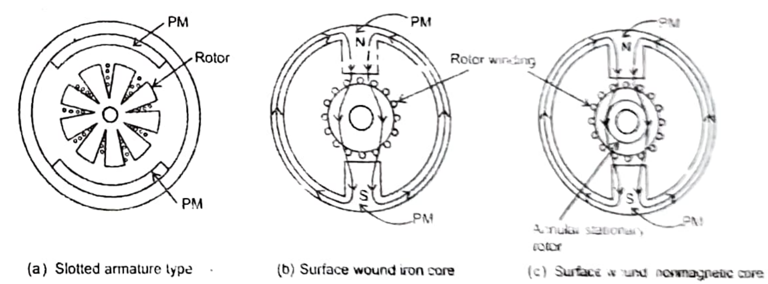


Fig. 1.312

From the control point of view, an armature voltage controlled dc servomotor is similar to a conventional dc motor with fixed excitation. We obtained mathematical model of an armature control dc motor in section 1.13. In this section let us see the constructional features of dc servomotors. Due to high residual flux density and coercivity, permanent magnets (PM) are used in the construction of dc servomotors. This property gives a higher torque-inertia ratio and high efficiency.

The speed of a PM dc motor is directly proportional to the armature voltage at a given load torque. A PM dc motor has much more flatter speed torque characteristics than a field wound motor which has severe armature reaction effects. There are three types of dc servomotors. They are slotted armature type, surface wound iron core type and surface wound nonmagnetic core type as shown in Fig 1.312. (a), (b) and (c) respectively.

Fig 1.312 (a) shows the slotted armature type. Armature windings are placed in the slots similar to a conventional dc motor. Slotted armature type has large inertia. To reduce this, surface wound iron core is used as shown in Fig 1.312 (b). The main draw back in the surface wound iron core is that it requires a strong PM for its construction. The inertia may be further reduced by having surface wound nonmagnetic core as shown in Fig 1.312 (c). This nonmagnetic core rotates on an annular stationary rotor.

1.18.5 *AC Servomotors*

JNTU April / May 2007 & 2009

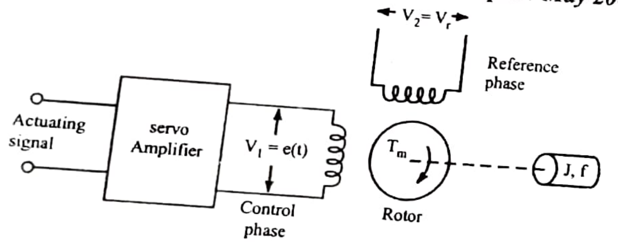


Fig. 1.313 Speed Torque characteristics.

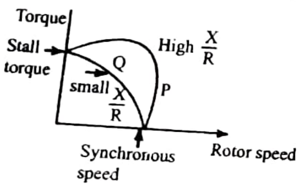


Fig. 1.314 Schematic diagram of an AC servo motor.

It is a two phase induction motor with little modifications. The schematic diagram of a two phase induction motor is shown in Fig. 1.313. In this case V_1 and V_2 are equal in magnitude but they are 90° out of phase. Therefore the two phase windings are producing a rotating magnetic field. The direction of rotation

depends upon the phase shift (lag or lead) between V_1 and V_2. The rotor is placed in the rotating magnetic field and so an emf is induced in it, causing a current in the short circuited rotor. Now, the rotating magnetic field interacts with the rotor current and produces a torque in the rotor in the direction of rotation of field. Normally the X/R ratio will be high in the case of an induction motor. But it exhibits highly nonlinear speed-torque characteristics. For a servomotor, linear torque-speed characteristic is required. So in the case of AC servomotor small X/R is considered. Speed-torque characteristics (High X/R and low X/R) are presented in Fig. 1.314.

The size of the rotor is inversely proportional to the acceleration of the motor. To obtain good accelerating characteristics, size of the rotor is considered small. The rotor construction is usually squirrel cage or drag cup type. In a servomotor, as shown in Fig. 1.313, both voltages are not equal in magnitude as we have seen in the induction motor. The reference phase voltage is kept constant and the control phase voltage is actuated by the actuating error signal. The torque of the motor is the function of the speed and error signal. That is $T_m = f(\dot{\theta}, e)$. The torque of the motor can be varied by varying the magnitude of the control phase voltage. Similarly, the direction of the rotation can be changed by changing the sign of the control phase voltage (either lead or lag).

Mathematical modelling of AC servomotor

The torque generated by the motor is a function of both the speed $\dot{\theta}$ and error signal e. That is $T_m = f(\dot{\theta}, e)$ where f is a function of the speed $\dot{\theta}(t)$ and error signal (control phase) $e(t)$. By using Taylor's series we have,

$$T_m = T_m(0) + \left.\frac{\partial T_m}{\partial e}\right|_{e=e(0)} (e(t) - e(0)) + \ldots + \left.\frac{\partial T_m}{\partial \dot{\theta}}\right|_{\dot{\theta}=\dot{\theta}(0)} (\dot{\theta}(t) - \dot{\theta}(0)) + \ldots$$

By neglecting higher order terms and considering only changes, we have

$$T_m = K(e(t) - e(0)) - f(\dot{\theta}(t) - \theta(0))$$

where

$$K = \left.\frac{\partial T_m}{\partial e}\right|_{e(t)=e(0)}$$

$$f = \left.\frac{-\partial T_m}{\partial \dot{\theta}}\right|_{\dot{\theta}=\dot{\theta}(0)}$$

The equation for changes(or with zero initial conditions) is

$$T_m = Ke(t) - f\dot{\theta}(t) \tag{1.306}$$

We know that the mechanical relations for a motor is

$$T_m = J\ddot{\theta} + B\dot{\theta} \tag{1.307}$$

From Eq. (1.306) and Eq. (1.307) we have

$$Ke(t) - f\dot{\theta}(t) = J\ddot{\theta}(t) + B\dot{\theta}(t)$$

Taking Laplace transform on both sides we have

$$(Js^2 + Bs)\theta(s) = KE(s) - fs\theta(s)$$

$$\frac{\theta(s)}{E(s)} = \frac{K}{Js^2 + (B+f)s} = \frac{K}{s(Js + B + f)} = \frac{Km}{s(T_m s + 1)}$$

where $Km = \frac{K}{B+f}$ and $T_m = \frac{J}{B+f}$ are gain and time constant of the servo motor respectively.

1.18.6 *Stepper motor*

Stepper motors can be classified in two ways. One is permanent magnet type and the other is variable reluctance type. Variable reluctance motors usually have three (or four) windings with a common terminal. On the other hand, permanent magnet motors usually have two independent windings. A stepper motor rotates in steps. It may vary from as large step size of 90° to as the smallest size of 0.72°. It is also possible to turn the stepper motor in half-steps as well as fractional steps with the help of controllers.

Variable Reluctance Stepper Motor

A typical variable reluctance stepper motor windings and its construction are shown in Fig. 1.315a and Fig. 1.315b respectively. This motor rotates steps of 30°. It consists of 6 wound stator poles. Poles are grouped into three. 1 and 4 are called p, poles 2 and 5 are called q. Similarly poles 3 and 6 are called r. One end of the three windings are connected to a common terminal e. The rotor in this motor has 4 teeth made up of ferromagnetic material. When the winding p is energised, the rotor teeth marked x are attracted by the poles (that is by 1 and 4). If the current through p is turned off and windings q is turned on then the rotor will rotate 30 degrees clockwise so that the poles marked y line up with the poles 2 and 5. This is because as soon as q is turned on and p is turned off, the teeth y nearest to poles 2 and 5 are attracted than the far away teeth (x). To rotate this motor continuously , we have to apply power to the three windings in sequence. In 12 steps of sequential turning on will rotate the rotor one complete revolution. Sequence of operation for one complete rotation is as follows

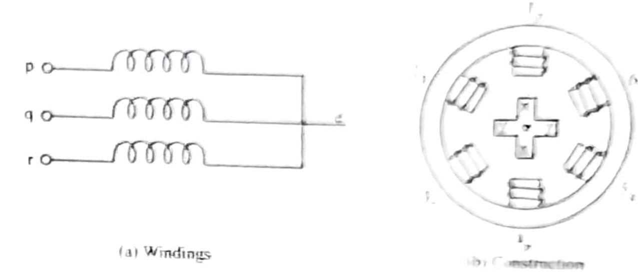


Fig. 1.315 Variable Reluctance Stepper Motor

Step No	1	2	3	4	5	6	7	8	9	10	11	12
Coil p	ON	OFF	OFF	ON	OFF	OFF	OFF	OFF	OFF	ON	OFF	OFF
Coil q	OFF	ON	OFF	OFF	ON	OFF	OFF	ON	OFF	OFF	ON	OFF
Coil r	OFF	OFF	ON	OFF	OFF	ON	OFF	OFF	ON	OFF	OFF	ON

Rotation of stator flux appears anticlockwise for a clockwise rotation of rotor.

More number of motor poles and more number of rotor teeths allow construction of motors with smaller step angles.

Unipolar stepper motors

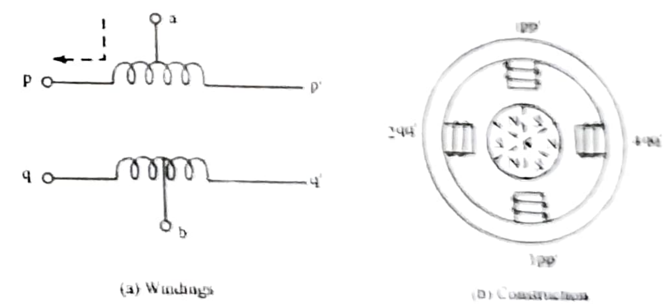


Fig. 1.316 Unipolar Stepper Motor.

In a typical unipolar stepping motor as shown in Fig. 1.316, the stator winding has 6 terminals. The winding pp' has a center tap a and the other winding qq' has a center tap b. The center taps a and b are permanently connected to the positive terminal of the dc source. If p is connected to the negative terminal of the dc source, then pole 1 becomes north pole and pole 3 becomes south pole. Alternatively if p' is connected to the negative terminal of the dc source, then pole 1 becomes south pole and pole 3 becomes north pole. Let us assume that the current is flowing from a to p (p is grounded). Then pole 1 becomes north and pole 3 becomes south. One of the south poles of the rotor is attracted by stator pole 1. Similarly, the opposite side of this rotor (north pole) is attracted by stator pole 3. As a next step, bq is energised (b is connected to the positive terminal and q is connected to the negative terminal) and ap is deenergised. Then stator pole 2 becomes north and 4 becomes south pole. Stator pole 1 and 3 are demagnetised. The nearest south pole is attracted by stator pole 2 (on the other hand nearest north pole is attracted by the stator pole 4). So, the rotor is turned by 30° clockwise. Similarly the following sequence of energisation is followed continuously to rotate the rotor.

Step	1	2	3	4	5	6	7	8	9	10	11	12
Winding ap	ON	OFF	OFF	OFF	ON	OFF	OFF	OFF	ON	OFF	OFF	OFF
Winding bq	OFF	ON	OFF	OFF	OFF	ON	OFF	OFF	OFF	ON	OFF	OFF
Winding ap'	OFF	OFF	ON	OFF	OFF	OFF	ON	OFF	OFF	OFF	ON	OFF
Winding bq'	OFF	OFF	OFF	ON	OFF	OFF	OFF	ON	OFF	OFF	OFF	ON

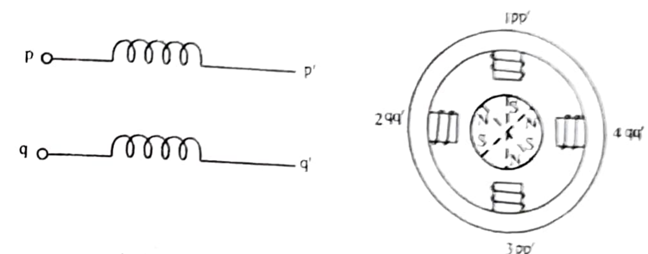
The above mentioned sequence will rotate the rotor once in steps of 30°. In the above sequence two halves of each windings are never energised at the same time. An alternate method can be used to get more power. In that case two half coils will be energised at a time but rotate at the same angle of 30° per step. The following sequence gives 1.4 times higher power for the same angle of rotation.

Step	1	2	3	4	5	6	7	8	9	10	11	12
Winding ap	ON	ON	OFF	OFF	ON	ON	OFF	OFF	ON	ON	OFF	OFF
Winding bq	OFF	ON	ON	OFF	OFF	ON	ON	OFF	OFF	ON	ON	OFF
Winding ap'	OFF	OFF	ON	ON	OFF	OFF	ON	ON	OFF	OFF	ON	ON
Winding bq'	ON	OFF	OFF	ON	ON	OFF	OFF	ON	ON	OFF	OFF	ON

In the above sequence at step one, ap and bq' are energized. Let us assume that stator poles 1 and 2 are north and south poles respectively. So, one of the south poles of the rotor is locked with stator pole 1. A point mid way between two rotor poles is aligned with stator pole 2 (alternatively a north pole may be locked with

stator pole 2 and a midway between two rotor poles aligned with stator pole 1 is also possible). Then as step two, bq' is deenergised and bq is energised by keeping ap energised turn the rotor in the anticlockwise direction by 30°. The two coils energised at a time gives 1.4 times more power than previous one.

Bipolar Stepper Motors



(a) Windings

(b) Construction

Fig. 1.317 Bipolar Stepper Motor.

Bipolar permanent magnet stepper motors are constructed with exactly the same mechanism we saw in the unipolar motors; but the two windings do not have any center taps. So it requires a circuit to reverse the polarity of the coils. Suppose the direction of current is from pp'; then let us assume that the polarity of stator poles 1 and 3 are north and south respectively. If the direction of the coils are reversed as $p'p$, then the polarity of the stator poles 1 and 3 becomes south and north respectively.

Similarly if the direction of current is from q to q' then the polarity of stator poles 2 and 4 becomes north and south respectively. On the other hand, if the direction is q' to q then the polarity of stators poles 2 and 4 becomes south and north respectively. The operation of a bipolar motor is energisation of only one coil at a time as mentioned below:

If the coil pp' is energised by making p as positive, then stator pole 1 will attract south pole of the permanent magnet (6 poles) rotor. As a next step, if we energise the coil qq' by making q as positive, then the stator pole 2 will become north and in turn it will attract the nearest south pole. Therefore the rotor will turn by 30° in clockwise direction. The following sequence will rotate the rotor in steps of 30°. The following table shows which terminal is positive when energised.

Step No	1	2	3	4	5	6	7	8	9	10	11	12
Coil p	+	-	-	-	+	-	-	-	+	-	-	-
Coil q	-	+	-	-	-	+	-	-	-	+	-	-
Coil p'	-	-	+	-	-	-	+	-	-	-	+	-
Coil q'	-	-	-	+	-	-	-	+	-	-	-	+

The above 12 steps of energisation will turn the rotor one complete rotation. Other variations in permanent magnet stepper motors are bifilar motors and multi-phase motors. Bifilar motors are a combination of unipolar and bipolar windings. It can be operated either as unipolar mode or bipolar mode. In the multiphase motor, all the windings of the motor are in a cyclic series. Only one terminal changes its polarity in one step. This method is less common in practice.

1.19 Potentiometers

AU Nov / Dec 2009

Potentiometer is a simple and reliable device to measure mechanical displacement. It is a voltage dividing resistor with two fixed terminals A & B and one movable terminal M attached to a movable arm as shown in Fig 1.318. The total resistance R_{AB} of the potentiometer is spread out uniformly and linearly between the points A and B. If a fixed voltage is applied between AB, then the output voltage across BM is proportional to the displacement of movable arm M. The main disadvantage in this arrangement is that the output will be affected by the load resistance connected across BM.

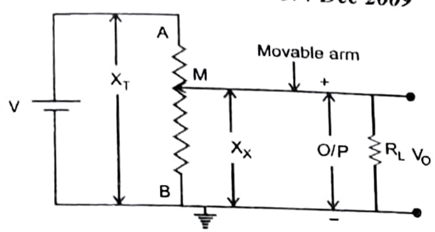


Fig. 1.318 Linear Potentiometer

For better results, the load resistance across BM should be very high.

Let R_{AB} be the total potentiometer resistance, X_T be the distance between A and B and X_X be the distance between B and M.

Let the load resistance be R_L, V be the constant voltage across A and B, V_O be the output voltage, and R_M be the resistance between B and M.

We have

$$\therefore \quad R_M = R_{AB}\left(\frac{X_M}{X_T}\right)$$

$$R_{eq} = R_M \| R_L = \frac{R_M\, R_L}{R_M + R_L}$$

$$= \frac{\left(\frac{X_M}{X_T}\right) R_{AB}\, R_L}{\left(\frac{X_M}{X_T}\right) R_{AB} + R_L} = \frac{X_M R_{AB} R_L}{X_M R_{AB} + R_L X_T}$$

$$\therefore \quad V_O = V\left(\frac{R_{eq}}{R_{eq} + (R_{AB} - R_M)}\right) \quad \text{where} \quad R_{eq} = \frac{X_M R_{AB} R_L}{X_M R_{AB} + R_L X_T}$$

Note that the potentiometer can also be used to measure angular measurements by modifying the construction as shown in the Fig 1.319. For circular displacement the output V_O is proportional to the angle θ.

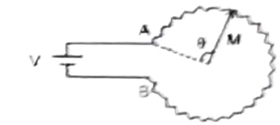


Fig. 1.319 Angular Potentiometer

1.20 Encoders

Encoders are used in control systems to convert linear or rotational displacement into digital code or pulse signals. Encoders are classified into two types. They are (i) Incremental encoders, and (ii) Absolute encoders.

1.20.1 *Incremental encoders*

It consists of a light source (LED), a rotating disk, a reference disk and a sensor (LDR) as shown in Fig 1.320. Let the disk have holes and opaques. Whenever the holes in both rotating and reference disk coincide, then the sensor receives the light from the source. The resolution of the disk of the encoder depends upon the number of holes in the disk.

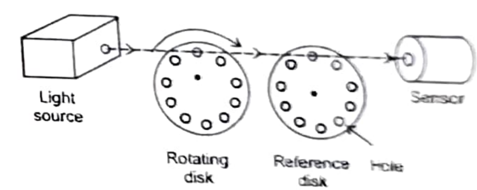


Fig. 1.320 Incremental encoder

$$\text{Basic resolution of the encoder} = \frac{360^\circ}{N}$$

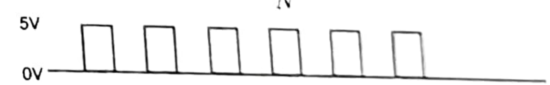


Fig. 1.321 Output of encoder

where N is the number of holes in the disk. If the displacement of the rotating disk is continuous, then we will get a continuous pulse as shown in Fig 1.321.

1.20.2 *Absolute encoders*

Absolute encoder generates a particular code for each position. For example,let the rotating disk have 16 positions; then it can generate 16 distinct codes for each position. For this, the rotating disk has many tracks as the number of bits in the code as shown in Fig 1.322. In the present case it has 4 tracks, and 16 positions. This can be achieved by a 4 bit number. To set a 4 bit code,4 light sources and 4 sensors are required. By increasing number of bits, the resolution may be improved.

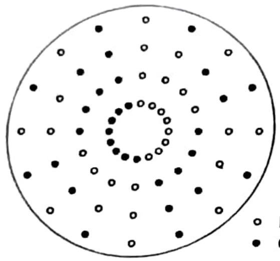


Fig. 1.322 Rotating disk in absolute encoder

Example 1.37:

Find the transfer function relating displacement 'y' and 'x' for the following system shown in Fig.1.323. ***JNTU April / May 2007***

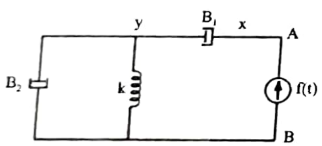


Fig. 1.323

Solution

Let M_1 and M_2 are two negligible masses having displacement x and y respectively. ∴ $M_1 = M_2 = 0$. Free body diagram of negligible mass M_1 and M_2 are as follows:

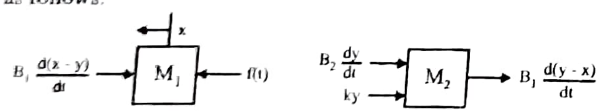


Fig. 1.324

The mathematical model for this mass M_1 & M_2 are

$$M_1\frac{d^2x}{dt^2} + B_1\frac{d(x-y)}{dt} = f(t) \tag{1.308}$$

$$M_2\frac{d^2y}{dt^2} + ky + B_2\frac{dy}{dt} + B_1\frac{d(y-x)}{dt} = 0 \tag{1.309}$$

But $M_1 = M_2 = 0$ and taking Laplace transform by assuming zero initial conditions, we have

$$B_1 s[X(s) - Y(s)] = F(s) \tag{1.310}$$

$$kY(s) + B_2 sY(s) + B_1 s[Y(s) - X(s)] = 0 \tag{1.311}$$

$$[(B_1 + B_2)s + k]Y(s) = B_1 sX(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{B_1 s}{(B_1 + B_2)s + k} \tag{1.312}$$

Substituting Eq (1.310) in Eq (1.311) we have,

$$kY(s) + B_2 sY(s) - F(s) = 0 \Rightarrow \frac{Y(s)}{F(s)} = \frac{1}{B_2 s + k}$$

$$\text{Similarly} \quad \frac{X(s)}{F(s)} = \frac{(B_1 + B_2)s + k}{(B_2 s + k)B_1 s}$$

Example 1.38:

Determine the transfer function $\frac{C(s)}{R(s)}$ for the following block diagram. ***JNTU April / May 2007***

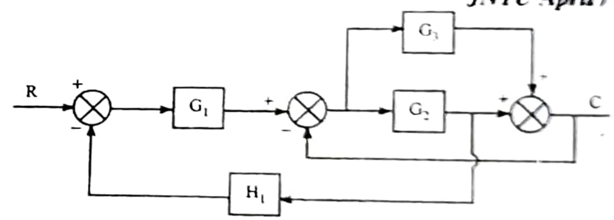


Fig. 1.325

Solution

$$\frac{C(s)}{R(s)} = \frac{\frac{G_1(G_2+G_3)}{1+G_2+G_3}}{1 + \frac{H_1 G_2}{G_2+G_3} \cdot \frac{G_1(G_2+G_3)}{1+G_2+G_3}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1(G_2 + G_3)}{1 + G_2 + G_3 + H_1 G_1 G_2}$$